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Disappointment Aversion as a Solution to the Equity Premium and the Risk-Free Rate Puzzles*

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Résumé / Abstract

In this paper, we match both the first and the second moments of the equity premium and the risk-free rate by endowing the agents in the economy with disappointment aversion preferences and by making the joint process of consumption and dividends follow a Hamilton's (1989) Markov switching model. The interesting feature about the model proposed in this paper is that we need both disappointment aversion and a Markov switching endowment to match the first and second moments of both real and excess returns. With disappointment averse agents but a joint random walk for consumption and dividend growth rates, the average equity premium produced by the model is in the order of 2.5% compared with 5.3% in our sample. With isoelastic preferences but a bivariate three-state Markov switching model for consumption and dividend growth rates, the equity premium is 1.7% for a coefficient of relative risk aversion of 8 and a discount factor of 0.98, while the standard deviations for both the equity premium and the risk-free rate are close to the observed ones. The mean of the risk-free rate stands however very high at 13%. For a disappointment averse consumer, who weights more bad outcomes than good ones (where bad and good are defined with reference to a certainty equivalent measure of a gamble), it is precisely the existence of a bad

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state that lowers the equilibrium risk-free rate and increases the mean stock return, thereby producing the desired equity premium.

Keywords: equity premium puzzle, risk-free rate puzzle, disappointment aversion, Markov switching models, asset pricing, recursive utility.

Dans le présent article, nous reproduisons les premier et deuxième moments de la prime de risque sur les actions et du taux de risque en dotant les agents dans notre économie de préférences exhibant de l'averison pour la déception et en adoptant un modèle à changements de régime markoviens (Hamilton (1989)) pour le processus conjoint de la consommation et des dividendes. Le modèle proposé a la particularité intéressante de devoir combiner l'aversion pour la déception et une dotation à changements de régime markoviens pour pouvoir reproduire les premier et deuxième moments des rendements réels et excédentaires. Avec des agents dotés d'averison pour la déception mais une promenade aléatoire conjointe pour les taux de croissance de la consommation et des dividendes, la prime de risque moyenne sur les actions produites par le modèle est de l'ordre de 2,5 % par rapport à 5,3 % dans notre échantillon. Avec des préférences isoélastiques mais un modèle bivarié à changement de régime markovien à 3 états pour les taux de croissance de la consommation et des dividendes, la prime de risque sur les actions est de 1,7 % pour un coefficient d'aversion relative pour le risque de 8 et un facteur d'escompte de 0,98, tandis que les écarts-types de la prime de risque et du taux sans risque sont proches des valeurs observées. La moyenne du taux sans risque est toutefois très élevée à 13 %. Pour un consommateur ayant de l'aversion pour la déception, qui accorde un poids plus important aux mauvaises réalisations de la nature qu'aux bonnes (où bon et mauvais se définissent par un rapport à une mesure d'équivalence certaine d'un enjeu), c'est précisément l'existence d'un mauvais état de la nature qui, à l'équilibre, fait baisser le taux sans risque et augmenter le rendement moyen sur les actions, ce qui produit la prime de risque désirée sur les actions.

Mots clés : énigme de la prime de rendement sur les actions, énigme du taux de l'actif sans risque, aversion pour la déception, modèles à changements de régime markoviens, valorisation des actifs financiers, utilité récursive.

1. Introduction

The equity premium puzzle put forward by Mehra and Prescott (1985) — the fact that an exchange economy equilibrium model could not reproduce the secular difference between the average return on stocks and the average return on Treasury bills for reasonable configurations of the preferences and the endowment —, triggered a thorough specification search to come up with the various pieces that will fit the puzzle. The successful attempts are based on models which generate sufficient variation in the intertemporal marginal rate of substitution. This variation might come either from the endowment process as in Rietz (1988), with an economy where consumption may fall by as much as 25% in one year, or from the specification of preferences as in Constantinides (1990), with habit persistence in the form of a subsistence level for consumption. In the latter, the large variations in the marginal rate of substitution are due to the fact that small changes in consumption generate large changes in consumption net of the subsistence level. Unfortunately, this variability in the marginal rate of substitution generates too much variation in the risk-free rate. In models with levered economies — where equity is a levered claim to firms' production — Kandel and Stambaugh (1990, 1991) claim that both the first and second moments of the return data can be matched. As Cecchetti, Lam and Mark (1993) have shown however, when the leverage ratio is set to values that make the share of the endowment to equity holders close to the average observed value, the amount of bonds required to match the second moments is too high to be consistent with the data.

In this paper, we match both the first and second moments of the equity premium and the risk-free rate by endowing the agents in the economy with disappointment aversion preferences and by making the joint process of consumption and dividends follow a Hamilton's (1989) Markov switching model.

Preferences that exhibit disappointment aversion have been axiomatized by Gul (1991) to offer a solution to the so-called Allais paradox. Since this paradox manifests itself for choices involving sure lotteries, the potential for reproducing features corresponding to the risk-free asset — the asset producing one unit of consumption for sure in the next period — seems intuitively promising. Epstein and Zin (1989, 1991b) integrate these generalized preferences in an intertemporal asset pricing model within a recursive utility framework and show in the latter work that disappointment aversion helps to satisfy the bounds for the marginal rate of intertemporal substitution proposed by Hansen-Jagannathan (1991).

Bivariate Markov switching models for consumption and dividend growth rates have been proposed by Bonomo and Garcia (1991) and Cecchetti, Lam and Mark (1993) to model the endowment process in an exchange economy with time separable isoelastic preferences. The first authors have shown that the best model in the class of joint Markov switching models allows for three states where both the means and the variances change with the state. In the original Lucas (1978) model, consumption is equal to dividends and to output but in the actual data the series of consumption growth rates is very different from the series of dividend growth rates. Therefore in this specification equity prices are determined solely by the dividends that accrue to the stockholder discounted with a marginal rate of intertemporal substitution based itself on consumption. Bonomo and Garcia (1991) show that this separation of consumption and dividends is useful to reproduce stylized facts associated with real returns (first and second unconditional moments, negative serial correlation, forecastability of multiperiod returns by the dividend-price ratio) but does not bring anything in explaining the facts related to excess returns, the equity premium being one of them.

The interesting feature about the model proposed in this paper is that both disappointment aversion and a Markov switching endowment are necessary to match the first and second moments of real and excess returns. With disappointment averse agents but a joint random walk for consumption and dividend growth rates, the average equity premium produced by the model is in the order of 2.5% compared with 5.3% in our sample. On the other hand, with time-additive power utility but a bivariate three-state Markov switching model for consumption and dividend growth rates, the equity premium is 1.7% for a coefficient of relative risk aversion of 8 and a discount factor of 0.98, while the standard deviations for both the equity premium and the risk-free rate are close to the observed values. This premium is produced however with a mean risk-free rate of 13%, which is what Weil (1989) dubbed the risk-free rate puzzle.[1] For a disappointment averse consumer, who weights more bad outcomes than good ones (where bad and good are defined with reference to a certainty equivalent measure of a gamble), it is precisely the existence of a bad state that lowers the equilibrium risk-free rate (to 3.5%) and increases the mean stock return (to 8.7%), thereby producing the desired equity premium (5.2%). It has to be emphasized that the selected three-state Markov switching joint process for consumption and dividends comes out of an estimation and testing proce-

[1] As a matter of fact, Abel (1992) shows that the Markov regime-switching process exacerbates the equity premium puzzle and the risk-free rate puzzle. It should be noted however that his assumptions about the preferences and the information avail able to the agents a re different from the ones made in this paper.

dure and it is therefore the model in the class of Markov switching models that best fits the data.

To assess how well the population moments produced by the model match the sample moments estimated from the data, we follow Cecchetti, Lam, and Mark (1993) and use chi-square tests on various sets of unconditional moments of the equity premium and the risk-free rate, accounting for the uncertainty present in the estimation of the empirical moments. We also study the dynamics of the model by looking at its ability to forecast the future multi-period excess returns based on the current dividend yield, as it seems that such forecastability is present in actual data.

Given his failure to solve the equity premium puzzle or the risk-free puzzle even in an intertemporal non-expected utility framework, Weil (1989) was concluding that the misspecification of the preferences could not be held responsible for the existence of the equity premium puzzle. However, his specification of preferences was limitative. Although his representative agent was not indifferent to the temporal resolution of uncertainty, he was still using expected utility for evaluating timeless gambles. As a contrast, our disappointment averse agent has non-expected utility preferences for atemporal lotteries.

Benartzi and Thaler (1993) also use asymmetric preferences over good and bad results to match the equity premium, but instead of having an intertemporal asset pricing framework with preferences defined over consumption streams, they start from preferences defined over one-period returns based on Kahneman and Tversky (1979)'s 'prospect theory' of choice. By defining preferences in this way directly over returns, they avoid the challenge of reconciling the behavior of asset returns with aggregate consumption.

Finally, similarly to Epstein and Zin (1991b), our results tend to bring supportive evidence in an intertemporal asset pricing context for such disappointment averse preferences, whereby previous empirical evidence for such theories were limited to experimental studies.

The plan of the paper is as follows. Section 2 presents the asset pricing model, with a detailed account of the disappointment aversion preferences and of the bivariate Markov switching model for the endowment, along with the return formulas for the stock and the risk-free asset. The estimation and testing results associated with the choice of the best Markov model for the endowment are reported in Section 3. Section 4 compares the GMM estimates of the first and second unconditional moments of the real and excess stock returns to the same statistics obtained from the model. It also assesses whether the model can reproduce the predictability of future returns by current dividend yields. Section 5 concludes.

2. The Asset Pricing Model

Many identical infinitely lived agents maximize their lifetime utility and receive each period an endowment of a single nonstorable good. Following Epstein and Zin (1989), we specify a recursive utility function of the form:

$$U_t = W(C_t, \mu_t) \tag{2.1}$$

where W is an aggregator function that combines current consumption C_t with $\mu_t = \mu(\tilde{U}_{t+1} \mid I_t)$, a certainty equivalent of random future utility $\tilde{U}_{t+1}$, given the information available to the agents at time t , to obtain the current-period lifetime utility U_t. Epstein and Zin (1989) propose the CES function as the aggregator function, i.e.:

$$U_t = [C_t^{\rho} + \beta \mu_t^{\rho}]^{\frac{1}{\rho}}$$

The way the agents form the certainty equivalent of random future utility is based on their risk preferences, which are assumed to be disappointment averse (Gul, 1991). These preferences differ from the expected utility framework in that they are consistent with Allais type behavior, i.e. people will prefer p_1, a degenerate lottery which yields an amount m , to a lottery that yields an amount m' (much greater than m) say with probability 0.9 and 0 dollars with probability 0.1 , but will prefer a lottery p_4 which yields m' with probability 0.45 and 0 with probability 0.55 to a lottery p_3 which yields m with probability 0.5 and 0 with probability 0.5.[2]

Intuitively, many people will prefer a much smaller gain for sure to a small risk of getting nothing, yet when confronted with two almost equally risky prospects, they will choose the one that promises the much higher gain. In terms of security returns, the agent might settle for less return on the risk-free asset - an asset which gives one unit of consumption for sure, to avoid being disappointed with a stock, even if the latter promises a much higher return but runs a small chance of bringing nothing. This type of preferences appears therefore as a potentially relevant candidate to offer a solution to the risk-free rate puzzle.

Formally, the certainty equivalent function μ is defined implicitly by:

$$\int \phi_{DA}(x/\mu(p))dp(x) = 0 \tag{2.2}$$

where:

$$\phi_{DA}(x) = \begin{cases} v(x) - v(1) & x \geq 1 \\ A(v(x) - v(1)) & x \leq 1 \end{cases} \tag{2.3}$$

[2]This behavior violates expected utility maximization since under the latter $p_1 \succ p_2$ would imply $u(m) > 0.9u(m')+0.1u(0)$, while $p_4 \succ p_3$ would imply $0.5u(m)+0.5u(0) < 0.45u(m')+0.55u(0)$, hence a contradiction if we divide both sides of the last inequality by 0.5.

with:

$$v(x) = \begin{cases} (x^\alpha - 1)/\alpha & \alpha \neq 0 \\ \log(x) & \alpha = 0 \end{cases} \tag{2.4}$$

This certainty equivalent function μ is a special case of the Chew-Dekel mean value functional presented in Epstein and Zin (1989). It should be emphasized that this functional evaluates timeless wealth gambles in a non-expected utility fashion and therefore generalizes Kreps-Porteus (1978) preferences ($A = 1$) and the expected utility preferences ($A = 1, \alpha = \rho$). Epstein and Zin (1991) have used some members of the family of semi-weighted utility functions (Chew (1989)), among them disappointment aversion, in the context of an intertemporal asset pricing model to investigate the empirical relevance of such theories for explaining the relationship between consumption and asset returns. Intertemporal asset pricing models with expected utility or Kreps-Porteus preferences have fared poorly in explaining, for example, the first and second unconditional moments of real or excess stock returns.[3] They also produced marginal rates of substitution that did not pass the Hansen-Jagannathan (1991) test for the ratio of the standard deviation to the mean of the marginal rate of substitution. Epstein and Zin (1991) show also that the Hansen-Jagannathan bounds are satisfied for a large set of values of α, β, and ρ when A is smaller than one, that is when the agent exhibits first-order risk aversion [4] (captured by the parameter A).

There are N risky assets and one safe asset in the economy. Below are the first-order conditions for an interior maximum of the consumption and portfolio decisions as derived in Epstein and Zin (1989 and 1991b):

$$E_t\left[\Phi_{DA}(\tilde{z}_{t+1})\right] = 0 \tag{2.5}$$

$$E_t\left[I_A(\tilde{z}_{t+1})h(\tilde{z}_{t+1})\frac{\tilde{r}_{i,t+1}}{\tilde{M}_{t+1}}\right] = E_t\left[I_A(\tilde{z}_{t+1})\right] \qquad i = 1, \ldots, N \tag{2.6}$$

[3]Epstein and Zin (1990) show that a model based on Yaari's (1987) dual theory of choice is able to lower the volatility of the risk-free rate but still produces a low equity premium. These preferences share in common with semi-weighted utility functions the fact that they exhibit first-order risk aversion (see footnote 4).

[4]First-order risk aversion refers to the fact that the risk premium on a small gamble about certainty is proportional to the standard deviation of the gamble, and not to its variance as with second-order risk aversion. Segal and Spivak (1990) have used the term first-order risk aversion to characterize indifference curves (between consumption in two mutually exclusive states) that display a kink at the certainty line. This implies that an individual might not invest in a risky asset with a posit ive expected return if the latter is sufficiently small . On the contrary, individuals exhibit second-order risk aversion if their indifference curves are differentiable at the point of certainty (the intersection with the 45 degree line), which is typically the case when the individual maximizes expected util ity.

with $\tilde{z}_{t+1} = \beta^{1/\rho} \left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{\frac{\rho-1}{\rho}} \tilde{M}_{t+1}^{1/\rho}$(for $\rho \neq 0$); $\tilde{M}_{t+1}$ is the return on the market portfolio, which pays off C_t in period t, $\tilde{r}_{i,t+1}$ is the real gross return on the i^{th} asset, $I_A(x)$ is the indicator function with value 1 when $x < 1$ and value A when $x \geq 1$, and $h(x) = \frac{x^\alpha}{\alpha}$ for $\alpha \neq 0$, 1 otherwise. After replacing the various expressions in equations (2.5) and (2.6) rearranging, the following equation is obtained for the return on the market portfolio:

$$E_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{\tilde{C}_{t+1}}{C_t} \right)^{\frac{\alpha(\rho-1)}{\rho}} \tilde{M}_{t+1}^{\frac{\alpha}{\rho}} \right] \tag{2.7}$$

$$+(A-1)E_t \left[I_B \left(\beta^{\frac{\alpha}{\rho}} \left(\frac{\tilde{C}_{t+1}}{C_t} \right)^{\frac{\alpha(\rho-1)}{\rho}} \tilde{M}_{t+1}^{\frac{\alpha}{\rho}} - 1 \right) \right] = 1$$

where $I_B(x)$ equals 0 when x< 0 and x when x$\geq$ 0. The first line of (2.7) gives the Euler equation for the optimal consumption decision for Kreps-Porteus preferences (when A=1 and $\alpha \neq \rho$) or for expected utility (when $A = 1$ and $\alpha = \rho$). The Euler equation for the disappointment averse consumer (when $A \neq 1$) is obtained by adding the second line to the formula. Both for Kreps-Porteus and disappointment averse preferences, the intertemporal marginal rate of substitution depends on consumption growth as well as on the market portfolio return.

Similarly, for the equity return, we obtain the following equation:

$$E_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{\tilde{C}_{t+1}}{C_t} \right)^{\frac{\alpha(\rho-1)}{\rho}} \tilde{M}_{t+1}^{(\frac{\alpha}{\rho}-1)} \tilde{R}_{t+1}^e \right] \tag{2.8}$$

$$+(A-1)E_t \left[I_B \left(\beta^{\frac{\alpha}{\rho}} \left(\frac{\tilde{C}_{t+1}}{C_t} \right)^{\frac{\alpha(\rho-1)}{\rho}} \tilde{M}_{t+1}^{(\frac{\alpha}{\rho}-1)} \tilde{R}_{t+1}^e - 1 \right) \right] = 1$$

The difference with equation (2.7) is the presence of $\tilde{R}_{t+1}^e$, the return on equity, defined by:

$$\tilde{R}_{t+1}^e = \frac{P_{t+1}^e + D_{t+1}}{P_t^e}$$

In (2.9) below, the return on the risk-free asset is given by the payoff at time t+1, one unit of the consumption good, divided by the price at time t, P_t^f.

$$E_t\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{\frac{\alpha(\rho-1)}{\rho}}\tilde{M}_{t+1}^{(\frac{\alpha}{\rho}-1)}\frac{1}{P_t^f}\right] \tag{2.9}$$

$$+(A-1)E_t\left[I_B\left(\beta^{\frac{\alpha}{\rho}}\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{\frac{\alpha(\rho-1)}{\rho}}\tilde{M}_{t+1}^{(\frac{\alpha}{\rho}-1)}\frac{1}{P_t^f}-1\right)\right]=1$$

In this economy, the equity pays off a dividend D_t which is different from the aggregate payoff C_t on the market portfolio. We will therefore assume an exogenous joint process for consumption and dividends.

We postulate that the logarithms of consumption and dividends growth follow a bivariate process where both the means and the variances change according to a Markov variable S_t which takes the values $0, 1, \ldots, K-1$ (if K states of nature are assumed for the economy). The sequence $\{S_t\}$ of Markov variables evolves according to the following transition probability matrix P :

$$P=\begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0(k-1)} \\ p_{10} & p_{11} & \cdots & p_{1(k-1)} \\ \vdots & \vdots & \vdots & \vdots \\ p_{(k-1)0} & p_{(k-1)1} & \cdots & p_{(k-1)(k-1)} \end{bmatrix} \tag{2.10}$$

The bivariate consumption-dividends process can then be written as:

$$\begin{aligned} c_t - c_{t-1} &= \alpha_0^c + \alpha_1^c S_{1,t} + \cdots + \alpha_{k-1}^c S_{k-1,t} + (\omega_0^c + \omega_1^c S_{1,t} + \cdots + \omega_{k-1}^c S_{k-1,t})\epsilon_t^c \\ d_t - d_{t-1} &= \alpha_0^d + \alpha_1^d S_{1,t} + \cdots + \alpha_{k-1}^d S_{k-1,t} + (\omega_0^d + \omega_1^d S_{1,t} + \cdots + \omega_{k-1}^d S_{k-1,t})\epsilon_t^d \end{aligned} \tag{2.11}$$

where $S_{i,t}$ is a function of the state of the economy, S_t, taking the value 1 whenever $S_t = i$ and 0 otherwise; c_t and d_t are $\ln C_t$ and $\ln D_t$ respectively; ϵ_t^c and ϵ_t^d are $\aleph(0,1)$ error terms with correlation ρ_{cd}. Therefore, in state i, the means and standard deviations of the growth rates of consumption and dividends will be given respectively by $(\alpha_0^c + \alpha_i^c, \omega_0^c + \omega_i^c)$ and $(\alpha_0^d + \alpha_i^d, \omega_0^d + \omega_i^d)$.

The choice of such a process can be justified on various grounds. The reason for disentangling the consumption and dividend processes is first and foremost an empirical one: the series are very different in terms of mean, variance, and other moments. It seems therefore empirically sound to choose a model that does not impose the equality between consumption and dividends as the simple Lucas model does.

Tauchen (1986) proposes an extension to the Lucas asset pricing model where-by consumption is the payoff on the market portfolio (the sum of the payoffs of all assets), while the dividends only accrue to the owners of the stock. Following Abel (1992), the difference between aggregate consumption and aggregate dividends may be interpreted as labor income in an underlying model where randomness comes from technology shocks.

Another justification for the choice of a bivariate process is that a model based on either one of the series fails in explaining the observed features of the data. It is well-known that the simple Lucas asset pricing model calibrated to the series of consumption is unable to account for the large equity premium observed historically (Mehra and Prescott (1985), Weil (1989)). Regarding the apparent negative autocorrelation present in the series[5], Cecchetti, Lam, and Mark (1990) and Kandel and Stambaugh (1990) propose simple equilibrium asset pricing models that generate negative autocorrelation of the magnitude found in the data. Bonomo and Garcia (1993) have also shown however that their results rest on a misspecification of the Markov switching models chosen for the endowment. Once the proper specification is chosen, the negative autocorrelation effect disappears.[6]

Bonomo and Garcia (1991) use specification (2.11) for the joint consumption-dividends process to investigate if an equilibrium asset pricing model with isoelastic preferences can reproduce various features of the real and excess return series. Cecchetti, Lam, and Mark (1991) use a two-state homoskedastic specification of (11) for the endowment and similar preferences to try to match the first and second moments of the return series.[7] As we will see in section 4.3, both models fail to reproduce some moments of the risk-free rate or the excess returns, the homoskedastic specification failing especially to match the standard deviation of the risk-free rate.

Given the process specified by (2.11), equation (2.7) for the market portfolio can be rewritten as follows:

$$\sum_{l=0}^{K-1} p_{kl} \left[\left\{ \beta^{\frac{\alpha}{\rho}} \exp[\alpha m_l^c + \frac{\alpha^2}{2} \sigma_l^{c^2}] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}} \right\} + \right.$$

[5] Various authors have challenged this evidence. Kim, Nelson, and Startz (1991) argue that negative autocorrelation over long horizons is a pre-war phenomenon. Richardson (1988) challenges the statistical evidence of serial correlation.

[6] Ceccheti, Lam and Mark (1990) select a two-state Markov switching model with two means and one variance for the growth rate of the endowment process (represented by consumption or dividends), while Kandel and Stambaugh (1990) choose a four-state model with two means and two variances. Bonomo and Garcia (1993) show that, in the class of Markov switching models, the best model is a two-state model with one mean and two variances.

[7] The authors use two models, one with a leverage economy, another with a pure exchange economy without bonds. In both instances, they are unable to replicate the first and second moments taken together.

$$\int_{B(k,l)}^{\infty} (A-1) \left\{ \beta^{\frac{\alpha}{\rho}} \exp[\alpha m_l^c + \alpha \sigma_l^c \epsilon_{t+1}^c] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}} - 1 \right\} f(\epsilon^c) d\epsilon^c \Bigg] = 1 \tag{2.12}$$

for $k = 0, \ldots, K-1$. The parameters m_l^c and σ_l^c denote the mean and the standard deviation of consumption growth in state l, and $f(\epsilon^c)$ is the density function of ϵ_{t+1}^c, which is assumed to be normal. The λs are the ratios of the market portfolio price to consumption (the payoff of the market portfolio) in the various states. Finally:

$$B_{(k,l)} = \frac{1}{\sigma_l^c} \left(-\frac{1}{\rho} \log \beta - m_l^c - \log \frac{\lambda(l)+1}{\lambda(k)} \right)$$

For the equity equation (2.8), we obtain[8]:

$$\sum_{l=0}^{K-1} p_{kl} \Bigg[\{ \beta^{\frac{\alpha}{\rho}} \exp[\mu_o + \mu_l \mathbf{1}(l>0)] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}-1} \frac{\varphi(l)+1}{\varphi(k)} \}$$

$$+ \int_{B(k,l)}^{\infty} (A-1) \{ \beta^{\frac{\alpha}{\rho}} \exp[(\alpha-1) m_l^c + m_l^d + (\alpha-1) \sigma_c^l \epsilon_{t+1}^c] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}-1} .$$

$$\frac{\varphi(l)+1}{\varphi(k)} . \exp[\rho_{cd} \sigma_l^d \epsilon_{t+1}^c + \frac{1}{2}(1-\rho_{cd}^2) \sigma_l^{d^2}] - 1 \} f(\epsilon^c) d\epsilon^c \Bigg] = 1 \tag{2.13}$$

where:

$\mu_o = (\alpha-1)\alpha_o^c + \alpha_o^d + \frac{1}{2} \left[(\alpha-1)^2 \omega_o^{c^2} + \omega_o^{d^2} + 2(\alpha-1)\omega_o^c \omega_o^d \rho_{cd} \right]$

$\mu_j = (\alpha-1)\alpha_j^c + \alpha_j^d + \frac{1}{2}[(\alpha-1)^2 \omega_j^{c^2} + \omega_j^{d^2} + 2(\alpha-1)(\omega_o^c \omega_j^d + \omega_j^c \omega_o^d +$
$\omega_j^c \omega_j^d) \rho_{cd} + 2(\alpha-1)^2 \omega_o^c \omega_j^c + 2\omega_o^d \omega_j^d]$

$\mathbf{1}(l>0)$ is an indicator function taking value 1 when l is greater than zero and 0 otherwise, and $\varphi(l)$ is the price-dividend ratio for the stock. Finally, the equation for the risk-free rate is given by :

$$\sum_{l=0}^{K-1} p_{kl} \Bigg[\left\{ \beta^{\frac{\alpha}{\rho}} \exp[(\alpha-1) m_l^c + \frac{(\alpha-1)^2}{2} \sigma_l^{c^2}] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}-1} (P^f(k))^{-1} \right\}$$

$$+ \int_{B(k,l)}^{\infty} (A-1) \left\{ \beta^{\frac{\alpha}{\rho}} \exp[(\alpha-1) m_l^c + (\alpha-1) \sigma_l^c \epsilon_t^c] \left(\frac{\lambda(l)+1}{\lambda(k)} \right)^{\frac{\alpha}{\rho}-1} (P^f(k))^{-1} - 1 \right\} f \epsilon^c) d\epsilon^c \Bigg] = 1 \tag{2.14}$$

[8] In this equation, we integrate out ϵ_{t+1}^d to avoid keeping a double integral.

Given estimates of the endowment process parameters, equations (2.12), (2.13), and (2.14) can be solved numerically for the λ, φ, and P^f functions. The return formulas for the equity return and the safe asset return will finally be given by:

$$R^e_{t+1} = \frac{\varphi(S_{t+1}) + 1}{\varphi(S_t)} \exp(\alpha^d_0 + \cdots + \alpha^d_{k-1} S_{k-1,t+1} + (\omega^d_0 + \cdots + \omega^d_{k-1} S_{k-1,t+1})\epsilon^d_{t+1}) \tag{2.15}$$

$$R^f_{t+1} = \frac{1}{P^f_t(S_t)} \tag{2.16}$$

In the next section, we estimate the parameters for the joint consumption-dividend process by maximum likelihood.

3. Maximum Likelihood Estimation of the Joint Consumption-Dividend Process Parameters

Since Mehra and Prescott (1985), most equilibrium models attempting to solve the equity premium puzzle were based on the equality of consumption and dividends in equilibrium, since dividends were the total payoff on the market portfolio and were equal to consumption because of the single good non-storability assumption. We have explained in the previous section why this assumption is neither appropriate nor necessary, and have proposed a joint Markov switching model for the growth rates of consumption and dividends. The Markov structure is useful not only because it offers closed-form solutions for the asset returns, but mainly because it fits better the skewness and kurtosis present in the series. Table 1 reports the estimation results of the joint random walk and of the two-state Markov switching model. Judging by the large increase in the likelihood function value (more than 40), it seems that the null of a random walk is overwhelmingly rejected for these series, but the standard $\chi^2(1)$ is not the appropriate asymptotic distribution in this context to judge the significance of the likelihood ratio. This problem is by now well-known in the Markov switching literature and is due to the fact that, under the null hypothesis, some parameters are not identified and the rank condition for the information matrix is violated. Hansen (1991) and Garcia (1992) have recently addressed these problems. The first bounds the asymptotic distribution of a standardized likelihood ratio statistic under these two non-standard conditions, while the second derives analytically the asymptotic null distribution of the likelihood ratio for two-state Markov switching models. Garcia (1992) reports the critical values for the likelihood ratio statistic for a null hypothesis of a one-state model against various two-state alternative hypotheses. For example, the 5% critical value for a null of a linear model against a heteroskedastic (two means and two variances) Markov switching model is 14.11. Even if this value is not directly applicable here since it is derived in a univariate context, the value of 84.5 obtained for the likelihood ratio statistic should make us confident about rejecting the null of a random walk.

Looking at the estimates for the two-state model in table 1, we notice that the second state is mainly characterized by a high volatility of both consumption and dividend growth. This state is mainly present before 1950, but there are some short lapses of the good state before the Second World War.

In the next step, we estimate the three-state Markov switching model. The results are also presented in Table 1. The main difference with the two-state model is the presence of a new state (state 2) with intermediate volatility of consumption growth and a low mean of dividend growth. The presence of this new state adds a lot of dynamics before the Second World War, as the transition probabilities between state 1 and 2 indicate. After the mid-fifties, the series stay mainly in state 0, the good state, as in the two-state model. The value of the likelihood ratio statistic between the two-state model and the three-state model is 14.5. Since no critical values are available for such a statistic, it is difficult to judge if the two-state model should be rejected in favor of the three-state model. However, even if this full three-state model does not pass the test, the rejection of a constrained version of it, with $\alpha_2^c = 0$ and $\omega_2^d = 0$, should be much less likely. Since the estimates of α_2^c and ω_2^d are small in magnitude, the constrained and unconstrained versions of the model should produce comparable results. We chose to use the unrestricted model of the endowment process to assess the ability of the equilibrium asset pricing model to reproduce various statistics of the return series in the next section.

4. Assessment of the Equilibrium Asset Pricing Model

In this section, we want to address various issues that are currently unresolved. First, we want to see if the model is capable of resolving both the equity premium puzzle and the risk-free puzzle at the same time. That is we want to reproduce the level of the equity premium (5.28% in our sample) while maintaining the risk-free rate close to its historical value (2.12%). We want also to reproduce the second moments of the equity premium and the risk-free rate series since the solutions to the equity premium puzzle have fared poorly in this respect (Constantinides (1991) with habit formation, and Abel (1992) with "Catching up with the Joneses"[9]). The model proposed by Epstein and Zin (1990), based on first-order risk aversion preferences, could reproduce the second moments but not the equity premium mean. We finally want to see if the model is able to replicate the forecastability of the future multi-period excess returns by the dividend yield, an empirical fact originally put forward by Fama and French (1988) and shown to be statistically significant by Nelson and Kim (1991) and Hodrick (1991).[10] This evidence has been interpreted by Fama and French (1988) as support for a cyclical behavior of expected returns. More generally, one can interpret this result, combined with the little variability of the risk-free rate, as time-varying risk premia. To date, no equilibrium model has been able to reproduce such forecastability, leaving the proponents of an inefficient market explanation unchallenged.

4.1. The Unconditional Moments

Given the endowment process parameter estimates obtained by maximum likelihood for the three-state model, we can compute the unconditional moments of the stock and risk-free asset return series by taking the unconditional expectation of the return formulas in (2.15) and (2.16). The formulas for the various moments are given in the appendix.

Table 2 reports the first and second moments of the equity premium and risk-free rate series estimated from our sample (1889-1987 at annual frequency) and produced by the model for various combinations of the utility function parameters. We have selected the values that best reproduced the various moments. In these disappointment aversion preferences, both A and α determine the level of risk aversion. To increase risk aversion, one can lower α or A, but by lowering A it is especially the risk aversion towards small gambles that is increased. It is really this specificity that sets apart the disappointment aversion preferences from the regular time-additive preferences. We will elaborate more on this aspect when we will discuss the reasonableness of these parameter values in section 4.4.

The sample moments have been estimated by a generalized method of moments and robust standard errors are reported in parentheses. Looking first at the mean of the equity premium, we see that for all selected com-

[9] Under habit formation, the representative agent derives its utility from its consumption relative to last period's individual consumption. For catching up with the Joneses, utility is based on consumption relative to last period 's aggregate consumption.

[10] In a recent paper however, Goetzmann and Jorion (1992) fail to reject the null hypothesis that future returns are unrelated to past dividend yields at conventional significance levels. The difference with previous studies is that they use a bootstrap methodology to generate returns instead of Monte-Carlo simulations.

binations of A, α,and ρ values [11] the model is able to reproduce the equity premium mean within one standard error. The mean of the safe asset return tends to be overestimated but stays within two standard errors. To test for the equality of the estimated means and the means produced by the model, we can compute the following statistic:

$$(\mu_a - \mu_m)' \sum^{-1}(\mu_a - \mu_m)$$

where μ_a are the moments estimated from the data, μ_m are the moments generated by the model, and Σ is the variance-covariance matrix of the estimates μ_a. Assuming asymptotic normality for the mean estimates, this statistic is distributed as a $\chi^2(2)$. For all selected sets of parameter values, one cannot reject the equality between the estimated sample means and the means generated by the model. These sets of values therefore solve simultaneously the risk premium and the risk-free puzzles.

The values produced for the standard deviations tend to be lower than the actuals and are not within two standard errors of the estimated values. Given the values obtained for the $\chi^2(4)$ statistic, taking into account this time both the means and the standard deviations of the equity premium and the risk-free rate, we fail however to reject at the 1% level, for $A = 0.2, \alpha = -6$,and $\rho = -1$, that the moments produced by the model are equal to the moments computed from the data. We are close to the non-rejection at this level for the other sets. It should be emphasized that these tests account only for the uncertainty present in estimating the sample moments, but not for the uncertainty involved in estimating the parameters of the joint consumption-dividend process. Taking into account both sources of uncertainty, we could probably increase the probability of accepting the null hypothesis that the actual moments were generated by our model. For that purpose, we would need to estimate jointly the sample moments and the parameters of the endowment. Cecchetti, Lam, and Mark (1993) use a GMM method to carry out such a joint estimation, but its implementation in the context of the three-state model chosen for the endowment is quite requiring.

We finally add as a fifth moment the correlation between the equity premium and the risk-free rate. In the data this correlation is close to zero and not significantly different from it, but the model tends to estimate a negative correlation in the order of -0.27. Although this value is within two standard errors of the estimated value, the $\chi^2(5)$ tests carried out with the five moments reject the equality of all five moments for the four sets of parameter values.

[11] For all models, we have used 0.97 as a value for β.

4.2. Predicting Future Excess Returns by Past Dividend Yields

Regressing one- to four-year returns on the dividend-price ratio, Fama and French (1988) found evidence of a predictable component for long-horizon returns. Table 3 confirms their results for our sample period (1871-1987) with excess returns. The actuals in the first column show that the regression coefficients increase with the horizon, but slightly less than in proportion, and that both the t-values and the R^2 also increase with time. Fama and French (1988) have provided a rationale for the fact that the coefficients do not increase in proportion with the horizon. Since multiperiod returns are cumulative sums of one-period returns, it indicates that the dividend yield does not predict as much variation in the distant one-period expected returns, an indication of slow mean reversion in short term expected returns. This slow mean reversion means that short term expected returns are persistent and therefore that the variance of multiperiod expected returns grows more than in proportion with the return horizon. Since the variance of the regression residuals grows much less with time, it explains why the forecasting power increases with the horizon.

To perform the same regressions in the context of our model, we generate return and dividend yield series from our artificial economy in the following way. Given a randomly drawn vector of $\aleph(0,1)$ errors ϵ^d_{t+1} and a randomly drawn vector of $S_{0,t}$, $S_{1,t}$, and $S_{2,t}$ according to the transition probabilities estimated in Section 3, we generate series of excess returns according to formulas (2.15) and (2.16) with the estimates obtained in Section 3 for the α^c, ω^c, α^d, ω^d,and ρ^{cd} parameters. For the dividend-price ratio series, it should be noted that if we assumed that the agent knows the state at time t, the model would give us only three values for the dividend-price ratio. To obtain a continuous variable for the latter, we therefore assume that the state is not directly observable and allow the agent to make an optimal inference about the probabilities of states 0, 1, and 2 at time t given his information up to time t and the values of the parameters of the model. The inferred probabilities are what is called in the Markov switching literature the filter probabilities.[12] We therefore obtain the continuous price-dividend series by weighing the three values of $\varphi(S_t)$ by these filter probabilities. We repeat the procedure a 1,000 times and run each time five regressions for the 1 to 5 year future multiperiod excess returns on current dividend yields. The medians of the distributions so obtained for the regression coefficients, the t-statistics and the R^2 are reported in Table 4 for the set of parameter values that produces the best results. We can see that, apart from the magnitude of the coefficients, all the features present in the actual data

[12] For a description of these filter probabilities and their computation, see Hamilton (1989).

statistics are reproduced by the model. All the coefficients are positive and grow with the return horizon, as does the forecasting power.

In Bonomo and Garcia (1991), with the same model for the endowment but with time-additive isoelastic preferences, we could not reproduce at all these predictability patterns. Disappointment aversion seems therefore to be a determining factor in the forecastability of future excess returns by past dividend yields.

4.3. The Respective Role of the Heteroskedastic Markov Switching Endowment and of Disappointment Aversion Preferences

To disentangle the effects of the disappointment aversion preferences from the effects of the heteroskedastic Markov switching endowment in our results, we will keep in turn either the endowment or the preferences fixed and vary the other.

We will first keep the joint heteroskedastic consumption-dividend three-state Markov switching specification and use as preferences constrained versions of our general preference specification. We start by setting A equal to 1 and α equal to ρ. We are therefore in the isoelastic time-additive utility case investigated among others by Cecchetti, Lam and Mark (1993). We show in Table 4 the best results we could obtain in terms of matching the means and variances of the equity premium and the risk-free rate. For $\alpha = -8$ and $\beta = 0.98$, we see that the mean of the safe asset is too high and the mean of the equity premium is too low. These well-known results constitute precisely the two puzzles we are addressing. However, we are better able to match the standard deviations. By comparison with the results reported by Cecchetti, Lam and Mark (1993)[13] for a homoskedastic endowment process and comparable parameter values, this matching of the second moments is solely due to our heteroskedastic specification. Lowering α will not in our specification improve the results since rapidly we do not obtain anymore a solution for the price-dividend ratio. With the two-state homoskedastic specification of Cecchetti, Lam and Mark (1993), lowering α increases the standard deviation of the risk-free rate. This result illustrates the interplay that takes place between the choice of the endowment process specification and the values of the preference parameters that will achieve the matching. A misspecification of the endowment process might translate into a higher than needed risk aversion parameter. Finally, as shown also in other studies, increasing β over 1 will lower the mean of the risk-free rate but will leave intact the equity premium puzzle. As shown in Table 5, a value of β of 1.08 will give a mean of 2.62 % for the risk-free asset.

[13] In their Table 3.

Setting A to 1 and not restricting α to be equal to ρ, we obtain the Kreps-Porteus preferences. The results are presented in the right panel of Table 4. To match the first moments, we have to increase the coefficient of relative risk aversion to 28 ($\alpha = -28$), similarly to Kandel and Stambaugh (1991) in an asset pricing model with levered equity and a calibrated four-state Markov endowment involving only consumption. For a ρ of -1 (i.e. an intertemporal elasticity of substitution of 0.5) we also obtain the magnitude of the standard deviations for both the equity premium (13.70 %) and the risk-free asset (4.84 %). Comparatively, for the same values of the parameters, Kandel and Stambaugh (1991) obtain respectively 6.67 % and 8.06 %. To match the second moments, the last authors need to lower the elasticity of intertemporal substitution to a very low value in the order of 0.05. Moreover, their matching is achieved by assuming too high a leverage ratio - the ratio of debt to the market value of the firm- compared to what has been observed in the last century. We can therefore conclude that allowing for a high risk aversion as Kandel and Stambaugh (1991) do, we are able to match the first and second moments of the equity premium and the risk-free rate in a model with no such counterfactual assumption. The estimated joint consumption-dividend heteroskedastic Markov switching in our model is sufficient to achieve the same results.

To see the importance of this specification of the endowment process in the case of disappointment aversion preferences, we will compute the moments for the four sets of parameter values selected in Table 2 with restricted versions of the general bivariate model (11) for consumption and dividend growth. Table 5 presents the results for four processes: a random walk for consumption growth alone, a one-mean two-variance two-state Markov switching model for consumption growth alone[14], a joint random walk for consumption and dividend growth, and finally a joint homoskedastic two-state model for consumption and dividend which is the process selected by Cecchetti, Lam and Mark (1993).

With the simplest model, the random walk for consumption, we achieve an equity premium of at most 1.9 % with a standard deviation of 4%, which is quite far from the actual values. The mean of the risk-free rate is much lower than in a time-additive utility framework, but still in the order of 4 %. The risk-free rate being deterministic in this case, its variance is zero. By allowing the variance of consumption growth to differ in two states, we increase by about one percent the equity premium, while not changing much the mean of the risk-free rate and the standard deviation of the equity premium. The standard deviation of the risk-free rate is too low at about 2.5%.

[14]Bonomo and Garcia (1993) have shown that this is the best univariate specification in the class of Markov switching models.

For the joint random walk, we maintain the equity premium mean at about 2.5% but we increase its standard deviation to 12.8%, a sizable improvement. The mean of the risk-free rate is still in the order of 4% . Finally, for the joint two-state process estimated by Cecchetti, Lam and Mark (1993), we increase the equity premium mean to about 7% and its standard deviation to 14%, while we lower the risk-free rate mean to about 3%. The main shortcoming is the severe underestimation of the standard deviation of the risk-free rate. These results show that the joint heteroskedastic specification selected as the best in the class of bivariate Markov switching models for consumption and dividends is essential for matching the means and the variances of the equity premium and the risk-free rate. Disappointment aversion alone is not enough.

4.4. Reasonableness of the Selected Parameters in terms of Risk Aversion

With expected utility preferences, we are used to judge the reasonableness of the model by the magnitude of the coefficient of relative risk aversion α. Although Kandel and Stambaugh (1990) argued forcefully that a value of α of -29 should not be seen as excessive, it is common wisdom since Mehra and Prescott (1985) to limit α to a value of less than ten. For disappointment averse preferences, it is harder to form a judgment since there is another parameter A which affects the risk aversion of the individual. To give an idea of the risk aversion entailed in the parameter values selected to match the moments, we follow Epstein and Zin (1991b) and report in Table 6 the willingness to pay for various configurations of preferences given a simple gamble. Assuming a level of wealth of 75000\$, we compute the amount an individual endowed with these preferences is willing to pay to avoid the gamble. For small gambles, the disappointment averse individual is willing to pay much more than an expected utility maximizer, but as the size of the gamble increases the magnitudes of the willingness to pay of the two types of agents tend to move closer together. The parameters chosen for the disappointment aversion preferences in our model tend to place the risk aversion at a level between a α of -9 and a α of -29 for expected utility preferences. This is certainly not a small level of risk aversion, but the disappointment aversion preferences give more reasonable amounts for the willingness to pay to avoid small gambles. As mentioned by Epstein and Zin (1990), the gamble $\epsilon = 2,500$ has a coefficient of variation close to that of the U.S. per capita consumption growth rate series used in our study and is therefore the gamble that matters for the equity premium and the risk-free rate puzzles.

5. Conclusion

The results presented in this paper bring some evidence in favor of disappointment aversion as a characterization of attitudes towards risk. When these preferences are coupled with an endowment where bad states of nature can happen with probabilities estimated from the historical series of consumption and dividends over the last century, we can solve both the equity premium and the risk-free rate puzzles and reproduce closely the variability of the equity premium and the risk-free rate series. We are also able to predict future excess returns at long horizons by past dividend yields, as observed in the data.

These results show the importance of the interplay between the endowment process and the preferences to arrive at a model that reproduces the features observed in the excess returns series. In Bonomo and Garcia (1991), the same joint heteroskedastic process for consumption and dividends, coupled with time-separable isoelastic preferences, was shown to reproduce the features of real returns but not of excess returns. Based on a model which also distinguishes consumption from dividends and assumes time separable preferences, Abel (1992) concludes that a Markov switching structure for the endowment exacerbates the equity premium puzzle. Our results show that his conclusion is specific to both his information assumption in the endowment specification[15] and to time-separable preferences.

Abandoning the time separability of preferences and the equality between the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution that they entail, Kandel and Stambaugh (1990) could, with a high coefficient of relative risk aversion and a counterfactually high leverage ratio, reproduce the first and second moments of the excess returns. Our joint heteroskedastic process for consumption and dividends coupled with the same preferences can also reproduce these moments in an unlevered economy. This result highlights that a realistic specification of the endowment process helps in explaining the actual pattern of returns. The fact that with these preferences we could not reproduce the dynamics of excess returns, as captured by the forecastability of future excess returns by the current dividend yield, stands as additional evidence in favor of disappointment averse preferences.

Finally, we showed that disappointment aversion or, more generally, first-order risk aversion alone cannot reproduce the magnitude of the equity premium and that the heteroskedastic endowment is essential to achieve the matching.

Our results bring therefore a better understanding of the interaction

[15] In Abel's (1992) model, the agent knows in time t the dividend and consumption in t+1, except for the realization in t+1 of a mean-zero i.i.d. term.

between the characterization of the endowment process and the specification of preferences to successfully explain the behavior of excess returns. However, our results are not entirely satisfactory since the risk aversion implicit in our model is high. This may suggest that another line of investigation, as the heterogeneity of agents in an incomplete markets framework, could be fruitful. Constantinides and Duffie (1992) have shown that heterogeneity in the form of uninsurable, persistent, and heteroskedastic labor income shocks can resolve the empirical problems of representative agent models. Future research based on micro studies should therefore investigate the possible explanations for the different agent behaviors observed empirically and see to what extent they account for the seemingly puzzling aggregate stylized facts.

Appendix

Derivation of Formulas for the Unconditional Moments

First moment of the equity return:

$$E[R_e] = \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \exp[\alpha_0^d + \frac{\omega_0^{d^2}}{2}] \pi_i p_{ij} f_{ij}^1$$

where: $\pi_i = \frac{C_{ii}}{\sum_{j=0}^{K-1} C_{jj}}$,

C_{ii} being the cofactor of the i, i element of the transition probability matrix P;

and

$$f_{00}^1 = 1$$

$$f_{ij}^1 = \frac{\varphi(j)+1}{\varphi(i)} \exp[\alpha_j^d + \omega_0^d \omega_j^d + \frac{\omega_j^{d^2}}{2}]$$

Second moment of the equity return:

$$E[R_e^2] = \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \exp[2(\alpha_0^d + \omega_0^{d^2})] \pi_i p_{ij} (f_{ij}^2)^2$$

$$f_{00}^2 = 1$$

$$f_{ij}^2 = \frac{\varphi(j)+1}{\varphi(i)} \exp[\alpha_j^d + 2\omega_0^d \omega_j^d + \omega_j^{d^2}]$$

First and second moments of the risk-free rate:

$$E[R_f] = \sum_{i=0}^{K-1} \pi_i \frac{1}{P_f(i)}$$

$$E[R_f^2] = \sum_{i=0}^{K-1} \pi_i \frac{1}{P_f(i)^2}$$

Covariance of the equity return with the risk-free rate:

$$E[R_e R_f] = \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \exp[\alpha_0^d + \frac{\omega_0^{d^2}}{2}] \pi_i p_{ij} f_{ij}^{ef}$$

with:

$$f_{00}^{ef} = 1$$

$$f_{ij}^{ef} = \frac{\varphi(j)+1}{\varphi(i)} \frac{1}{P_f(i)} \exp[\alpha_j^d + \omega_0^d \omega_j^d + \frac{\omega_j^{d^2}}{2}]$$

All the moments calculated in the paper can be derived from this set of formulas.

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TABLE 1
Estimation Results for the Joint
Consumption-Dividend Models (1889-1987)

	Random Walk		Two-State Markov Switching Model		Three-State Markov Switching Model	
	Coeff. Estimate	Stand. Error	Coeff. Estimate	Stand. Error	Coeff. Estimate	Stand. Error
α^c_0	0.0182	0.0039	0.0206	0.0026	0.0210	0.0027
α^c_1	-	-	-0.0041	0.0069	-0.0123	0.0113
α^c_2	-	-	-	-	0.0022	0.0106
ω^c_0	0.0371	0.0027	0.0159	0.0023	0.0107	0.0019
ω^c_1	-	-	0.0341	0.0055	0.0310	0.0069
ω^c_2	-	-	-	-	0.0274	0.0074
α^d_0	-0.0018	0.0124	-0.0039	0.0060	-0.0029	0.0061
α^d_1	-	-	0.0035	0.0236	-0.0287	0.0454
α^d_2	-	-	-	-	0.0272	0.0148
ω^d_0	0.1196	0.0088	0.0377	0.0041	0.0388	0.0044
ω^d_1	-	-	0.1167	0.0159	0.1685	0.0330
ω^d_2	-	-	-	-	0.0166	0.0146
p_{01}	-	-	-	-	0.0000	0.0000
p_{02}	-	-	-	-	0.0309	0.0312
p_{11}	-	-	0.9660	0.0370	0.5374	0.1436
p_{12}	-	-	-	-	0.4202	0.1421
p_{21}	-	-	-	-	0.3753	0.1738
p_{22}	-	-	0.9548	0.0467	0.6247	0.1737
ρ_{cd}	0.4407	0.0840	0.4858	0.0851	0.4947	0.0838
L	443.31		485.55		492.82	

TABLE 2
First and Second Moments of Equity Premium and Treasury Bill Rates

		Actuals (in %)	A=0.2 α=-6 ρ=-1	A=0.35 α=-9 ρ=-1	A=0.2 α=-9 ρ=-0.5	A=0.28 α=-8 ρ=-1
Mean Equity Premium		5.28 (1.40)	5.22	4.84	5.85	5.28
Mean Safe Asset		2.12 (0.87)	3.49	3.88	3.48	3.78
Std. deviation Equity Premium		18.43 (1.81)	13.60	13.23	13.44	13.40
Std. deviation Safe Asset		5.75 (0.74)	3.39	3.10	2.29	3.21
Correlation Equity Premium and Safe Asset		-0.065 (0.16)	-0.32	-0.28	-0.28	-0.30
Chi Square Tests	$\chi^2(2)$	-	3.10	4.62	4.10	4.69
	$\chi^2(4)$	-	12.88	15.77	17.38	15.32
	$\chi^2(5)$	-	25.49	22.42	28.16	24.63

Notes: 1. Estimated by GMM over the period 1889-1987 with robust standard errors in parentheses.
2. One years Treasury Bills.
3. The 1% critical values for the $\chi^2(2)$, $\chi^2(4)$, $\chi^2(5)$ are 9.21, 13.28, and 15.09 respectively.

TABLE 3
Simulated statistics on the Forecastability
of Future Excess Returns by Current Dividend Yields
Regression: $R_{t,t+k}=\alpha+\beta(D_t/P_t)+u_{t,t+k}$

Median of Distribution of various regression statistics

	Actual			A=0.2 α=-9 ρ=-0.5		
k	Coef.	t	R^2	Coef.	t	R^2
1	3.40	2.29	0.04	7.12	2.53	0.05
2	6.49	3.05	0.08	13.42	3.31	0.09
3	8.12	3.31	0.09	19.17	3.96	0.12
4	10.75	3.87	0.12	24.18	4.32	0.15
5	13.88	4.69	0.17	28.81	4.61	0.16

TABLE 4
Comparison with Other Preference Specifications
for the Same Joint Heteroskedastic Three-State
Consumption-Dividend Markov Switching Model

	Isoelastic Time-Additive Utility		Kreps-Porteus Preferences
	α=-8 β=0.98	α=-8 β=1.08	α=-28 ρ=-1
Mean Equity Premium	1.72	1.52	6.50
Mean Safe Asset	13.09	2.62	1.13
Standard Deviation Equity Premium	15.40	21.98	13.70
Standard Deviation Safe Asset	5.14	4.67	4.84

TABLE 5
Comparison with Other Endowment Models
for the Same Disappointment Aversion Preferences

Panel A - A=0.2 α=-6 ρ=-1				
	Random Walk Consumption	Random Walk Consumption-Dividends	Two-Variance, One-Mean Markov Switching Model for Consumption	Joint Two-State Homoskedastic Consumption-Dividend Markov Switching Model
Mean Equity Premium	1.72	2.26	2.96	6.85
Mean Safe Asset	4.33	4.49	4.47	3.59
Standard Deviation Equity Premium	4.02	12.82	4.76	13.81
Standard Deviation Safe Asset	--	--	2.74	1.08

Panel B - A=0.35 α=-9 ρ=-1				
	Random Walk Consumption	Random Walk Consumption-Dividends	Two-Variance, One-Mean Markov Switching Model for Consumption	Joint Two-State Homoskedastic Consumption-Dividend Markov Switching Model
Mean Equity Premium	1.91	2.56	3.25	7.11
Mean Safe Asset	4.15	4.27	4.41	3.28
Standard Deviation Equity Premium	4.02	12.83	4.85	13.58
Standard Deviation Safe Asset	--	--	2.73	0.75

TABLE 5 (Continued)
Comparison with Other Endowment Models
for the Same Disappointment Aversion Preferences

Panel C - A=0.2 α=-9 ρ=-0.5				
	Random Walk Consumption	Random Walk Consumption-Dividends	Two-Variance, One-Mean Markov Switching Model for Consumption	Joint Two-State Homoskedastic Consumption-Dividend Markov Switching Model
Mean Equity Premium	1.68	2.26	2.35	7.84
Mean Safe Asset	3.98	4.05	4.55	2.88
Standard Deviation Equity Premium	4.00	12.76	4.64	13.82
Standard Deviation Safe Asset	--	--	2.60	0.61

Panel D - A=0.28 α=-8 ρ=-1				
	Random Walk Consumption	Random Walk Consumption-Dividends	Two-Variance, One-Mean Markov Switching Model for Consumption	Joint Two-State Homoskedastic Consumption-Dividend Markov Switching Model
Mean Equity Premium	1.87	2.49	2.88	7.57
Mean Safe Asset	4.18	4.31	4.02	3.26
Standard Deviation Equity Premium	4.02	12.82	4.51	13.77
Standard Deviation Safe Asset	--	--	1.87	0.87

TABLE 6
Willingness-to-Pay for Different Risk Preferences

ϵ	μ_{EU}^{α} (α=-1)	μ_{EU}^{α} (α=-9)	μ_{EU}^{α} (α=-29)	$\mu_{DA}^{A,\alpha}$ (A=0.20) (α=-6)	$\mu_{DA}^{A,\alpha}$ (A=0.35) (α=-9)	$\mu_{DA}^{A,\alpha}$ (A=0.20) (α=-9)	$\mu_{DA}^{A,\alpha}$ (A=0.28) (α=-8)
250	1	4	12	168	124	169	143
2500	83	410	1091	1814	1491	1867	1636
25000	8333	21009	23791	23484	23309	23979	23440
40000	21333	37198	39153	38921	38813	39284	38903
50000	33333	47999	49395	49229	49152	49488	49217
60000	48000	58799	59637	59537	59491	59693	59530
74000	73013	73920	73976	73969	73966	73980	73969