VAR network models to measure contagion between Bitcoin market exchanges

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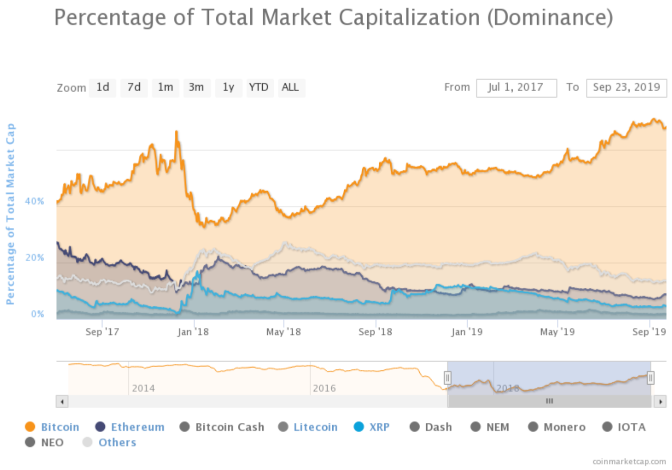
Figure: Bitcoin historical price series (USD)


Percentage of Total Market Capitalization (Dominance)
Zoom 1d 7d 1m 3m 1y YTD ALL
From Jul 1, 2017 To Sep 23, 2019
Percentage of Total Market Cap
40%
20%
0%
Sep '17
Jan '18
May '18
Sep '18
Jan '19
May '19
Sep '19
2014
2016
2018
Bitcoin
Ethereum
Bitcoin Cash
Litecoin
XRP
Dash
NEM
Monero
IOTA
NEO
Others
coinmarketcap.com

Main research questions:

- How much are cryptocurrencies interconnected? And which are the ones showing high/low degree of interconnectedness among each other?
- How do shocks in cryptocurrency returns propagate to the others in the short and medium/long term?
- Which are the leading cryptocurrencies in the price discovery process and which are the followers?

Literature:

- Connectedness measures:
  - Billio et al. (2012), Diebold & Yilmaz (2012, 2014), Barunik & Krehlik (2018)
- Price discovery and connectedness of Bitcoin exchanges:
  - Brandvold et al. (2015), Corbet et al. (2017), Pagnottoni & Dimpfl (2018), Giudici & Pagnottoni (2019)
- interconnectedness, spillovers and shock transmissions in the cryptocurrency market:
  - Fry & Cheah (2016), Yi et al. (2018), Koutmos (2018), Ji et al. (2019), Zieba et al. (2019), Antonakakis et al. (2019)

Vector AutoRegression (VAR)

$$x_t = \sum_{i=1}^{k} \Phi_i x_{t-i} + \varepsilon_t$$

- x_t : cryptocurrency returns at time t
- k : autoregressive order
- Φ_i : $(n \times n)$ VAR parameter matrices
- ε_t : zero-mean white noise process, with variance-covariance matrix Σ_ε

Vector Moving Average (VMA)

$$x_t = \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \ldots$$

- Ψ_i :$(n \times n)$ VMA parameter matrices

- Barunik and Krehlik (2018) propose a technique to estimate unconditional connectedness relationships in time frequency domain
- This methodology allow us to evaluate connectedness at short and long frequency
- We consider the frequency response function $\Psi\left(\mathrm{e}^{-\mathrm{i}\omega}\right)=\sum_h \mathrm{e}^{-\mathrm{i}\omega h}\Psi_h$ which can be retrieved as a Fourier transform of the coefficients Ψ_h, with $i=\sqrt{-1}$.
- The spectral density of x_t at frequency ω can be defined as a Fourier transform of MA(∞) filtered series as:

$$S_{\mathbf{x}}(\omega)=\sum_{h=-\infty}^{\infty} E\left(\mathbf{x}_t\mathbf{x}'_{t-h}\right)\mathrm{e}^{-\mathrm{i}\omega h}=\Psi\left(\mathrm{e}^{-\mathrm{i}\omega}\right)\Sigma\Psi'\left(\mathrm{e}^{+\mathrm{i}\omega}\right) \quad (1)$$

- The unconditional generalised forecast error variance decomposition (GFEVD) on a particular frequency ω is specified as:

$$(\Theta(\omega))_{i,j} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{\infty} \left(\Psi\left(e^{-ih\omega}\right) \Sigma\right)_{i,j}^{2}}{\sum_{h=0}^{\infty} \left(\Psi\left(e^{-ih\omega}\right) \Sigma \Psi\left(e^{ih\omega}\right)\right)_{i,i}} \qquad (2)$$

  - σ_{jj} : standard deviation of the innovation for equation j
  - e_i : selection vector with one as element i and zeros elsewhere
- The equation from above can be standardized as:

$$(\tilde{\Theta}(\omega))_{i,j} = (\Theta(\omega))_{i,j} / \sum_{j=1}^{k} (\Theta(\omega))_{i,j} \qquad (3)$$

- The accumulative connectedness table over an arbitrary frequency band $d = (a; b)$ can be expressed as:

$$\left(\tilde{\Theta}_d\right)_{i,j} = \int_a^b (\tilde{\Theta}(\omega))_{i,j} d\omega \qquad (4)$$

- From the use of the total contributions to the forecast error variance decomposition we estimate the overall within connectedness within the frequency band d as:

$$C^d = \frac{\sum_{i=1, i \neq j}^{k} \left(\tilde{\Theta}_d\right)_{i,j}}{\sum_{i,j} \left(\tilde{\Theta}_d\right)_{i,j}} = 1 - \frac{\sum_{i=1}^{k} \left(\tilde{\Theta}_d\right)_{i,i}}{\sum_{i,j} \left(\tilde{\Theta}_d\right)_{i,j}} \tag{5}$$

- Moreover, we obtain the within "from", "to" and "net" connectedness within the frequency band d respectively as:

$$C^d_{i\leftarrow} = \sum_{j=1, i\neq j}^{k} \left(\tilde{\Theta}_d\right)_{i,j} \tag{6}$$

$$C^d_{i\rightarrow} = \sum_{j=1, i\neq j}^{k} \left(\tilde{\Theta}_d\right)_{j,i} \tag{7}$$

$$C^d_{i,net} = C^d_{i\rightarrow} - C^d_{i\leftarrow} \tag{8}$$

- Finally, the pairwise connectedness between market i and j can be specified as:

$$C_{i,j}^{d} = \left(\tilde{\Theta}_d\right)_{j,i} - \left(\tilde{\Theta}_d\right)_{i,j} \qquad (9)$$

- The measures from above are estimated for positive, negative and full sample time series of returns, defined as:

$$\begin{aligned} R(+) &= \begin{cases} R_t, & \text{if } R_t > 0 \\ 0, & \text{otherwise} \end{cases} \\ R(-) &= \begin{cases} R_t, & \text{if } R_t < 0 \\ 0, & \text{otherwise} \end{cases} \\ R_t &= R(+) + R(-) \end{aligned} \qquad (10)$$

- (Spectral) variance decompositions can be seen as weighted, directed networks:

	x_1	x_2	$\dots$	x_N	From Others
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\Sigma_{j=1}^N d_{1j}^H, j \neq 1$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\Sigma_{j=1}^N d_{2j}^H, j \neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\Sigma_{j=1}^N d_{Nj}^H, j \neq N$
To Others	$\Sigma_{i=1}^N d_{i1}^H$ $i \neq 1$	$\Sigma_{i=1}^N d_{i2}^H$ $i \neq 2$	$\cdots$	$\Sigma_{i=1}^N d_{iN}^H$ $i \neq N$	$\frac{1}{N}\Sigma_{i,j=1}^N d_{ij}^H$ $i \neq j$

- 4 intraday price series (USD) belonging to selected cryptocurrencies (listed on Kraken exchange):
  - Bitcoin
  - Ethereum
  - Litecoin
  - Ripple
- Time period analyzed: 1 July 2017 - 23 September 2019
- sampling interval: hourly
- number of observations: 19,536

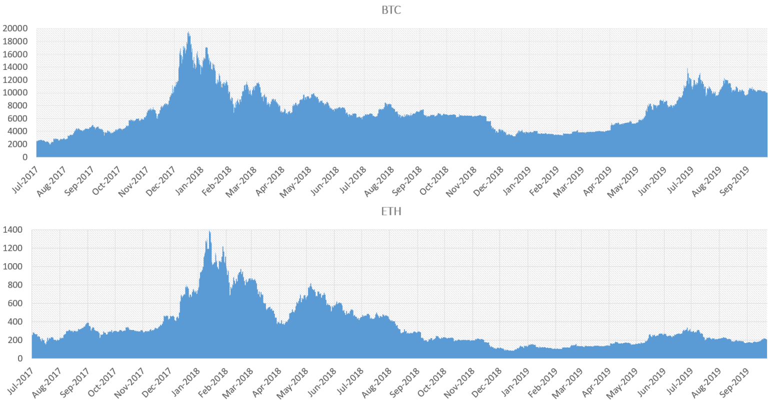
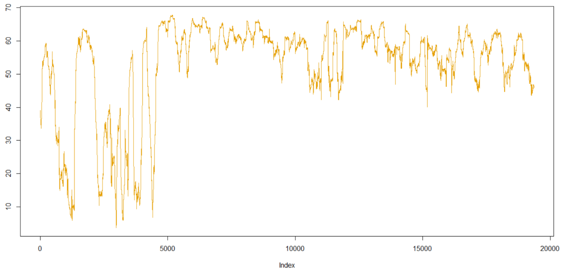


Figure: Bitcoin and Ethereum price series

- Preliminary tests
  - ADF Test for (non-)stationarity
- Rolling window estimation: 2 weeks (1 and 3 weeks for sensitivity)
- Forecast horizon: 12 hours (6 and 18 hours for sensitivity), following Diebold and Yilmaz (2012)
- VAR lags: 2 for the full sample, according to BIC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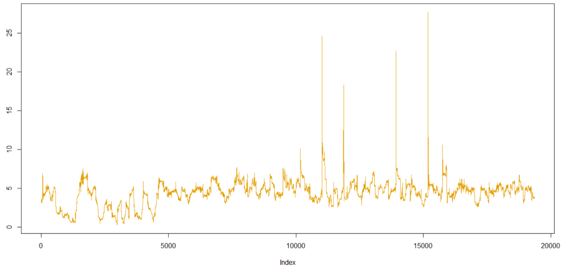
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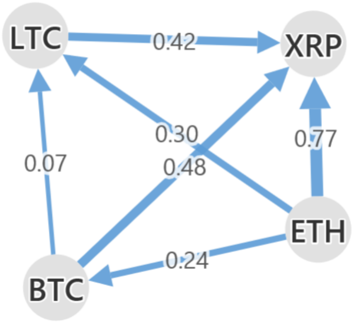

Figure: Full sample spillovers 1-12 hrs

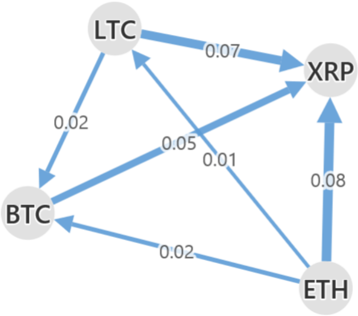

Figure: Full sample spillovers over 12 hrs

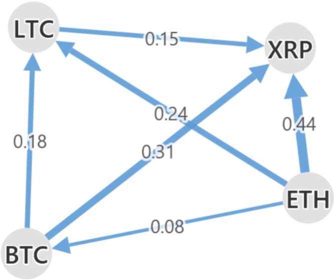

Figure: (+) return spillovers 1-12 hrs

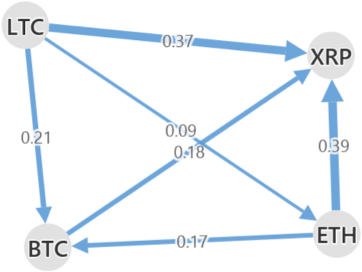

Figure: (+) return spillovers over 12 hrs

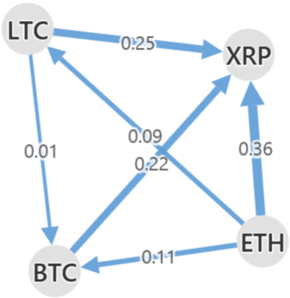

Figure: (-) return spillovers 1-12 hrs

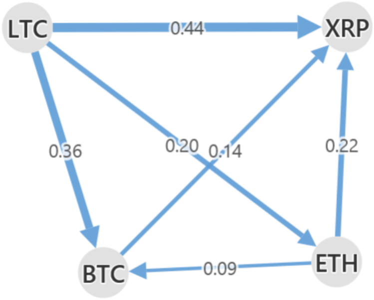

Figure: (-) return spillovers over 12 hrs

To conclude:

- We employ the Barunik & Krehlik (2018) spectral variance decomposition approach to derive weighted, directed econometric networks describing major relationships among cryptocurrency returns
- The methodology is able to distinguish between high and low frequency band effects
- During bull times, crypto interconnectedness is generally asymmetric, whereas during bear times, interconnectedness stays generally steady
- Ethereum is identified as the biggest spillover transmitter, with Bitcoin maintaining its relative importance, while Ripple as receiver
- Further analysis is on-going