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**On the Elasticity of Effort for Piece
Rates: Evidence from the British
Columbia Tree-Planting Industry**

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On the Elasticity of Effort for Piece Rates: Evidence from the British Columbia Tree-Planting Industry*

Harry J. Paarsch[†], Bruce S. Shearer[‡]

Résumé / Abstract

Pour que les prévisions politiques à l'égard des systèmes de compensation soient utiles, il faut qu'elles soient basées sur des analyses empiriques des effets incitatifs, i.e. l'élasticité de l'effort du travailleur par rapport aux changements dans le système de compensation. Nous mesurons l'élasticité de l'effort du travailleur par rapport aux changements dans la rémunération à la pièce en utilisant des données longitudinales que nous avons colligées à partir des archives d'une compagnie qui s'occupe de plantation d'arbres en Colombie-Britannique. Nos données contiennent de l'information sur la productivité quotidienne des travailleurs ainsi que sur le taux de rémunération à la pièce pendant une période de 5 mois. Nous nous intéressons plus particulièrement aux problèmes d'endogénéité inhérents à l'analyse empirique traditionnelle des systèmes de compensation. En employant des méthodes de régression qui utilisent la covariance de l'échantillon entre la rémunération à la pièce et la productivité quotidienne pour identifier l'effet incitatif, nous estimons que l'élasticité de l'effort par rapport aux changements dans la rémunération à la pièce est négative. En employant un modèle structurel qui contrôle l'endogénéité de la rémunération à la pièce, nous estimons que l'élasticité est d'environ 2,2. L'application des méthodes structurelles nous permet également de faire des expériences politiques et de comparer les profits de l'entreprise sous différents systèmes de compensation. Nos résultats démontrent que les profits augmenteraient de 17 % si l'entreprise adoptait le contrat optimal prédit par la théorie du principal-agent.

If policy prescriptions for compensation systems are to be useful, then they must be based on the empirical analysis of incentive effects; i.e., the elasticity of worker effort with respect to changes in the compensation system.

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We measure the elasticity of worker effort with respect to changes in the piece rate using panel data collected from the payroll records of a British Columbia tree-planting firm. Our data contain information on daily worker productivity and the piece rate received over a five-month period. We highlight the endogeneity problems inherent in traditional empirical analyses of compensation systems. In particular, employing regression methods, which use the sample covariance between piece rates and daily productivity to identify the incentive effect, we consistently estimate the elasticity of effort with respect to changes in the piece rate to be negative. Using a structural model to control for the endogeneity of the piece rate, we estimate the elasticity to be approximately 2.2. Structural estimation also allows us to perform policy experiments and to compare firm profits under alternative compensation systems. Our results suggest that profits would increase by at least 17 percent were the firm to implement the optimal contract as predicted by principal-agent theory.

Mots Clés : Systèmes de compensation, effet incitatif, modèles principal-agent

Keywords : Compensation Systems, Incentive Effect, Principal-Agent Models

JEL : D2, J3, L2

1. Introduction and Motivation

The role of economic incentives in determining behaviour is of major interest to economists. Within the domain of labour economics, much theoretical attention has been focussed on the optimal form of contracts between the firm and its workers; see, for example, Hart and Holmstrom (1985), Holmstrom and Milgrom (1990), Milgrom and Roberts (1992), and Baker (1992). The related and recently-developed field of personnel economics (see Lazear [1995,1998]) considers compensation systems as policy instruments of the firm which can be used to achieve optimal productivity on the part of the worker. Some economists (*e.g.*, Blinder [1990]) have also argued that the increase in worker productivity in response to the widespread adoption of performance-based pay would result in macroeconomic benefits.¹ In order for normative policy prescriptions to be valuable, however, they must be based on empirical analyses of the benefits accruing to changes in compensation systems. Empirically analyzing compensation policies and evaluating these benefits requires measuring incentive effects; *i.e.*, how workers react to changes in their economic incentives.

In the past, empirical work concerning incentive models has typically involved cross-sectional or longitudinal comparisons of wages among workers who do and do not receive incentive pay; see, for example, Pencavel (1977), Seiler (1984), and Parent (1997) as well as Booth and Frank (1997). The strength of this approach is that it is based on a wide sample of observations from different sectors of the economy and therefore provides “general” results. Yet, while the results of these studies are usually consistent with incentive models (thus supporting the existence of incentive effects), problems exist with their interpretation. In particular, workers who do not receive explicit incentive pay may be provided with incentives through other mechanisms, such as the promise of future promotions (as in Lazear and Rosen [1981] or Goldin [1986]) or termination contracts (as in Shapiro and Stiglitz [1984] or Macleod and Malcolmson [1989]). This inability to document and to understand fully the personnel policies implemented by different firms in a cross section of data makes it difficult to identify incentive effects using these methods.

An alternative approach is to concentrate on industry or firm-level

¹ At least one country has taken these notions seriously. Booth and Frank (1997) report that the government of the United Kingdom has introduced tax policy aimed at inducing firms to implement incentive pay.

data. Such an approach combines elements of the traditional case-study methodology, once popular in the industrial organization literature (see, for example, Wallace [1937]), with econometric estimation. Examples of this approach can be found in the recent work of Ferrall and Shearer (1996), Shearer (1996), Lazear (1996), Paarsch and Shearer (1996), and Treble (1997). Within this approach, the detailed study of the personnel policies of the firm or firms in question admits knowledge of the incentive system determining worker behaviour. Measuring worker reaction to variation in the compensation system then admits identification of incentive effects. Furthermore, access to firm archives often yields direct measures of worker productivity, so that the presence or absence of incentive effects does not have to be inferred indirectly through a comparison of wages.

One potential problem with both approaches is that the changes in the compensation system may not be exogenous (Ehrenberg [1990], Brown [1990]). To wit, the firm may select a compensation system based on elements which are unobservable to the econometrician, but which affect worker productivity. This suggests that regression methods, which use the observed covariation between worker productivity and the payment system, may fail to provide consistent estimates of the incentive effect.

In this paper, we measure incentive effects with particular emphasis on piece-rate workers, those workers whose pay is proportional to their output. We use data on daily individual productivity and piece rates to measure how workers react to changes in their compensation system. Knowledge of the elasticity of effort with respect to changes in the piece rate has important implications for firms who are paying or considering paying their workers piece rates. Stiglitz (1975) has shown that the optimal piece rate for a firm to set is an increasing function of this elasticity. Intuitively, the higher is the elasticity of effort, the more beneficial is it for the firm to set a high piece rate. While this elasticity may depend on the technology employed in a particular industry or firm, the case-study approach can still be useful as long as the characteristics of the firm are taken into account for policy proposals.

Our data were collected from the payroll records of a tree-planting firm in the province of British Columbia, Canada. This firm paid its workers exclusively piece rates and workers received no base wage. The tree-planting industry has many advantages as a laboratory within which to estimate labour market incentive models. Worker output is easily observable on

a daily basis and compensation systems vary within firms. Moreover, the compensation systems are relatively simple, permitting straightforward analyses of incentives. For example, in the firm that we study no team production existed and workers were not unionized. Because our data are panel in nature, we observe the daily productivity of each worker as well as the piece rate received by that worker over a period of approximately five months.

There are also practical reasons for studying the British Columbia tree-planting industry. British Columbia produces around twenty-five percent of the softwood lumber in North America.² The success that this province has at managing its timber affects the supply of timber to North America as well as to many other parts of the globe. In addition, the scope of reforestation in British Columbia is huge. At its peak, between 1981 and 1985, almost 2 billion seedlings were planted. This pace has slowed down somewhat but still remains important. Today, about 200 million seedlings are planted per year. An average seedling cost about \$0.50 to plant. Thus, a ten percent improvement can yield savings of about \$10 million per year. Small improvements in personnel policy can result in large savings because of the enormous scope involved.

Using our data, we highlight the endogeneity problems inherent in the empirical analysis of compensation systems. In particular, employing regression methods, which use the observed covariance between piece rates and productivity to identify the incentive effect, we consistently estimate the elasticity of effort with respect to changes in the piece rate to be negative. While this result seems nonsensical from the point of view of incentive theory, it obtains because piece rates are determined endogenously by the firm in response to the relative difficulty of planting in different areas. In particular, the firm chooses the observed piece rate to satisfy the labour-supply constraint of the worker, the amount the firm must at least pay the worker to induce him to accept the contract, implying that piece rates are negatively correlated with average planting conditions. Since these planting conditions are unobservable to the econometrician, they enter the error term of the regression model, so the piece rate is, in fact, a statistically endogenous variable and the estimate of the elasticity of effort with respect

² When statistics are reported for Canada, they are reported as “East of the Rockies” and British Columbia. British Columbia is broken up into three regions — the coast, the southern interior, and the northern interior — each of which produces more timber than any province of Canada or state of the Union.

to the piece rate is inconsistent.

Obtaining a consistent estimate of the effort elasticity requires controlling for the unobservable planting conditions during estimation. We accomplish this by explicitly modelling the firm's choice of the piece rate as a function of planting conditions and worker behaviour as a function of the piece rate. In our model, we incorporate asymmetric information between the firm and the worker over planter effort and planting conditions. We also allow for individual-specific heterogeneity among workers. We estimate the elasticity of effort with respect to the piece rate is 2.2, which implies that an increase in the piece rate of one cent from the sample mean of 25 cents would increase average daily output by 70 trees when planting conditions are held fixed.

Estimating the model structurally has benefits beyond controlling for endogenous regressors. In particular, using estimates of the structural parameters, we can investigate how the observed contract departs from the optimal contract as predicted by theoretical incentive models. In particular, given risk-neutral workers, the optimal contract involves the workers paying the firm a fixed fee to plant trees and then receiving a piece rate equal to the price of output. The inclusion of the base fee gives the firm two instruments in the contract: one to provide incentives (the piece rate) and the other to extract rents from the worker (the base fee). In contrast, the observed contract contains only one instrument and therefore allows workers to earn rents. If the firm could charge the workers an up-front fee to plant trees, then our results suggest that firm profits would increase, on average, by at least \$31.77 per worker per day, an increase on the order of 17.8 percent. We offer institutional and practical reasons for why such a contract is not implemented.

The paper is organized as follows: In the next section, we describe the tree-planting industry in British Columbia as well as the compensation system with which we are concerned. In section 3, we describe the sample data and present some regression results which illustrate our point concerning endogeneity. In section 4, we develop and estimate a simple theoretical model of worker-effort choice for a given piece rate, and then the choice of piece rate chosen by the firm in response to worker behaviour. We use the estimated parameters from the structural model of section 4 to investigate alternative contracts in section 5, and we conclude in section 6.

2. Tree Planting in British Columbia

While timber is a renewable resource, active reforestation can increase the speed at which forests regenerate and also allows one to control for species composition, something that is difficult to do in the case of natural regeneration. Reforestation is central to a steady supply of timber to the North-American market. In British Columbia, extensive reforestation is undertaken by both the Ministry of Forests and the major timber-harvesting firms who hold Tree Farm Licenses.³

The mechanics of this reforestation are relatively straightforward. Prior to the harvest of any tract of coniferous timber, random samples of cones are taken from the trees on the tract, and seedlings are grown from the seeds contained in these cones. This ensures that the seedlings to be replanted are compatible with the local micro-climates and soil as well as representative of the historical species composition.

Tree planting is a simple, yet physically exhausting, task. It involves digging a hole with a special shovel, placing a seedling in this hole, and then covering its roots with soil, ensuring that the tree is upright and that the roots are fully covered. The amount of effort required to perform the task depends on the terrain on which the planting is done. In general, the terrain can vary a great deal from site to site. In some cases, after a tract has been harvested, the land is prepared for planting by burning whatever slash timber remains and by “screefing” the forest floor. Screefing involves removing the natural build-up of organic matter on the forest floor so that the soil is exposed. Screefing makes planting easier because seedlings must be planted directly in the soil. Sites that are relatively flat or that have been prepared are much easier to plant than sites that are very steep or have not been prepared. The typical minimum density of seedlings is about 1,800 stems per hectare, or an inter-tree spacing of about 2.4 meters, although this can vary substantially.⁴ An average planter can plant between 700

³ In British Columbia, nearly 90 percent of all timber is on government-owned (Crown) land. Basically, the Crown, through the Ministry of Forests, sells the right to harvest the timber on this land in two different ways. The most common way is through administratively set prices to thirty-four firms who hold Tree Farm Licenses. These licenses have been negotiated over the last three-quarter century, and require that the licensee adopt specific harvesting as well as reforestation plans. About 90 percent of all Crown timber is harvested by firms holding Tree Farm Licenses. The second, and less common way, to sell timber is at public auction through the Small Business Forest Enterprise Program. In this case, the Ministry of Forests assumes the responsibility of reforestation.

⁴ One hectare is an area 100 metres square, or 10,000 square metres. Thus, one hectare is approximately 2.4711 acres.

and 900 trees per day, about half a hectare, depending on conditions. An average harvested tract is around 250 hectares.

Typically, tree-planting firms are chosen to plant seedlings on harvested tracts through a process of competitive bidding. Depending on the land-tenure arrangement, either a timber-harvesting firm or the Ministry of Forests will call for sealed-bid tenders concerning the cost per tree planted, with the lowest bidder being selected to perform the work. The price received by the firm per tree planted is called “the bid price.” Bidding on contracts takes place in the late autumn of the year preceding the planting season, which runs from early spring through to late summer. Before the bidding takes place, the principals of the tree-planting firms typically view the land to be planted and estimate the cost at which they can complete the contract. This estimated cost depends on the expected number of trees that a planter will be able to plant in a day which, in turn, depends on the general conditions of the area to be planted.

Workers are predominantly paid using piece-rate contracts, although fixed-wage contracts are sometimes used as well. Under piece-rate contracts, workers are paid in proportion to their output. Generally, no explicit base wage or production standard exists, although firms are governed by minimum-wage laws. Output is typically measured as the number of trees planted per day, although some area-based schemes are used as well. An area-based scheme is one under which workers are paid in proportion to the area of land they plant in a given day, based on a particular stem density.

Our data were collected from a medium-sized, tree-planting firm that employed a total of 155 workers throughout the 1994 tree-planting season. This firm paid its workers exclusively piece rates; daily earnings for a worker were determined by the product of the piece rate and the number of trees the worker planted on that day. Sites to be planted were divided into plots. For each plot, the firm decided on a piece rate. This rate took into account the expected number of trees that a planter could plant in a day and the expected wage the firm wanted to pay. Thus, the piece rate should be negatively correlated with planting conditions. All workers planting on the same plot received the same piece rate; no matching of workers to planting conditions occurred, so even though workers may be heterogeneous, the piece rate received was independent of worker type. Planters were assigned to plots as they disembarked from the ground transportation that took

them to the planting site. Thus, to a first approximation, planters were randomly assigned to plots.

3. Sample Data and Regression Results

Our dataset contains information on the piece rate received by each worker as well as that planter's daily productivity. We considered only those workers who received the same piece rate for the whole day of planting. This eliminated the problem of aggregating trees planted under different piece rates. The summary statistics for the entire dataset, which contains 4573 observations on 155 individual planters planting for 31 different contracts over a five-month period in the spring and summer of 1994, are presented in table 1. A contract was identified by a unique value for the piece rate on a particular tract. The average piece rate received by planters was 24.6 cents per tree and workers planted, on average, 764 trees per day. The average wage was \$178 per day.

Table 1 also suggests that outliers exist in the data. For example, the recorded minimum number of trees planted in one day was 30 and the recorded minimum daily wage was \$9.30. While we have no data available concerning daily hours worked, we suspect that these low observations were for planters working fewer than the usual 8 hours per day. In figure 1, we present a histogram of the logarithm of trees planted daily. The presence of outliers is clearly evident from the long left-hand tail.

Since we want to compare worker productivity under homogeneous conditions, our strategy for dealing with outliers is to eliminate them from the sample (see Donald and Maddala [1993]). Yet, simply eliminating low productivity observations may lead to misleading results since low productivity may reflect difficult planting conditions as well as few hours worked. Since the piece rate is adjusted by the firm to compensate for planting conditions, we identify outliers on the basis of total daily earnings. In particular, we eliminated from our sample those observations for which the planter earned less than \$48.00 per day, the minimum daily earnings permitted by government minimum-wage law for an 8-hour workday in 1994.⁵ A histogram of the logarithm of trees planted daily for this restricted

⁵ An alternative possible reason for low productivity is that certain workers were of very low ability. By law, the firm is required to pay workers at least \$48.00 per day; workers who are incapable of earning this amount through the piece-rate system are fired. In this sense, our sample can be considered an equilibrium employment sample in which all workers have an ability level that is satisfactory to the firm.

Table 1.
Summary Statistics: Full Sample.

Variable	Observations	Mean	St.Dev.	Minimum	Maximum
Number of Trees	4578	764.26	319.30	30	2260
Piece Rate	4578	0.25	0.06	0.13	0.48
Daily Earnings	4578	178.32	62.09	9.30	530.00

Table 2.
Summary Statistics: Daily Earnings above Minimum-Wage Daily Earnings.

Variable	Observations	Mean	St.Dev.	Minimum	Maximum
Number of Trees	4473	778.54	309.03	120	2260
Piece Rate	4473	0.25	0.06	0.13	0.48
Daily Earnings	4473	181.66	58.64	48.00	530.00

Table 3.
Summary Statistics: Random Sample of Planters Used.

Variable	Observations	Mean	St.Dev.	Minimum	Maximum
Number of Trees	1059	757.62	299.37	160	2120
Piece Rate	1059	0.24	0.05	0.13	0.40
Daily Earnings	1059	177.58	62.27	48.00	530.00

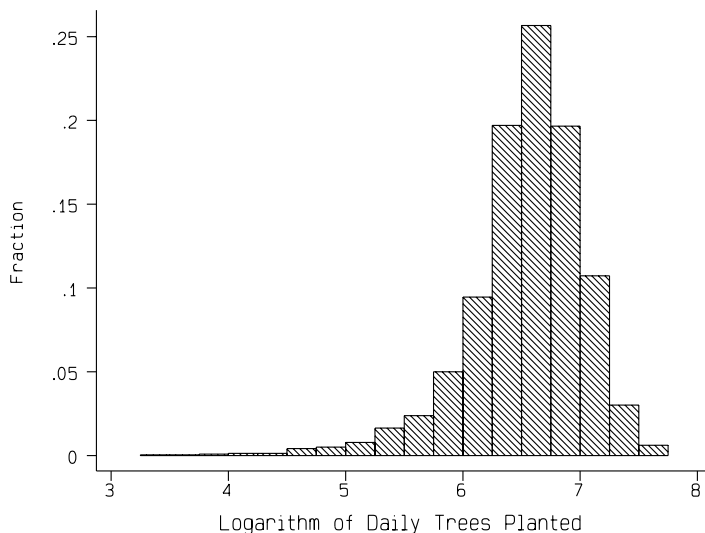
sample is presented in figure 2, while the summary statistics for this sample are presented in table 2. Note that the average number of trees planted daily increases slightly when compared to table 1, as does the average wage.

For the purposes of estimation, which will require estimating individual-specific effects and contract-specific variances, we restricted our sample to include only those contracts with at least 5 observations. We then restricted ourselves to a sample of 40 planters randomly selected from the set of planters who are observed at least twice.⁶ This yielded a sample of 1059 observations on 23 contracts. The summary statistics of the final sample used are provided in table 3.

We first considered regression methods as a way of measuring the

⁶ For reasons which will become clear later in this section, we ensured that the planter with the lowest average productivity in the firm was included in our sample. Thus, we randomly selected 39 additional planters to complete the sample.

Figure 1.
Histogram of the Logarithm of Daily Trees Planted.
Full Sample; Sample Size = 4578.



elasticity of effort. We estimated the following “log-log” regression model:

$$\log Y_{i,t} = \beta_{0,i} + \beta_1 \log r_t + U_{i,t} \quad (3.1)$$

where $Y_{i,t}$ is the daily productivity of worker i on contract t , $\beta_{0,i}$ is a (possibly individual-specific) constant term, r_t is the piece rate received by the worker on contract t , and $U_{i,t}$ is a zero-mean error term that in traditional analyses is assumed to have zero covariance with r_t .

We estimated equation (3.1) in two different ways. First, we included as explanatory variables only a constant and the piece rate. These results are presented in column (a) of table 4. Note that the estimate of β_1 is negative, equal to -0.712 , and has a p-value which is virtually zero. Second, we included individual-specific intercepts to control for heterogeneity across individuals. These results are presented in column (b) of table 4. Again, the estimate of β_1 is negative, but now equal to -0.856 , and has a p-value which is also virtually zero.

Figure 2.
 Histogram of the Logarithm of Daily Trees Planted.
 Sample Attaining Minimum-Wage Daily Earnings; Sample Size = 4473.

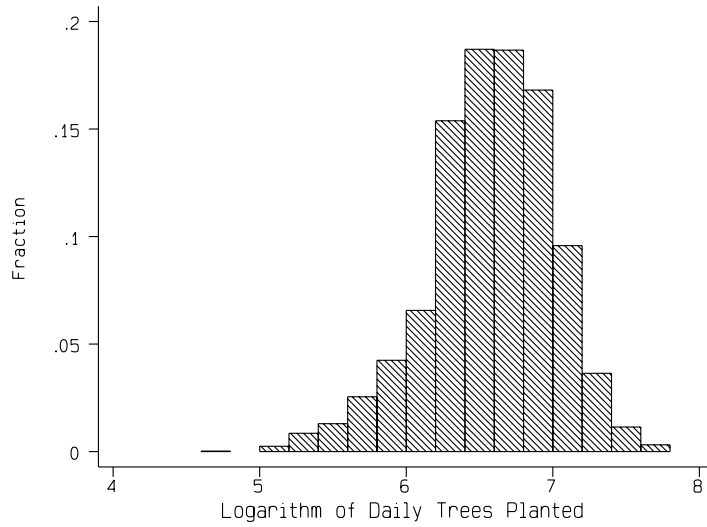
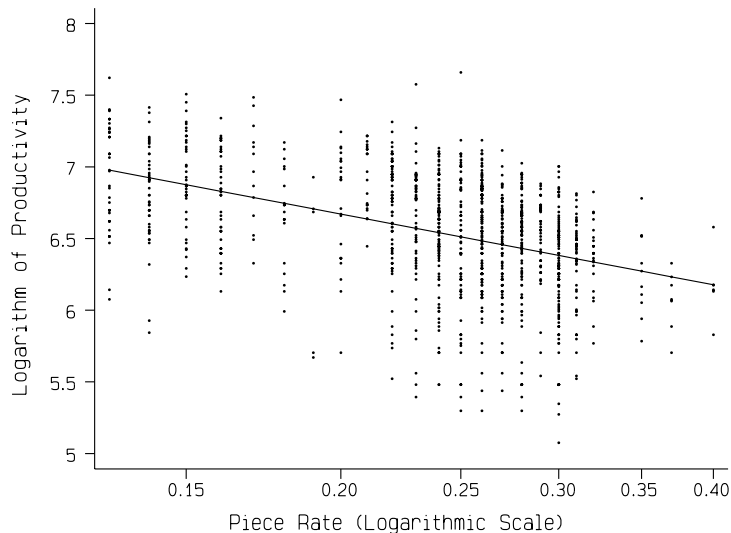


Table 4.
 Simple Regression Results.
 Dependent Variable: Logarithm of Daily Production.
 Sample Size = 1059.

Independent Variable	(a)	(b)
Constant	5.525 (0.069)	5.086 (0.087)
Logarithm of Piece Rate	-0.712 (0.048)	-0.856 (0.043)
Maximum Individual-Specific Effect		0.717
Minimum Individual-Specific Effect		-0.826
Average Individual-Specific Effect		0.105
R^2	0.174	0.606

To provide visual confirmation of our regression results, we present in figure 3 a scatterplot of the logarithm of the number of trees planted daily

Figure 3.
 Scatterplot of Daily Trees Planted and Piece Rates.
 Randomly Selected Sample; Sample Size = 1059.



versus the logarithm of the piece rate, along with the estimated regression line. Note the strikingly negative relationship.

The negative coefficient estimate on the logarithm of the piece rate paid to workers is troubling from the perspective of incentive theory. Taken literally, it suggests that when the piece rate is high workers work less intensively than when the piece rate is low: this seems counter-intuitive. An alternative explanation is that the piece rate is endogenous to the statistical model. In particular, if piece rates are correlated with unobservable variables which also affect worker productivity, then the observed piece rate will be correlated with the error term $U_{i,t}$ in (3.1). This correlation will result in biased estimates of the elasticity of effort with respect to piece rates because one of the assumptions maintained by least squares estimation has been violated.

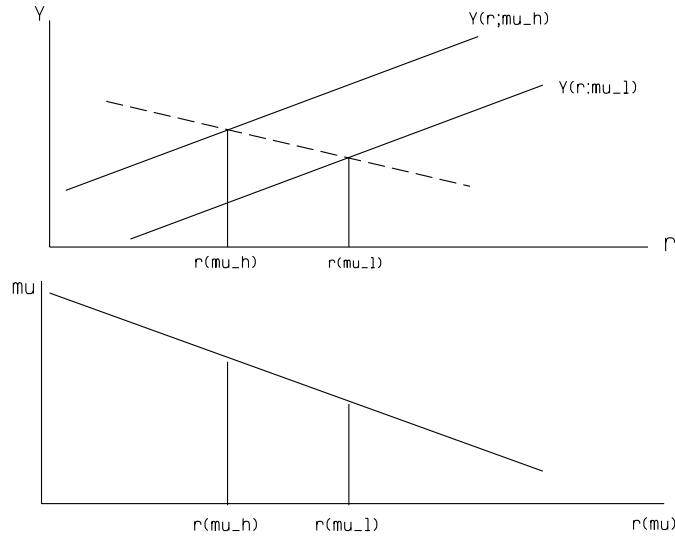
Discussions with firm principals revealed that piece rates are chosen by the firm after average planting conditions have been observed. The actual piece rate is chosen to ensure that the worker's labour-supply constraint, the

amount the firm must pay the worker to induce him to accept the contract, is satisfied. A worker's productivity is a function of how hard he works and the conditions under which he is planting: it is easier to plant trees on flat terrain that is covered in loose soil than on steep rocky hillsides. For planting conditions that are favourable to productivity, worker output will be higher for any given level of effort. Since planters are paid in proportion to the number of trees they plant and since effort is costly, these workers prefer planting in favourable conditions: they can plant lots of trees for little effort. Therefore, in order to induce workers to plant under unfavourable conditions the firm must increase the piece rate.

The effect of this process on regression models is illustrated graphically in figure 4. In the bottom panel, we represent the inverse relationship between the piece rate r and average planting conditions μ caused by the labour-supply constraint. In the upper panel of figure 4, we illustrate the relationship between productivity Y and the piece rate for two different levels (low μ_ℓ and high μ_h) of planting conditions μ . The slope of the line $Y(r; \mu)$, the (productivity, piece-rate) locus, represents the incentive effect that we seek to estimate. The fact that high piece rates $r(\mu_\ell)$ are associated with poor planting conditions μ_ℓ implies that productivity is lower for any value of r , shifting down the (productivity, piece-rate) locus. Since average planting conditions are unobservable to the econometrician, we have no way of controlling for the fact that the locus has shifted down and a simple regression of productivity on the piece rate connects the points on two separate loci producing the dotted line with a negative slope.

To obtain a consistent estimate of β_1 requires controlling for planting conditions. In general, three possible ways exist to do this. First, one could collect data on the planting conditions that affect the firm's choice of a piece rate. Note, however, that one would have to have *all* the information that the firm has when the firm makes its choice. If the econometrician observed only a subset of the planting conditions, then biased estimates of the incentive effect would still obtain because the set of conditions that were unobserved could still be correlated with both a worker's output and the piece rate. Gaining access to such data has proven impossible. A second possibility would be to use an instrumental variable; *i.e.*, a variable that is correlated with the piece rate, but uncorrelated with the planting conditions. While such a variable is easy to define, in practice, finding such a variable has proven impossible in this situation. A final approach, the

Figure 4.
 Example of Low- and High-Productivity Plots and the Piece Rate.



one that we follow, is to model explicitly the firm's decision rule over r as a function of planting conditions and to incorporate this decision rule directly into the estimation procedure.

4. Deriving and Estimating a Structural Model

We model unobserved planting conditions as productivity shocks that affect the output which obtains for a given level of effort on the part of the planter. We assume that productivity shocks S are draws from a particular distribution $F(s)$ having parameters μ and σ^2 . The firm's decides on the piece rate r by considering the parameters μ and σ^2 . A contract is defined by the pair (μ, σ^2) , and a unique value of r . We model the firm's choice of r as satisfying the worker's labour-supply constraint, conditional on average planting conditions. Thus, the firm chooses r to induce the worker to participate in planting. Note that changes in average planting conditions lead to changes in r .

We develop a simple model of worker-effort determination under piece

rates with risk-neutral workers.⁷ We assume for planters the following utility function defined over earnings W and effort E :

$$U(W, E) = W - C(E),$$

where the earnings function is given by

$$W = rY$$

with Y being worker output and the function $C(E)$ denoting the worker's cost of effort which is parameterized as

$$C(E) = \frac{\kappa}{\eta} E^\eta \quad \eta > 1, \kappa > 0.$$

Output is assumed to be determined by the following function:

$$Y = ES$$

where S is a random productivity shock drawn from the distribution $F(s)$ having parameters μ and σ^2 and represents planting conditions that are beyond the worker's control, such as the slope of the terrain, hardness of the ground, and the amount of ground cover. We assume that s , a realization of S , is observed by workers before they choose their effort levels, but after they accept a contract. Note that the firm does not observe s , but only the parameters of the distribution of S ; *viz.*, μ and σ^2 . Thus, while a planter can observe average planting conditions before he begins to plant, he only learns of the exact nature of the terrain to be planted once planting begins. The logarithm of the productivity shock is assumed to follow a normal distribution with mean μ and variance σ^2 , so the probability density function of S takes the form

$$f_S(s) = \frac{1}{s\sigma_S} \phi\left(\frac{\log s - \mu_S}{\sigma_S}\right) \quad (4.1)$$

where ϕ represents the standard normal probability density function.

The timing of the model is as follows:

1. For a particular contract to be planted, Nature chooses the pair (μ, σ^2) , the parameters of the distribution of S ;
2. the firm observes (μ, σ^2) , and then chooses a piece rate r ;

⁷ Interviews with planters suggest that variation in daily earnings is a relatively minor concern.

3. the worker observes (μ, σ^2, r) , and accepts or rejects the contract;
4. if the worker accepts the contract, then he is randomly assigned to plant a particular plot of the contract;
5. for each plot, Nature chooses s , a particular value of S ;
6. the worker observes s , and chooses an effort level e producing output y ;
7. the firm observes y , and pays earnings ry .

To solve the model, we work backwards. First, we solve for the worker's optimal effort level conditional on a given piece rate and productivity shock. Then we solve for the firm's choice of the piece rate, taking the reaction of the worker as given. Note that in order to induce the worker to accept the contract, the contract must satisfy the worker's labour-supply constraint.

Conditional on s , a particular realization of S , planters choose effort to maximize their utility, so the optimal level of effort e is

$$e = \left(\frac{rs}{\kappa} \right)^{\left(\frac{1}{\eta-1} \right)}$$

To simplify resulting expressions, let γ denote $[1/(\eta - 1)]$. Note that the second-order condition of the worker's problem is satisfied as long as γ exceeds zero, η exceeds one. Making the appropriate substitutions, we write the expressions for optimal effort and output on the part of the planter in response to a particular piece rate r as

$$\begin{aligned} e &= \left(\frac{rs}{\kappa} \right)^\gamma \\ y &= \left(\frac{r}{\kappa} \right)^\gamma s^{\gamma+1}. \end{aligned}$$

Taking logarithms of both sides of the second equation above yields

$$\log y = \gamma \log r - \gamma \log \kappa + (\gamma + 1) \log s$$

or, in terms of random variables,

$$\log Y = \gamma \log r - \gamma \log \kappa + (\gamma + 1) \log S \tag{4.2}$$

where

$$(\gamma + 1) \log S \sim N[(\gamma + 1)\mu, (\gamma + 1)^2\sigma^2].$$

Note that the parameter γ gives a direct measure of the elasticity of worker effort with respect to the piece rate.

Note too that (4.2) has the same form as the regression model (3.1) estimated above. From equation (4.2), it is also clear why regression methods fail to provide a consistent estimate of the incentive effect. To convert (4.2) into an equation with a mean-zero error term, we add and subtract $(\gamma + 1)\mu$, which yields

$$\log Y = \gamma \log r - \gamma \log \kappa + (\gamma + 1)\mu + V \quad (4.3)$$

where V now equals $(\gamma + 1)(\log S - \mu)$, which is distributed normally with mean zero and variance $(\gamma + 1)^2\sigma^2$. Comparing (4.3) to (3.1), one notes immediately that $U_{i,t}$, the error term in (3.1), equals $(\gamma + 1)\mu + V_{i,t}$, but from figure 4 we know that $\text{cov}(\mu, r)$ does not equal zero.⁸ Thus, one of the assumptions maintained in least-squares estimation (*viz.*, the weak exogeneity of the covariates) has been violated.

We assume that workers have an alternative utility given by \bar{u} , so the labour-supply constraint is

$$\mathcal{E}[W - C(e)] = \bar{u}$$

where \mathcal{E} is the expectation operator taken with respect to the random variable S . Substituting optimal effort into the labour-supply constraint yields

$$\frac{r^{\gamma+1}}{(\gamma + 1)\kappa^\gamma} \mathcal{E}(S^{\gamma+1}) = \bar{u}.$$

Using the properties of the log-normal distribution, we obtain

$$(\gamma + 1)\mu = \log \bar{u} + \log(\gamma + 1) + \gamma \log \kappa - (\gamma + 1) \log r - (\gamma + 1)^2 \frac{\sigma^2}{2}. \quad (4.4)$$

Substituting (4.4) into (4.3) yields an equation for the daily productivity of individual i on contract t

$$\log Y_{i,t} = \log(\gamma + 1) + \log \bar{u} - \log r_t - (\gamma + 1)^2 \frac{\sigma_t^2}{2} + V_{i,t}. \quad (4.5)$$

Incorporating the worker's expected-utility constraint eliminates the endogeneity problem because V represents only unexpected deviations from average conditions. Therefore, it is a mean-zero error term that is uncorrelated with r . Estimation based on equation (4.5) can provide consistent estimates of γ .

⁸ Note that while μ and σ^2 are fixed for a given contract, they vary across contracts — causing the correlation between μ and r .

4.1. Parameter Identification and Estimates

Our data contain observations on 40 workers planting under 23 different contracts. Each contract t is specified by a pair (μ_t, σ_t) , which in turn determines the piece rate r_t through (4.4). Therefore, the structural model consists of the parameter vector $(\gamma, \kappa, \mu_1, \sigma_1, \dots, \mu_{23}, \sigma_{23})^\top$.

Estimating equation (4.5) requires a measure of alternative utility \bar{u} . We used the daily British Columbia welfare payments to a single individual with no dependants in 1994. This measure captures what an individual would receive were zero effort supplied. In 1994, daily welfare payments were \$18.53 per day.

Defining

$$Y_{i,t}^* \equiv \log Y_{i,t} + \log r_t - \log \bar{u},$$

we can then re-write (4.5) as

$$Y_{i,t}^* = \log(\gamma + 1) - (\gamma + 1)^2 \frac{\sigma_t^2}{2} + V_{i,t}.$$

Estimating the above specification by the method of maximum likelihood is similar to estimating a linear regression with the added complication that the contract-specific variance of the productivity shock σ_t^2 enters the conditional-mean function. The elasticity of effort with respect to the piece rate is identified by the constant term.

Note that κ and the $\{\mu_t\}_{t=1}^{23}$ enter (4.5) and (4.4) additively. Thus, once (4.5) is estimated, we can recover an estimate of $[(\gamma + 1)\mu_t - \gamma \log \kappa]$ by substituting back into (4.4). However, separately identifying κ and the $\{\mu_t\}_{t=1}^{23}$ would require an additional identifying normalization, such as μ_1 equalling zero.

Results obtained by estimating equation (4.5) are given in column (a) of table 5. Our estimate of γ is 8.7999, suggesting a very large elasticity of effort with respect to the piece rate. Of the 23 contract-specific variances, we report only the maximum, minimum, and average values.

4.2. Introducing Individual-Specific Heterogeneity

Estimates of γ based on equation (4.5) neglect the fact that planters may be heterogeneous with respect to their ability. To capture individual-specific heterogeneity, we admit planters who have different costs of effort. We then assume that the firm chooses the piece rate to ensure that the least-able

Table 5.
Parameter Estimates of Structural Models:
With and Without Individual-Specific Heterogeneity.
Sample Size = 1059.

Parameter	(a)	(b)
γ	8.799 (0.143)	2.229 (0.647)
Maximum σ	0.080	0.179
Minimum σ	0.025	0.021
Average σ	0.039	0.079
Maximum Individual-Specific Effect		1.538
Minimum Individual-Specific Effect		0.124
Average Individual-Specific Effect		0.955
Logarithm of the Likelihood Function	-407.930	-27.116

(highest-cost) planter in the firm is indifferent between working and not working. Within this framework, all other planters earn rents.

Denoting the cost of effort for worker i by κ_i and the cost of effort for the least-able planter by k_h , which is the $\max\{\kappa_1, \kappa_2, \dots, \kappa_n\}$, piece rates are then determined by

$$\frac{r^{\gamma+1}}{k_h^\gamma (\gamma+1)} \mathcal{E}(S^{\gamma+1}) = \bar{u}, \quad (4.6)$$

while the output for individual i is determined by

$$Y_i = \left(\frac{r}{\kappa_i}\right)^\gamma S^{(\gamma+1)}. \quad (4.7)$$

Taking logarithms of both sides of equation (4.7) yields

$$\log Y_i = \gamma \log r - \gamma \log \kappa_i + (\gamma+1)\mu + V \quad (4.8)$$

where V equals $(\gamma+1)(\log S - \mu)$, which is normally distributed with mean zero and variance $(\gamma+1)^2 \sigma^2$. Taking logarithms of (4.6) and substituting for the term $(\gamma+1)\mu$ in (4.8) yields

$$\log Y_{i,t} = \log(\gamma+1) + \log \bar{u} - \log r_t + \gamma(\log k_h - \log \kappa_i) - (\gamma+1)^2 \frac{\sigma_t^2}{2} + V_{i,t} \quad (4.9)$$

A comparison of equation (4.9) to (4.5) suggests that ignoring individual-specific heterogeneity will lead to an over-estimate of γ because planters with low κ s will produce more output, on average, than those

planters with high κ s. In essence, by allowing planters to be heterogeneous, the term that was estimated as $\log(\gamma + 1)$ in (4.5) is now estimated as

$$\log(\gamma + 1) + \gamma(\log k_h - \log \kappa_i).$$

To estimate equation (4.9), we simply add to (4.5) individual-specific dummy variables for each planter in the sample, except the planter corresponding to k_h , for whom the term $(\log k_h - \log \kappa_i)$ equals zero. Note that we take the planter corresponding to k_h to be the planter with the lowest average productivity in the firm. Results from estimating equation (4.9) are presented in column (b) of table 5. Note that, after controlling for individual-specific effects, the estimate of γ falls to 2.2. That the individual-specific effects are jointly significant is evident from a comparison of the maximized values of the logarithm of the likelihood functions, -470.93 for the restricted model and -27.12 for the unrestricted model.

These results suggest that, holding planting conditions fixed, a 1 percent increase in the piece rate will increase productivity by 2.2 percent. In terms of measured output, an increase of the piece rate by \$0.01 from the sample mean of 25 cents will increase output by 70 trees if the conditions are held constant.

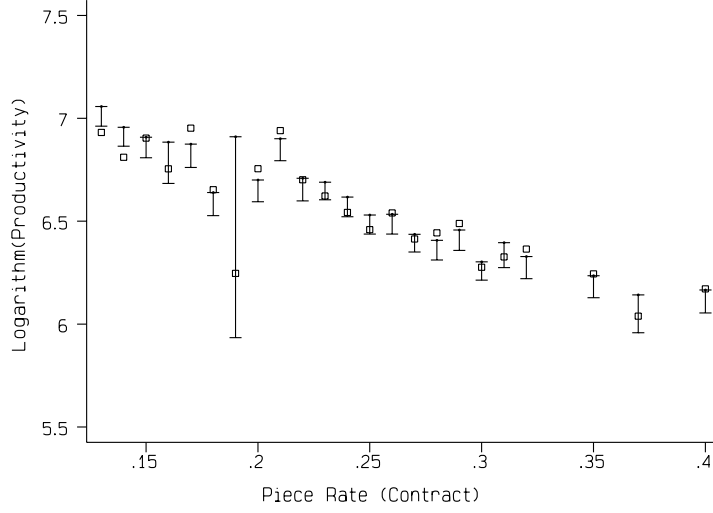
4.3. Prediction

In order to evaluate the performance of the structural model, we consider its ability to predict observed productivity for the different contracts. In figures 4 and 5, we present the average observed productivity per contract, denoted by the squares, and the 95-percent and 99-percent confidence intervals for the average predicted productivity, which are derived from the structural parameter estimates of the model. In general, the performance of the model appears quite good, 11 of the 23 95-percent confidence intervals and 17 of the 23 99-percent confidence intervals encompass the observed average productivity, suggesting that the predicted and actual values are quite close.

5. Alternative Contracts and Rents

The contract used by the firm we have studied is restrictive in that this firm only has one instrument (the piece rate) to accomplish two tasks: the provision of incentives, and the division of rents. With heterogeneous

Figure 5.
Ninety-Five Percent Confidence Intervals for Predicted Productivity.



workers, some of the workers will earn rents. The expected utility of worker i is

$$\mathcal{E}(U) = \frac{r^{\gamma+1}}{\kappa_i^\gamma(\gamma+1)} \mathcal{E}(S^{\gamma+1}).$$

Substituting for r from (4.6) yields

$$\mathcal{E}(U) = \bar{u} \left(\frac{k_h}{\kappa_i} \right)^\gamma.$$

Therefore, the rent earned by worker i is

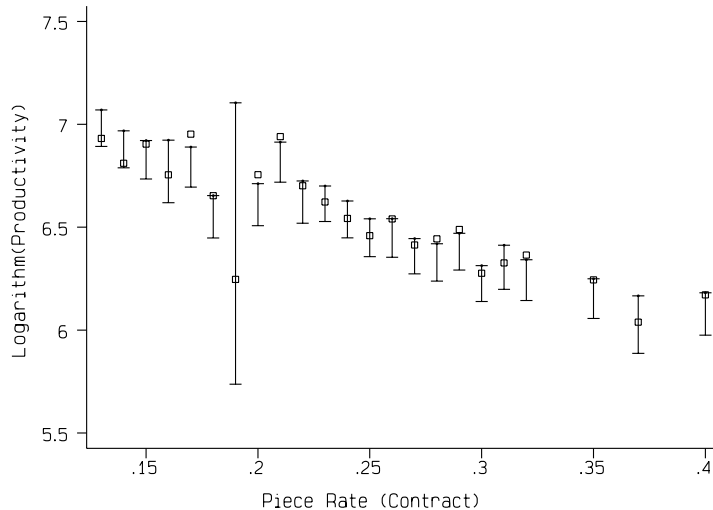
$$\left[\left(\frac{k_h}{\kappa_i} \right)^\gamma - 1 \right] \times \bar{u}. \quad (5.1)$$

An alternative contract, which nests the observed contract, pays earnings of the form

$$W = B + rY$$

where B is a base “wage” (or fee) that is independent of worker productivity. The advantage of introducing a base wage is that the firm can extract rents

Figure 6.
Ninety-Nine Percent Confidence Intervals for Predicted Productivity.



from the worker while still providing incentives. In particular, the optimal contract solves the following problem:

$$\max_{r, B} (P - r)Y - B \quad \text{subject to} \quad \mathcal{E}(U) = \bar{u}.$$

The solution to this problem is well known: With risk-neutral workers, the piece rate is set equal to the price of output and the base wage is adjusted to ensure the worker earns his alternative utility \bar{u} .

It is impossible to calculate the difference in profits between the fully optimal contract and the observed contract because the base wage under the optimal contract would vary across individuals and depend on each individual's cost of effort; these are not identified in our model. We do, however, identify each individual's cost of effort relative to k_h , so we can estimate the rent accruing to each individual in our sample. This allows us to estimate a lower bound to the increase in profits that would accrue from the optimal compensation system. Our estimate is a lower bound because we hold the piece rate fixed.

Table 6.
Estimates of Expected Worker Rents, Per Day.

Worker	1	2	3	4	5	6	7	8	9	10
Rent	23.89	51.21	36.41	2.45	19.90	63.31	4.00	35.98	8.45	39.27
Worker	11	12	13	14	15	16	17	18	19	20
Rent	38.87	38.74	28.18	67.78	18.88	53.81	8.70	43.03	52.13	37.59
Worker	21	22	3	24	25	26	27	8	29	30
Rent	47.03	59.00	54.96	36.17	22.09	38.09	16.51	40.73	24.29	46.75
Worker	31	32	33	34	35	36	37	38	39	40
Rent	17.80	10.04	39.56	21.11	20.42	14.66	15.40	0.00	32.89	40.60

To estimate the rents that accrued to each worker, we use the estimates of the structural model in (4.9). The estimated rents are presented in table 6. We estimate that the planters are earning, on average, a rent of \$31.77 per day. Thus, introducing a base wage could increase average firm daily profits by at least \$31.77 per worker.

Firm profits under the observed system are given by $(P - r)Y$. Interviews with the firm manager revealed that the bid price per tree received by the firm is typically twice the piece rate paid to the workers. This suggests that average firm profits are given by rY , which equals \$178, implying that the optimal contract would increase profits by at least $[100 \times (31.77/178)]$ or 17.8 percent.

The question that naturally arises is: “Why did our firm not implement the optimal contract?” A number of possible reasons can be given for why entry fees are not observed in the employment relationship. The fact that workers may be financially constrained and unable to afford a sufficiently large fee is an obvious one. Another concerns the transaction costs associated with the base fee which render it prohibitively expensive. Under the observed contract, the firm only has to set one instrument, the piece rate. Under the optimal contract the firm must decide on the piece rate and a base fee for each worker. Since the base fee will be a function of a worker’s characteristics as well as daily planting conditions, it will vary from contract to contract as well as within contracts from day to day, greatly increasing the accounting and measurement costs to the firm.

6. Summary and Conclusions

In this paper, we have investigated the sensitivity of worker performance to changes in the compensation system with particular emphasis on changes

in the piece rate. Using data from the payroll records of a British Columbia tree-planting firm, we have highlighted the econometric problems inherent in evaluating changes in firm compensation policy which arise due to the endogeneity of the compensation system. Explicitly modelling the decision rules of the worker, with regard to effort, and the firm, with regard to the parameters of the compensation system, controls for this endogeneity. We estimate the elasticity of effort with respect to the piece rate to be 2.2.

Structural analysis has benefits beyond being able to control for endogenous regressors. In particular, we are able to calculate how firm profits would change from the use of alternative compensation systems. We estimate that profits would increase by at least 17.8 percent if the firm were able to implement the optimal contract as predicted by agency theory.

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