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Nonparametric Tail Risk, Stock Returns and the Macroeconomy*

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Résumé/abstract

This paper introduces a new tail risk measure based on the risk-neutral excess expected shortfall of a cross-section of stock returns. We propose a novel way to risk neutralize the returns without relying on option price information. Empirically, we illustrate our methodology by estimating a tail risk measure over a long historical period based on a set of size and book-to-market portfolios. We find that a risk premium is associated with long-short strategies with portfolio sorts based on tail risk sensitivities of individual securities. Our tail risk index also provides meaningful information about future market returns and aggregate macroeconomic conditions. Results are robust to the cross-sectional information selected to compute the tail risk measure.

Mots clés/keywords : Tail Risk, Risk Factor, Risk-Neutral Probability, Prediction of Market Returns, Economic Predictability

Codes JEL/JEL Codes : G12, G13, G17

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1 Introduction

Following the 2007-2009 financial crisis, financial researchers as well as regulators devoted a lot of attention to the measure and analysis of tail risk and systemic risk and to the economic consequences of such risks. An important objective of this research agenda is to develop measures that are able to capture a common risk to many assets or institutions as opposed to individual risk measures such as VaR for one particular portfolio or firm. While systemic risk focuses on the financial institutions, tail risk looks more generally at the risk provoked by catastrophic events or disasters and affects all firms. It could be measured with the time series of one firm or one portfolio but rare occurrences of disasters makes this estimation at best very imprecise. Therefore, the most recent approach to measuring tail risk has been to exploit the richness of the cross section of returns. By proposing a power law model for the left tail of asset returns, Kelly and Jiang (2014) obtain a new measure of time-varying tail risk that captures common fluctuations in tail risk among individual stocks. Allen et al. (2012) measure systemic risk through the 1% VaR of several tail distributions of the cross section of returns of financial firms. The main additional advantage of these measures is the availability of long historical data for equity returns at the daily frequency. Both studies use the fact that tail risk plays an important role in explaining risk premia of equities and contains valuable information for predicting future economic conditions. One shortcoming is that this measure of tail risk is extracted directly from raw returns without risk neutralization, that is an adjustment of statistical returns for changes in risk aversion or time preferences or other changes in economic valuations.

Other measures of tail risk have sought for additional information by using prices in options markets. Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000) introduced an option-based methodology to recover the risk neutral densities for the S&P 500 index. In these papers, the authors stress that risk neutral information coming from option prices provides an additional, economically relevant, source of information to estimate tail risk, in their case value-at-risk. Recently, Bollerslev and Todorov (2011) proposed a model-free index of investors' fear adopting a database of intraday prices of futures and the cross section of S&P 500 options. They show that tail risk premia account for a large fraction of expected equity and variance risk premia as measured by the S&P 500 aggregate portfolio. In a complementary work, Bollerslev et al. (2015) provide empirical evidence that the jump risk component of the variance risk premium is a strong predictor of future market returns. Still relying on S&P 500 index options, based on its empirical distribution, Bali et al. (2011) propose risk neutral and objective measures of riskiness, and show that they are good predictors of future market returns. Siriwardane (2013) proposes another interesting method to estimate the risk of extreme events. He extracts daily measures of market-wide disaster risk from a cross-section of equity option portfolios (in particular from puts) using a large number of firms, and show that these measures are useful to predict business cycle variables and to construct profitable portfolios sorted by disaster risk. A major limitation of these studies is the short time span for the availability of option and high-frequency data, in the order of 20 years, especially when the focus is on rare events. Moreover several markets around the world do not have well-established and liquid option markets.

We propose an approach based on a cross-section of portfolio returns, together with a risk neutralization. Our aim is to develop a simple measure based on easily available data with a long history and applicable to a vast set of asset markets. This disqualifies option prices used in the previously cited studies. However we want to reinterpret the insight provided in Ait-Sahalia and Lo (2000) by eliciting state-price densities with a nonparametric methodology developed in Almeida and Garcia (2016). Based on a family of discrepancy functions, they derive nonparametric stochastic discount factor (SDF) bounds that naturally generalize variance (Hansen and Jagannathan (1991)), entropy (Backus et al. (2014)), and higher-moment (Snow (1991)) bounds. Implicit in these derivations is the computation of the SDF itself that provides the risk neutralization. Therefore our new tail risk measure is based on the risk neutral expected shortfall of a set of portfolio returns. Our aggregate tail risk measure will simply be the average of the individual portfolio expected shortfalls.

To assess the empirical relevance of our tail risk measure, we first look at how well it matches the main extreme market events and the business cycle during a long sample period (July 1926 to April 2014). It captures most events, is more volatile around these events, and appears as strongly counter-cyclical. We also verify that it correlates well with other measures of tail risk based on high-frequency and option-implied data as well as macroeconomic variables. Since our measure is based on risk-neutralized returns, it is essential to compare it with a measure using both returns and option data. We replicate the methodology used in Ait-Sahalia and Lo (1998, 2000) and Ait-Sahalia and Lo (2000) for the period January 1996-April 2014 and estimate their tail-risk measure. The time series from our own tail risk measure based only on the cross-section of returns follows closely their tail-risk measure time series. These basic properties are therefore reassuring in qualifying our measure as risk-neutral tail risk.

The second empirical test is to establish the existence and measure the magnitude of a tail risk premium in equity returns. Our main hypothesis, as in Kelly and Jiang (2014), is that investors marginal utility is increasing in tail risk. Therefore, we explore long-short portfolios based on tail risk hedging capacity exposures and show that, even after controlling for several benchmark factors, this strategy provides a sizable negative and statistically significant alpha for both one-month and one-year horizons. The results are also very similar to the option-based methodology of Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000) on the shorter sample.

Next, we provide evidence that our tail risk measure anticipates stock market movements for intermediate horizons, even after controlling for several known predicting variables. Our measure has also some predictive power for economic conditions. We use the predictive regressions framework of Allen et al. (2012) with forecasting horizons varying from one to twelve months. Our proposed tail risk measure reasonably anticipate future declines of the main macroeconomic condition indexes. To establish the "channels" through which tail risk affects the real economy we also consider Bloom (2009) vector auto-regressive framework. We show that short-term impulse responses to shocks in tail risk are associated with lower levels of employment and industrial production.

In a robustness section, we show how the risk premia and predictive properties vary with the proposed measure when we change such parameters as the number and nature of portfolios used to compute the excess expected shortfall, the window length and the VaR threshold as well as the discrepancy parameter that controls the risk neutralization. Overall our conclusion is that the results are robust in general and most notably when we use different portfolios such as industry portfolios or financial and real sector portfolios.

Brownlees and Engle (2015) also adopted a shortfall measure to assess systemic risk but proposed a market-based measure of conditional capital shortfall for individual financial firms (exposed to capital regulation). We differ by considering a risk-neutral measure for a larger cross section of returns. Of course, our measure could be specialized to financial firms to consider systemic risk and not simply tail risk. Our paper is also intimately related to Adrian and Brunnermeier (2014) by its use of cross-sectional information. However, they also consider a systemic risk measure that captures financial industry co-movements or financial contagion. In our methodology, interconnectedness of assets returns comes from the risk-neutral probability that gives more weight to states of nature where assets in the chosen panel have lower returns. Finally, this paper also expands the analysis performed by Bali et al. (2009) in several dimensions. First, while their risk measures are based on objective probabilities, our tail risk exploits the important information in terms of downside risk embedded in risk neutral probabilities. Also, while Bali et al. (2009) focus mainly on portfolio formation according to downside risk exposures, our paper puts more emphasis on analyzing the macroeconomic implications of tail risk and the channels through which it affects the real economy.

The rest of the paper is organized as follows. Section 2 describes how we construct a risk-neutralized shortfall measure of tail risk based on a panel of asset returns and explains how it differs from other available tail risk measures. We compute a long time series of such tail risk based on a set of size and book-to-market portfolios. Section 3 starts by analyzing the correlation of our tail risk measure with the main existing tail and systemic risk measures and market index returns. We analyze in detail the relationship of the proposed measure with its option counterpart using Ait-Sahalia and Lo (2000) methodology and with the Kelly and Jiang (2014) measure. In section 4, we study empirically the predictive properties of our tail risk measure for market returns and macroeconomic activity indicators. Section 5 analyzes the robustness of our tail risk measure in various dimensions. Section 6 concludes with a summary and some potential extensions.

2 A Nonparametric Tail Risk Measure

2.1 Building a Risk-Neutral Excess Expected Shortfall Measure

Historical Value-at-Risk (VaR) was considered for a long time as a good tool for managing the risk of a portfolio. However, given the relative scarcity of extreme events in historical samples, several authors proposed to build measures based on risk-neutral probabilities assigned to historical observations. These were generally computed from option prices on the underlying (generally the market) as in Ait-Sahalia and Lo (2000) and several other papers[1]. These probabilities incorporate economic conditions and the risk attitude of investors and are therefore more reliable than historical probabilities. However, since prices for liquid options are not readily available for many assets and countries, a cross-sectional approach based on returns has been suggested to compute risk associated with tails[2].

For both theoretical and empirical reasons we choose to base our measure on excess expected shortfall instead of value-at-risk[3] A main advantage of using threshold exceedances comes from the information it contains about the *whole* tail of the distribution instead of just a point-wise percentile as VaR.

For a particular asset i, we define the excess expected shortfall as follows:

$$TR_{i,t} = E^{\mathcal{Q}(R)}[(R_{i,\tau} - VaR_{\alpha}(R_{i,\tau}))|(R_{i,\tau} \leq VaR_{\alpha}(R_{i,\tau}))] \tag{1}$$

where t is the time period for which we are calculating the tail risk, τ denotes the possible states of nature, α is the VaR threshold and $\mathcal{Q}(R)$ indicates the risk neutral density. Our aggregate market tail risk measure will be based on the average of the expected shortfall $TR_{i,t}$ of several portfolios i. Note that in this design we model $\mathcal{Q}$ as a function of the observable returns. Throughout the paper we calculate tail risk at a monthly frequency. Thus, as it will become clearer further on, the states of nature (τ) will be captured by a number of past daily returns. By keeping this number relatively small our risk measure will react more quickly to changes in market conditions given that recent cross-sectional values of asset returns will be used to estimate the risk-neutral density. This is in contrast with usual Value-at-Risk measures based on statistical historical properties of asset returns[4].

[1]See in particular Ait-Sahalia and Lo (1998), Breeden and Litzenberger (1978), Bates (1991), Rubinstein (1994), Longstaff (1995).

[2]The most recent literature includes Kelly and Jiang (2014), Allen et al. (2012), and Adrian and Brunnermeier (2014) to cite a few.

[3]Basak and Shapiro (2001) provide an equilibrium analysis that puts forward various counter-intuitive implications of VaR risk management and show that expected shortfall avoids them. In addition, expected shortfall is a coherent measure of risk (see Artzner et al. (1999)).

[4]For an interesting exception, see Paul Glasserman (2015) for a data-driven selection of historical stress test scenarios that are both extreme and plausible based on empirical likelihood estimators.

2.2 A Non-Parametric Risk-Neutral Density

To circumvent the problems related to the availability of options prices, we propose to calculate the RND via nonparametric methods. To that end, Hansen and Jagannathan (1991) seminal paper proposes to minimize a quadratic loss function to estimate a stochastic discount factor that prices exactly a set of basis assets. This insightful approach has been used extensively to assess the adequacy of financial models with a mean-variance SDF frontier. Almeida and Garcia (2016) have generalized their methodology to frontiers that involve all moments of the return distributions and use them to gauge models involving tail events such as disaster and disappointment aversion models. Given a series of assets returns, in an incomplete market where there are more states of nature than assets, they find a family of SDFs that minimize convex functions defined in the space of admissible and strictly positive SDFs. These convex functions measure the distance between an admissible SDF and the constant SDF of a risk-neutral economy. Assuming a constant short-term rate and homogenous physical probabilities, just as in a VaR historical simulation, we are able to obtain a direct correspondence between SDFs and RNDs.

Given that RND are the building blocks of our tail risk procedure we succinctly expose the methodology adopted and its implications. Let $(\Omega, \mathcal{F}, P)$ be a probability space, and R denote a K-dimensional random vector on this space representing the returns of K primitive basis assets. In this static setting, an admissible SDF is a random variable m for which $E(mR)$ is finite and satisfies the Euler equation:

$$E(mR) = 1_K, \tag{2}$$

where 1_K represents a K-dimensional vector of ones.

As in Hansen and Jagannathan (1991), Almeida and Garcia (2016) are interested in the implications of Equation (2) for the set of existing SDFs. For a sequence of (m_t, R_t) that satisfy Equation (2) for all t, and observing a time series $\{R_t\}_{t=1,\dots,T}$ of basis assets returns, we assume that the composite process (m_t, R_t) is sufficiently regular such that a time series version of the law of large numbers applies[5]. Therefore, sample moments

[5]For instance, stationarity and ergodicity of the process (m_t, R_t) are sufficient (see Hansen and Richards, 1987). In addition, we further assume that all moments of returns R are finite in order to deal with general entropic measures of distance between pairs of stochastic discount factors.

formed by finite records of measurable functions of data R_t will converge to population counterparts as the sample size T becomes large.

Given a sample of basis assets returns, the set of admissible SDFs will depend on the market structure. The usual case is to have an incomplete market, i.e., the number of states of nature (T) larger than the number of basis assets K. In such case, an infinity of admissible SDFs will exist, and if there is no in-sample arbitrage on the basis assets payoff space (see Gospodinov et al. (2016)), there will exist at least one strictly positive SDF (see Duffie, 2001). For each strictly positive SDF there will be a corresponding risk neutral density.

Given a convex discrepancy (penalty) function ϕ the generalized, in sample, minimum discrepancy problem proposed by Almeida and Garcia (2016) can be stated as:

$$\begin{aligned} \hat{m}_{MD} = \underset{\{m_1,\dots,m_T\}}{\arg\min} \quad & \frac{1}{T}\sum_{i=1}^{T}\phi(m_i) \\ \text{subject to} \quad & \frac{1}{T}\sum_{i=1}^{T} m_i(R_i - \frac{1}{a}1_K)] = 0_K \\ & \frac{1}{T}\sum_{i=1}^{T} m_i = a \\ & m_i \geq 0(\text{or } m_i > 0)\ \forall\ i \end{aligned} \tag{3}$$

In this optimization problem, restrictions to the space of admissible SDFs come directly from the general discrepancy function ϕ. The conditions $E(m\left(R - \frac{1}{a}1_K\right)) = 0_K$ and $E(m) = a$ must be obeyed by any admissible SDF m with mean a. In addition, whenever there is a strictly positive solution the implied minimum discrepancy SDF is compatible with absence of arbitrages in an extended economy that considers derivatives over the underlying basis assets[6]. The choice to impose a non-negativity or strict positivity constraint in the optimization problem is dictated by the choice of the discrepancy function $\phi(.)$ (see Almeida and Garcia (2016) for a detailed analysis).

Despite the straightforward interpretation of the problem in (3), its solution is not easy given that the number of unknowns is as large as the size of the sample. Therefore, Almeida and Garcia (2016) show that one can solve an analogous simpler dual problem:

[6]It is important to note that the homogeneous probability assumption will not affect the key insights we derive from this methodology and, if desired, one could also consider a kernel density to model the physical probabilities without additional complications

$$\hat{\lambda} = \arg\sup_{\alpha\in\mathbb{R},\, \lambda\in\Lambda} a * \alpha + \frac{1}{T}\sum_{i=1}^{T}\phi^{*,+}(\alpha + \lambda'(R_i - \frac{1}{a}1_K)) \tag{4}$$

where $\Lambda \subseteq \mathbb{R}^K$ and $\phi^{*,+}$ denote the convex conjugate of ϕ restricted to the non-negative (or strictly positive) real line.

$$\phi^{*,+} = \underset{w\in[0,\infty)\cap\text{domain }\phi}{sup} zw - \phi(w) \tag{5}$$

In this dual problem λ can be interpreted as a vector of K Lagrange multipliers that comes from the Euler equations for the primitive basis assets in (3). Almeida and Garcia (2016) specialize the ϕ function to the Cressie Read family, $\phi^{\gamma}(m) = \dfrac{m^{\gamma+1} - a^{\gamma+1}}{\gamma(\gamma+1)}$, $\gamma \in \mathbb{R}$. This family captures as particular cases minimizations of variance (Hansen and Jagannathan, 1991), higher moments (Snow, 1991), and different kinds of entropy (Stutzer, 1996) estimators. With this family, closed-form formulas are obtained for λ and $\hat{m}_{MD}$:

i) if $\gamma > 0$,

$$\hat{\lambda}_{\gamma} = \arg\sup_{\lambda\in\Lambda_{CR}} \frac{1}{T}\sum_{i=1}^{T}\left(\frac{a^{\gamma+1}}{\gamma+1} - \frac{1}{\gamma+1}\left(a^{\gamma} + \gamma\lambda'\left(R_i - \frac{1}{a}1_K\right)\right)^{\dfrac{\gamma+1}{\gamma}} I_{\Lambda_{CR}(R)}(\lambda)\right) \tag{6}$$

where $\Lambda_{CR} = \{\lambda \in \mathbb{R}^K | \forall i = 1, \ldots, T\ (a^{\gamma} + \gamma\lambda'(R_i - \frac{1}{a}1_K)) > 0\}$, $I_A(x) = 1$, if $x \in A$, and 0 otherwise, and:

$$\hat{m}^{i}_{MD} = a\frac{\left(a^{\gamma} + \gamma\hat{\lambda}'_{\gamma}\left(R_i - \frac{1}{a}1_K\right)\right)^{\dfrac{1}{\gamma}}}{\frac{1}{T}\sum_{i=1}^{T}\left(a^{\gamma} + \gamma\hat{\lambda}'_{\gamma}\left(R_i - \frac{1}{a}1_K\right)\right)^{\dfrac{1}{\gamma}}} I_{\Lambda_{CR}(R)}(\hat{\lambda}_{\gamma}) \tag{7}$$

ii) if $\gamma \leq 0$,

$$\hat{\lambda}_{\gamma} = \arg\sup_{\lambda\in\Lambda_{CR}} \frac{1}{T}\sum_{i=1}^{T}\left(\frac{a^{\gamma+1}}{\gamma+1} - \frac{1}{\gamma+1}\left(a^{\gamma} + \gamma\lambda'\left(R_i - \frac{1}{a}1_K\right)\right)^{\dfrac{\gamma+1}{\gamma}}\right) \tag{8}$$

and:

$$\hat{m}^i_{MD} = a \frac{\left(a^\gamma + \gamma \hat{\lambda}'_\gamma \left(R_i - \frac{1}{a} 1_K\right)\right)^{\frac{1}{\gamma}}}{\frac{1}{T}\sum_{i=1}^{T} \left(a^\gamma + \gamma \hat{\lambda}'_\gamma \left(R_i - \frac{1}{a} 1_K\right)\right)^{\frac{1}{\gamma}}} \tag{9}$$

The implied SDF depends directly on the parameter γ. Different choices of γ will weight differently higher-order moments of returns (see Almeida and Garcia (2016)). The choice of γ and the robustness of results with respect to its value are therefore important elements of our empirical strategy. We discuss these aspects in Sections 2.4 and 5 respectively.

To obtain the risk neutral probabilities associated with each observation interpreted as a state of nature, we distort the usual $1/T$ measure by the computed SDF in (9) adjusted by the interest rate:

$$\pi_i^{RN} = \frac{m_i(1+r)}{T} \tag{10}$$

We illustrate the distortions in Figure 1 for a five-asset economy that we will make explicit in the next section. On the horizontal axis, we plot the returns on the optimal portfolio resulting from the optimization in (8), while the risk neutral probabilities are featured on the vertical axis for various values of the parameter γ. The different lines illustrate clearly the fact that for more negative values of γ the implied risk neutral probabilities give more weight to low returns or "bad" states of nature from an investor perspective. From the linear outcome generated by the unit value (Hansen and Jagannathan (1991)), the convexity increases as γ becomes more negative.

2.3 Choosing the basis assets and the estimation window

As mentioned earlier in this section our risk neutral density estimation methodology assumes an incomplete market framework. Empirically this translates into an upper bound in the number of cross sectional assets for a given number of states of nature T. More precisely, for a given number of days (states of nature) T we must consider a number of assets K such that $K < T$. Most papers (Kelly and Jiang (2014) in particular) compute a tail risk measure at a monthly frequency. To make our measure conditional on recent information, we therefore limit our sample to the former 30 days preceding the last day of each month[7]. Otherwise we would be using old information to predict current and future stock market and economic conditions. Therefore we cannot adopt a strategy similar to Kelly and Jiang (2014) who include all securities available in the CRSP data base at the end of each month and pool all their returns in the former 30 days to compute their tail risk measure based on the Hill estimator[8].

Given an upper bound of 30 days, we need to base our measure on portfolios. But we can ask ourselves whether we should limit ourselves to the market portfolio or exploit the cross-sectional information in say the usual twenty-five size and book-to-market portfolios [9]. Bali et al. (2014) argue that previous studies have shown that investors usually hold portfolios composed by some market proxy (e.g. stock funds) and individual stocks. In this environment individual stocks play a crucial role on the total portfolio tail risk. Empirically, they find evidence that market tail risk is not capable of providing useful information for return prediction whereas individual assets tail risk measures (idiosyncratic tail risk) is quite successful in predicting market returns. Expanding on this, Kelly and Jiang (2014) also argues that the whole cross-section of stocks contains valuable information regarding the tail behavior of the returns. Therefore it seems that we should rely on as much cross-sectional information as possible.

However, using the whole set of twenty-five portfolios as basis assets in our procedure might produce risk-neutral probabilities that result in sizable pricing errors on those basis assets[10]. Recently Kozak et al. (2015) show that a stochastic discount factor based on the first few principal components of asset returns explains many anomalies. Their argument rests on the fact that since asset returns have substantial commonality, absence of near-arbitrage opportunities implies that the SDF depends only on a few sources of return variation. Therefore, instead of considering the whole cross-section of the 25 size and book-to-market portfolios we extract the first five principal components from the 30-day panel[11]. To make our measure conditional, we compute the principal components for

[7]In the robustness section of the paper (section 5 we report the sensitivity of our empirical results to the panel length.

[8]In Almeida et al. (2016) we compute a tail risk risk measure using a similar methodology with individual assets in a high-frequency environment.

[9]For more details about these portfolios, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html.

[10]We also computed our tail risk measure for the whole set of 25 Fama-French portfolios. While the pricing errors where higher, the overall empirical findings were preserved.

[11]In our thorough comparison with the Kelly-Jiang measure in section 3.3, we also compute our measure with principal components computed from the whole cross-section of returns.

every end-of-month in the sample. Thus, for each t, we are not using future information in the tail risk calculation. Table 1 reveals that these principal components are responsible for much of the portfolio variations. With five principal components, close to 90% of the variation is explained for the full sample.

2.4 The choice of γ

To understand the effect of γ on our tail risk measure, we derive a Taylor expansion of the expected value of $\phi(m) = \frac{m^{\gamma+1}-a^{\gamma+1}}{(\gamma(\gamma+1))}$ around the SDF mean a. Noting that $\phi(a) = 0$, $\phi'(m) = \frac{m^{\gamma}}{\gamma}$, $\phi''(m) = m^{\gamma-1}$, $\phi'''(m) = (\gamma-1)m^{\gamma-2}$, $\phi''''(m) = (\gamma-1)(\gamma-2)m^{\gamma-3}$...,Taylor expanding ϕ and taking expectations on both sides we obtain:[12]

$$E(\phi(m)) = \frac{a^{\gamma-1}}{2}E(m-a)^2 + \frac{(\gamma-1)a^{\gamma-2}}{3!}E(m-a)^3 + \frac{(\gamma-1)(\gamma-2)a^{\gamma-3}}{4!}E(m-a)^4 + ... \tag{11}$$

Two important effects appear regarding the weights attributed to skewness and kurtosis in this expansion. First, for values of γ close to one, both skewness and kurtosis have small weights when compared to the variance that has a weight equal to one half in the expansion. This implies that discrepancies with values of γ close to one do not capture much of the higher moment activity of pricing kernels. Once we move to more negative values of γ both skewness and kurtosis receive considerable weights in the expansion. The second important aspect refers to the relative weights that are given to skewness and kurtosis by different Cressie Read functions. For $-2 < \gamma < 1$ note that the absolute weight given to kurtosis is smaller than the corresponding weight given to skewness. However, for values of $\gamma < -2$, kurtosis receives more weight than skewness. In fact all even higher-moments receive more absolute weight than their corresponding odd higher-moments in this region of γ.

In Almeida and Garcia (2016), we derive bounds for the disaster model in Backus et al. (2014). In this model the logarithm of consumption growth is given by:

$$g_{t+1} = \eta_{t+1} + J_{t+1} \tag{12}$$

[12]Note that all functions in the Cressie Read family are analytic, that is, their derivative of any order exists. For this reason, the only condition that is needed for the Taylor expansion to be valid is the existence of the first four moments of the MD SDF.

where η_{t+1} is the normal component $\aleph(\mu, \sigma^2)$ and J_{t+1} is a Poisson mixture of normals. The number-of-jumps variable j takes integer values with probabilities $e^{-\tau}\frac{\tau^j}{j!}$, where τ is the jump intensity. Conditionally on the number of jumps, J_t is normal:

$$J_t|j \sim \aleph(j\alpha, j\lambda^2). \tag{13}$$

In this model, the logarithm of the stochastic discount factor with power utility is:

$$\log m_{t+1} = \log \beta - \zeta g_{t+1} \tag{14}$$

where ζ is the coefficient of relative risk aversion. Therefore, the mean of the SDF is:

$$a = \exp\left\{\log \beta - \zeta\mu + \frac{1}{2}(\zeta\sigma)^2 + \tau(e^{-\zeta\alpha+0.5(\zeta\lambda)^2} - 1)\right\}. \tag{15}$$

The discrepancy bound for the Cressie-Read family is the expectation of $\phi^\gamma(m)$. It can easily be obtained by taking the expectation of $\exp{(\gamma+1)\log m}$, that is:

$$d = \exp{(\gamma+1)log(\beta) - \zeta\mu(\gamma+1) + 0.5 * (\zeta\sigma(\gamma+1))^2 + \tau(\exp{-\zeta(\gamma+1)\alpha\theta + 0.5(\zeta(\gamma+1)\lambda)^2 - 1)}} \tag{16}$$

These bounds have a direct link with the measure of entropy used in Backus et al. (2014). When $\gamma = -1$ our discrepancy function is $log(a) - E(log(m))$, which corresponds precisely to the entropy of the pricing kernel reported in their Equation (13).

For these bounds we are computing the Cressie-Read bounds with the returns on the S&P 500 index and equity options strategies on this index. We use four options portfolios that consist of highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 composite index trading on the Chicago Mercantile Exchange [13]. We want to illustrate here how the frontiers and the disaster model discrepancy vary with the values of γ. For the disaster model we use the parameter values reported in Table II of Backus et al. (2014), except for the risk aversion parameter that we set at 8 to match a slightly higher equity premium. For the values of γ, we chose three values that will illustrate the relation between the bounds and the model. In Figure 3 we plot the three frontiers corresponding to the three values of γ 0, -0.5 and -1. We

[13] These have been constructed by Agarwal and Naik (2004) to study performance of hedge funds

also plot in the corresponding colors the points corresponding to the discrepancy of the model at the mean SDF value 0.9946. For $\gamma = 0$ the model passes quite easily because the weights put on higher moments for building the frontier are relatively small. On the contrary, for $\gamma = -1$ the restrictions in the bound are more stringent and the model is far below the frontier. For the intermediate value of $\gamma = -0.5$, the model point is above the frontier but still closer than for the two other values[14]. Given this illustrative exercise we will choose $\gamma = -0.5$ as our base case.

There are also some statistical arguments for choosing this class of loss functions Kitamura et al. (2013) showed that, under some technical assumptions, the Hellinger ($\gamma = -0.5$) is the more robust one based on an asymptotic perturbation criterion[15].

Expanding on this argument, in the internet appendix we present a rich discussion on the importance of the non linearities in the RND and its effects on tail risk. Briefly, if we want a tail risk measure that incorporates higher-order moments information we must compute a SDF (and therefore a RND) that also considers this information, in contrast to the Hansen and Jagannathan one. For instance, Schneider and Trojani (2015) argue that investors' "fear" is an aversion to downside risk, exactly what we aim at capturing with our methodology. Furthermore, they link the concept of "fear" with investor prudence[16], which is intimately related to negative skewness aversion. Therefore, a complete and informative risk measure should be able to reflect the informational content on higher-order moments.

Therefore for the majority of the empirical section we adopt the Hellinger Tail Risk as the main benchmark,but we will still present robustness tests with different values of γ in section 5.

[14]We can use values close to -0.5 where the model point is event closer to the bound but one has to be careful because important discontinuities may suddenly occur and the frontier may get closer to the bound for $\gamma = -1$.

[15]In a recent paper, exploring the non-linearity implied by the Hellinger estimator, Schneider and Trojani (2015) introduce a class of trading strategies that might be used to trade higher moments of the returns. In the internet appendix we also explore the higher moments information contained in the estimated risk neutral density and its implications to tail risk.

[16](Almeida and Garcia, 2012) and (Almeida and Garcia, 2016) demonstrated that the methodology we consider to estimate the SDF is compatible with this concept.

2.5 A first look at the tail risk measure over time

Figure 2 plots the evolution of our tail risk measure, in blue, and a Hoddrick-Prescott filtered version, in red, from July 1926 to April 2014. Our measure is very volatile and features various peeks often coincident with extreme or significant financial, economic or political events. These events have affected negatively equity returns in important ways. Our tail risk measure captures most of these huge market drops in U.S. history and also appears to be more volatile in the periods before and after some of them. Of course two economic and financial crisis periods figure prominently, the Wall Street Crash of 1929 followed by the Great Depression of 1929-1939 on the left, the beginning of our sample, and the recent financial crisis of 2008-2010 on the right of the graph. In between these two major crises, the filtered series shows that the overall level of the tail risk remained below these extremes, except for occasional spikes that are triggered by some specific events. For these sporadic events the mean reversion is much stronger, as in the dot-com bubble of the early 2000s or the recent European debt crisis starting at the end of 2009.

Other tail risk measures have also captured these types of events but since many of them rely on option prices their span is much narrower, their sample starting at the beginning of the 1990s.

3 A comparison with other tail risk measures: correlations and risk premia

Since many tail risk or downside risk measures have been proposed in the last five years, we start by computing simple correlations with several of them over different sample periods, some based on equity returns starting in the 60s, others using options computed from the beginning of the 90s. We also consider correlations with market indexes and macroeconomic indicators. We investigate in more depth the relationship of our measure with two prominent tail risk measures, one based on option prices using the methodology of Ait-Sahalia and Lo (2000) and one recently proposed by Kelly and Jiang (2014) that uses the information in the whole panel of individual stock returns. In this section we will conduct the comparison in terms of risk premia associated with portfolios sorted according to the hedging capacity of individual securities against tail risk. We will extend the comparison in section 4 with the predictive properties of the measures for market returns and macroeconomic activity indicators.

3.1 Correlations with other indexes

In Table 2 we report the correlation coefficients between our tail risk measure and other tail risk measures as well as financial and macroeconomic indexes: the tail risk measure of Bollerslev et al. (2015) based on option prices, the Kelly and Jiang (2014) tail risk measure, the VIX, two stock market indexes, the S&P 500 and the CRSP portfolio, the systemic risk measure of Allen et al. (2012), and the macroeconomic conditions index of Bali et al. (2014).

The first interesting result is that our tail risk measure has a noticeable correlation (0.43) with Bollerslev et al. (2015) tail risk measure despite the fact that we do not include any option returns in the computation of our measure. Another important measure based on option prices is the VIX. Its correlation with our tail risk measure is 0.56. This measure is often interpreted as a good indicator for investors' crash fears. More generally, since option prices reflect investors' risk attitudes and economic conditions, it seems that our risk neutralization goes some distance in capturing this important information for extreme risk. In section 3.2, we will conduct a thorough comparison between a nonparametric measure incorporating option prices and our measure based only on risk-neutralized cross-sectional portfolio returns.

Another tail risk measure has been proposed recently by Kelly and Jiang (2014), thereafter KJ tail risk measure. Surprisingly, it is not at all correlated with our tail risk measure. From a time series point of view our measure is volatile and has little persistence, while the KJ measure is highly persistent. The nature of these measures is therefore very different. The source of this difference may be due to the amount of information used in constructing the two measures (the whole cross-section of returns for KJ, principal components of the 25 size and book-to-market portfolios in our measure), the fact that KJ is based on raw returns while ours is computed with risk-neutralized returns, or other differences regarding the VaR threshold and the estimating window. We will explore all aspects in a thorough comparison in section 3.3. It should be noted that the KJ measure has a negative correlation of -0.38 with the VIX and a small and positive correlation of 0.23 with the Bollerslev et al. (2015) measure of tail risk.

As it should be expected, our measure is counter-cyclical, with negative correlations of -0.32 and -0.24 with the S&P 500 and CRSP returns respectively. In 4.1, we will look more closely at the relation between tail risk and the future aggregate returns. Another important benchmark is the measure of systemic risk (CATFIN) based on the financial sector by Allen et al. (2012). The computation of CATFIN is related to our measure since it is estimated using both value-at-risk (VaR) and expected shortfall (ES) methodologies, with nonparametric and parametric specifications. The correlation with our tail risk measure is 0.45, which is reassuring since returns in the financial sector capture financial crisis risk which underlies a good portion of the movements in a more general tail risk measure based on all equity returns. Allen et al. (2012) show that high levels of systemic risk in the banking sector impact macroeconomic conditions. Similarly our tail risk measure is correlated with macroeconomic indicators of Bloom (2009) and Bali et al. (2014) (0.46 and 0.47 respectively). In section 4.2 we study in detail its relationship with the real economy through the uncertainty channel. The KJ measure of tail risk exhibits no counter-cyclical behavior nor does it relate to the macroeconomic indicators of Bloom (2009) and Bali et al. (2014).

These correlations suggest a common component between our tail risk measure based on the cross-section of risk-adjusted returns and option-based measures of tails risk measures. They also point out the distinctive behavior of the KJ tail risk measure despite the use of the cross-section of returns. We will probe further the relation between these two sets of measures in the next two sections.

3.2 A detailed comparison with an option-based tail risk measure

We have seen that our tail risk measure is reasonably well correlated with several tail risk and systemic risk measures. Among them, we want to probe further the relation with option-based measures. Since we claim that our procedure produces a risk-neutral measure of tail risk even though we are using only equity returns, it would be reassuring to show that the extracted measure has much in common with a measure incorporating option prices. Our empirical strategy consists in applying the methodology in Ait-Sahalia and Lo (2000) to estimate the excess expected shortfall in (1) using the S&P 500 index options and returns. Ait-Sahalia and Lo (2000) propose a nonparametric VaR measure that incorporates economic valuation through the state-price density associated with the underlying price processes. Ait-Sahalia and Lo (1998) propose to estimate an option-pricing formula $\hat{H}(.)$ which has the structure of the Black-Scholes formula but where the volatility is estimated nonparametrically, that is $H_{BS}(F_{t,\tau}, X, \tau, r_{t,\tau}; \sigma(X/F_{t,\tau}, \tau))$, where $F_{t,\tau}$ is the forward price of the asset (S&P 500 index) at time t for time-to-maturity τ, X is the exercise price, r denotes the interest rate, and $\sigma(.)$ is the volatility function. The estimated option-price function can be differentiated twice with respect to the strike price X to obtain $\partial^2 \hat{H}(.)/\partial X^2$, and then the state-price density by multiplying this second derivative by $\exp(r_{t,\tau}\tau)$. Ait-Sahalia and Lo (2000) used this estimated state-price density to compute an economic VaR but we can of course use it to compute the excess expected shortfall.

The main point of Ait-Sahalia and Lo (2000) was to arrive at a VaR value that is adjusted for risk and time preferences and for changes in economic conditions. We have the same objective but we differ from their work in one important aspect. We estimate the risk-neutral density from a panel of equity returns and distort the objective probabilities through a minimum-discrepancy procedure. We apply the two methodologies for the period January 1996 to April 2014. Figure 4 features both tail risk measures. Despite several methodological and empirical differences between the two measures, Figure 4 reveals that both time series tend to move together with many coincident peaks and troughs.

To analyze more precisely the similarities between the two measures we conduct an analysis of implied market premium. We first measure the hedging capacity (or insurance value) of all individual stocks with code 10-11 in CRSP over the period (i.e. their contemporaneous betas with respect to the tail risk measure, $r_{i,t} = \alpha_i + \beta_i TR_{i,t}$[17]) and then sort these betas into ten portfolios for the lowest hedging portfolio to the highest one. We then compute the post-formation returns associated with these portfolios over the next month. The results are reported in Table 3. There are two panels. The upper panel reports the option-implied tail risk while the lower one features our tail risk measure with the Hellinger discrepancy. For each panel, we report several measures of returns. In the first line we present the raw average returns for the ten portfolios and for the difference between the highest hedging and the lowest hedging portfolios (last column High-Low).

[17] This is to be distinguished from the predictive betas computed in Kelly and Jiang (2014), $r_{i,t+1} = \alpha_i + \beta_i TR_{i,t}$, which measure exposure to tail risk.

For the other lines we report the returns after controlling for other factors, namely the three Fama-French factors, to which we add momentum, liquidity and volatility one by one.

First, looking at the first line which reports the raw average returns, it is remarkable to see that the magnitudes of the High-Low column are very similar and share the same signs and statistical significance. The difference is -3.73 for the option-based measure while it is -3.06 for the Hellinger measure. This suggests that the crash perceptions embedded in the option prices are stronger than in our nonparametric distortion. After controlling for the other factors, all the signs remain negative except when we add volatility. In terms of statistical significance the option-implied high minus low returns are estimated more precisely than the nonparametric measure, which makes sense since we do not incorporate market prices of options.

There is evidently some short-run correlation between volatility and tail risk. One can think of an equilibrium explanation in a consumption-based asset pricing model as in Farago and Tedongap (2015). They propose an intertemporal equilibrium asset pricing model featuring disappointment aversion and changing macroeconomic uncertainty (consumption volatility) where besides the market return and market volatility, three disappointment-related factors are also priced: a disappointment factor, a market downside factor, and a volatility downside factor. The last factor represents changes in market volatility in disappointing states and calls for a particular premium. Our linear analysis cannot disentangle these downside effects, hence the correlation between the volatility factor and the tail risk factor.

We believe that these results show that our nonparametric tail risk measure goes a long way in capturing the risk adjustment ingrained in option prices. Therefore it can supplement the option-based tail risk measures for markets where liquid option markets are not present and for periods where these derivative securities did not exist but where tail risk was prominent, such as the Great Depression of 1929-1939. Since our measure is based on a panel of equity returns we need to understand its relation with the Kelly and Jiang (2014) tail risk measure, especially since they appear to be uncorrelated.

3.3 A detailed comparison with the Kelly and Jiang (2014) tail risk measure

Figure 5 plots the tail risk measure based on our expected excess shortfall methodology together with the Kelly and Jiang (2014) measure. The two series are quite distinct and seem to relate negatively when big shocks hit the market. Kelly and Jiang (2014) assume that the lower tail of any asset return i behaves according to the following law:

$$P(R_{i,t+1} < r \mid R_{i,t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t} \tag{17}$$

with $r < u_t < 0$. The authors estimate the time-varying tail exponent by the Hill estimator:

$$\lambda_t^{Hill} = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}, \tag{18}$$

where $R_{k,t}$ is the kth daily return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences in the cross-section within month t. They find that the tail exponent is highly persistent and features a high degree of commonality across firms.

While the KJ estimator is based on an assumption about the statistical behavior of the tail of the cross-section distribution of returns, our estimator relies on a risk adjustment that distorts the returns of the chosen basis assets according to a set of Euler conditions. This adds a lot of variability to our tail risk estimator.

Although our tail risk factor and the Kelly and Jiang (2014) tail risk measure are computed from a cross-section of equity returns they differ in several aspects. An important difference relates to the information used by each method. Each month, Kelly and Jiang (2014) compute their tail risk measure based on the whole set of individual raw returns below a threshold of 5% in the elapsed month. To set ideas if we have 5,000 individual securities, the procedure uses 5% of 100,000 observations $(5,000 \times 20 days)$, that is 5,000 observations. Our measure starts from a set of 25 portfolios sorted by size and book-to-market value, collects their returns over the last 30 days (a month and a half) and then extracts five principal components of these 25 series of 30 days, leaving us with five series of 30 observations. We then compute our excess expected shortfall with the risk-adjusted returns at a level of 10% for each principal component and take the average of the five shortfall measures. Therefore there is a fundamental difference since the Kelly and Jiang (2014) method extracts a common component among the say 5,000 tail observations while our procedure selects extreme observations after a double reduction in information by first looking at portfolios and then taking principal components of these portfolios. Is this contraction of information the main reason for the fundamental difference between the two measures of tail risk?

To answer this question we start by running our procedure with all individual securities. We compute the five principal components out of the whole universe of available stocks every month for a window of 30 days. The robustness of the procedure is evaluated as in section 3.2 with the returns of 10 portfolios sorted according to the tail risk hedging capacity and the resulting high minus low returns. To benchmark the results obtained with all securities, we first compute the portfolio returns over the 1926-2014 period with our original data set based on the 25 size and book-to-market portfolios. In Table 4, we report in the last column the high minus low returns earned post-formation for a one-month holding period (Panel A) and for a one-year holding period (Panel B). For the short-period returns, all differences are negative, of similar magnitude across the various controls for additional factors and statistically significant except for the last line where we add a volatility factor. The results are similar for the yearly returns but the difference is now positive and insignificant when we add the volatility factor. Table 5 contains the results of the same analysis when the five principal components are computed from the whole universe of equities available in CRSP. Results for both short and long holding periods are very similar. Moreover we have conducted the same analysis directly on the 25 size and book-to-market portfolios instead of their five principal components and results were also very similar. Therefore we can conclude confidently that our methodology is robust to the amount of cross-sectional information used to build the tail risk measure.

Another source of divergence could be due to our risk-neutralization. As a final exercise using this methodology, we define a standardized tail risk measure based on the average of the excess expected shortfall in equation (1) for the raw returns of the five principal components, a so-called objective counterpart to our risk-neutral measure. We proceed similarly to sort the cross-section of returns in ten portfolios according to their standardized tail risk hedging sensitivity and compute the post-formation returns of the portfolios as before. Table 6 presents the results with this alternative objective tail risk measure. We note that both in terms of magnitude and statistical significance results are very different from our reported figures in Table 4, providing some evidence of the important contribution of the risk-neutralization procedure.

We conclude this section by some interesting observations about the Kelly and Jiang (2014) tail risk measure. If we express the law of tail returns in terms of gross returns instead of net returns, that is $1 + R_{k,t}$, $1 + r$ and $1 + u$, then the Hill estimator will be approximately, for small daily returns:

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K_t} [R_{k,t} - u_t], \tag{19}$$

Based on this new Hill estimator, we compute the tail risk equivalent KJ series with the whole cross-section at any point in time. We plot in Figure 6 this series together with our Hellinger measure of tail risk based on the 5 principal components of the whole cross-section of returns. Despite the different methodologies and the risk-neutralization the two series behave much more similarly than before. Averaging the difference of the logarithms of the returns or the difference of the returns makes a huge difference on the property of the tail risk series. Of course this new Hill estimator is closer to our excess expected shortfall measure, the main difference being the fact that our measure is based on risk-adjusted returns of more aggregated information. The graphs suggest that these two elements, risk neutralization and aggregation of information, are important for the magnitude of the fluctuations in both series, sometimes enhancing them sometimes dampening them. There is not a clear consistent pattern over time.

4 Predictive Properties of Tail Risk

Predictability of market returns or other financial and macroeconomic variables is another way to assess the usefulness of our tail risk measure. A key recent paper by Welch and Goyal (2008) studies comprehensively the predictive power of variables that have been suggested by the academic literature to be good predictors of the equity premium. Their conclusions are rather negative both in-sample and out-of-sample. They find that models have a poor and unstable predictive performance. Therefore, we want to submit our measure to the same type of analysis and determine whether it adds to the main predictors established in the literature. We explore in turn market returns, macroeconomic responses to tail risk shocks, and finally how tail risk anticipates business cycle fluctuations.

4.1 Stock Markets Returns

We provide two sets of predictive regressions. First, in Table 7, we consider simple regressions where we control for market returns (12 lags): $r_{[t,t+k]} = \alpha + \beta TR_t + r_{[t-k,t]} + u_t$, where TR refers to the tail risk measure. Our goal is to compare the predictive power of our measure relative to the other proposed tail risk measures. This has two advantages. It is a test with respect to the direct competition but it is also a robustness test with respect to sub-samples of the long sample 1926-2014 since the various tail risk measures are available over different periods. We choose the CRSP value-weighted market returns index as our target[18]. Overall, we note that the estimated regression coefficients are positive, meaning that increases in current tail risk are associated with future higher market returns. Also, for most of the samples over which we perform the regressions the estimated coefficients are statistically significant for an interval of two to six months. The most robust performance comes from the long horizons starting in the 1960s. This is not the case of the other tail risk measures. The CATFIN index does not show any forecasting power while the Bloom volatility index is forecasting at a very short horizon of one or two months. The VIX has some predictive ability in the medium-term, while the Kelly-Jiang tail risk measure starts showing some predictive power after six months. The best predictor appears to be the BTX measure from Bollerslev et al. (2015). From the third month on, we see a statistically significant relation between future CRSP returns and the option-based tail risk measure. We also observe a growing pattern in the coefficients. This is also apparent in our Hellinger tail risk measure, which could be due to the period starting in 1996, because this pattern is also present for the regressions with the VIX and not for the other horizons.

We continue our predictive exercise in Table 8, by adding many of the predictive candidates used in Welch and Goyal (2008) to the previous regression with our Hellinger tail risk measure and lagged returns for the long sample (1962-2012), that is $r_{[t,t+k]} = \alpha + \beta TR_t + r_{[t-k,t]} + GW_t + u_t$. In Panel A, of the seven predictors considered (Book-to-Market, Dividend Payout, Earnings Price Ratio, Dividend Price Ratio, Dividend Yield, Stock Variance, Default Yield Spread), only the dividend price ratio and the dividend

[18]Since results with the S&P500 were very similar, we omitted for space considerations.

yield appear to be significantly related to future stock market returns for all horizons, as it has been often established. However it does take away the predictive power of tail risk for the three- to eight-month horizons. The results for the default yield spread are particularly interesting. It shows some significant predictive ability for the six- to twelve-month horizons, but it takes away the predictive significant relation between tail risk and future market returns. This reinforces the interpretation of our measure as tail risk since the default spread ought to widen in periods when tail risk is prevalent. It is therefore hard to capture each effect independently.

In Panel B of Table 8, we consider another set of seven predictors used in Welch and Goyal (2008): Default Return Spread, Inflation, Long Term Yield, Long Term Rate of Returns, Term Spread, Treasury Bill Rate and Net Equity Expansion. Only the long-term rate of bond returns shows a strong and statistically significant predictive power, but tails risk remains as a significant predictor three-to-six months ahead.

Overall, we can conclude that tail risk has a significant predictive power in the short run even after controlling for the main predictors of future stock market returns.

4.2 Economic Predictability

We now explore the relationship of our tail risk measure with the real economy. Recently, in the aftermath of the great recession, several papers put forward facts and patterns about economic uncertainty, its fluctuations during business cycles, and its empirical relation with micro and macro growth dispersion measures. While measures of volatility may capture uncertainty, it is often the case that big negative shocks play an important role in increasing economic uncertainty. This asymmetry occurs because good news seem to build up more gradually over a period of time. Therefore tail risk measures may help in measuring better the effects of economic uncertainty on macroeconomic variables such as output growth and employment. Both Bloom (2009) and Bali et al. (2014) propose measures of macroeconomic uncertainty based on volatility of macroeconomic variables. We have seen that our measure of tail risk has a correlation close to 0.5 with these two uncertainty indicators. We have also characterized the jumps in tail risk in Figure 2 and they often correspond to jumps in uncertainty identified by Bloom (2009)[19].

[19]Bloom (2009) identifies 17 uncertainty dates defined as events associated with stock market volatility in excess of 1.65 standard deviations above its trend.

In what follows we will follow the vector auto-regressive impulse response function approach of Bloom (2009) to distinguish between the effects of volatility and tail risk on macroeconomic aggregate variables.

4.2.1 Impulse Response Functions to Tail Risk Shocks

To assess whether uncertainty shocks influenced firm-level decisions, Bloom (2009) considered several variables in his VAR approach: stock market returns, volatility(either market variance or an indicator capturing the biggest volatility events), the Federal Fund rate, wages, inflation, hours, employment and production in manufacturing, in this order. A main conclusion was that a rise in economic uncertainty reduces industrial production and employment in the short run. We argue that some of the uncertainty effect captured by Bloom's study might actually be coming from tail risk, in the same line as Kelly and Jiang (2014)[20]. The main theoretical insight behind our argument comes from a firm's perspective on investment decisions in the presence of uncertainty. If firms look at their investment choices as real options, an increase in uncertainty may defer her decision to invest, to produce or to hire. Therefore a rise in volatility or tail risk may affect aggregate production or employment as well as investment[21].

Therefore, we expand Bloom (2009)'s vector auto-regressive approach by adding our measure of tail risk to the set of variables considered[22]. In terms of order we include it just after volatility. In Figure 7, we plot the impulse response functions on both employment and industrial production to a one standard deviation shock in the monthly estimated tail risk measure[23]. The gray area represents the 95% confidence interval for the response function while the solid line represents the response itself. In Figure 8, we plot the same response functions for a one standard deviation shock to volatility measured by stock market variance[24]. For the shocks to tail risk, the effect is less abrupt but it seems to be followed by a more positive recovery compared to a volatility shock where after 10

[20]These authors use the tail risk measure that we described earlier applied to the manufacturing firms in the sample. It is therefore a very specific measure that we want to compare to our general-purpose measure based on principal components. It is also instructive to see how our measure performs given the two very different properties of the two measures as illustrated in section 3.3.

[21]Kellogg (2014) and Jurado et al. (2015) explore in more depth this particular channel.

[22]To be precise, market returns measured by S&P 500 returns lagged, wages, hours, industrial production and employment are in logs, inflation is measured as the difference in log CPI. The variables are in the order indicated in the previous paragraph.

[23]The only difference between our approach and Bloom's one is that we perform the impulse response analysis with a additional tail risk factor.

[24]Bloom (2009) uses realized variance before the availability of VXO and VXO afterwards.

months the strong negative shock has been absorbed without any ensuing significant positive effect. Although the evidence is not strong, it seems that a tail risk shock has a recovery period after the initial negative shock. In Figures 9 and 10, we plot the same respective impulse response functions but this time with Bloom (2009) indicator of big volatility events. Although the shapes of the reaction functions are similar, the confidence intervals are now more telling about the effects of shocks to volatility and tail risk. The responses to tail risk are of greater magnitude, distinctively negative in the short run up to 10-15 months, and then becoming significantly positive at an horizon of 30-35 months. For shocks to the volatility indicator, small initial negative effects die after 5 months and never pick up significantly afterwards for employment. However, for industrial production, after a small contraction in the first three months, there is a small rebound at an horizon of 10-15 months. These results tend to show that it is difficult to disentangle uncertainty created by big events from the the uncertainty associated with more volatile environments.

4.2.2 Business Cycles

Bloom (2009) states that almost every macroeconomic indicator of uncertainty appears to be countercyclical. In Figure 2 we plot our tail risk, in blue, and highlight with shades of grey the NBER recession periods. To give a better idea of the relation between tail risk and recessions we also plot in red the same tail risk measure after passing the series through a Hodrick-Prescott filter to isolate the cyclical component. This makes clear that usually tail risk is higher whenever the economy is in a recession. Therefore, a natural question to ask is whether our measure of tail risk has some predictive power over economic downturns. Our approach is comparable to Allen et al. (2012) who consider measures of systemic risk for the financial sector as predictors for economic downturns. While they focus their argument on the special intermediation role of the financial industry, our measure is economy-wide since it is based on portfolios of all firms making up the stock market. Therefore in the same logic evoked in the previous section our tail risk should anticipate movements in the real activity or business cycle indicators.

We collect a large set of macroeconomic indicators and assess their predictability by our tail risk index at monthly horizons between one and twelve months. Overall, we have eight indices that are available for different samples: 1) ADS, the Aruoba, Diebold and

Scotti macroeconomic activity indicator (02/1960 - 04/2014); 2) KCFED, the Kansas City FED macroeconomic indicator index (01/1990 - 04/2014); the NBER recession indicator (a recession period dummy, 07/1926 - 04/2014); CFNAI, the Chicago FED National Activity Index (02/1967 - 04/2014); the financial and macroeconomic indexes of Jurando, Ludvigson and Ng (2015)(07/1960 - 04/2014), the St. Louis FED Financial Stress Index (01/1994 - 04/2014), and EPU, the Economic Policy Uncertainty Index of Backer, Bloom and Davis (2015) (01/1985 - 04/2014). We run the following regression:

$$I_{t+i} = \beta_0 + \beta TR_t + \sum_{k=0}^{11} \gamma_i I_{t-k} + \epsilon_t \tag{20}$$

where I stands for the activity index we consider and i indexes the forecasting horizon ($i = 1, \ldots, 12$). Note that since the NBER recession indicator is a binary variable we estimate a Probit model. Regression results are reported in Table 9. We observe that for all indicators except the KCFED there is a significant predictability at short horizons up to 3 to 4 months ahead. For the NBER and the financial uncertainty indicator the predictive power is strong and extends to all horizons up to twelve months ahead. The signs are of course negative to capture the negative effect of current tail risk increases on future activity, except of course for NBER recessions and EPU which are downturn and uncertainty measures respectively.

To conclude this section we discuss the inter-temporal relationship between our tail risk measure and four other tail risk measures over different time periods[25], namely the Bali et al. (2014) macroeconomic uncertainty index (01/1994 - 12/2013), the BTX tail risk index (01/19963 - 08/2013) in Bollerslev et al. (2015), the Kelly and Jiang (2014) tail risk measure (01/1963 - 12/2010), and finally the Allen et al. (2012) systemic risk measure (01/1973 - 12/2012) called CATFIN. We use the same prediction regression framework as in equation (20). In Table 10, it can be seen that we predict well the Macro and KJ indicators at all horizons. For the BTX index, the predictability is limited to the very short horizons, while for CATFIN we see some predictability in the first six months. We also note that the relationship with Bollerslev et al. (2015) is almost monotonically decreasing, but that it is the contrary for the Kelly and Jiang (2014) measure. This is consistent with the fact that the KJ tail risk is a slow moving, very persistent measure.

[25]These time periods are determined by the currently available series on the web sites of the researchers.

On the contrary, the BTX measure of Bollerslev et al. (2015), like ours, is very volatile and quickly mean reverts.

5 Robustness

To build our measure of tail risk, we have made several choices regarding the parameter of the Cressie-Read discrepancy family ($\gamma = -0.5$, so-called Hellinger discrepancy), the number of principal components of the 25 size and book-to-market portfolios, the number of days over which the excess expected shortfall is computed (30 days), and the threshold of the shortfall set at 10%. Therefore, we will first verify whether our results are sensitive to these choices. Second, we argued that there were good reasons for choosing an expected shortfall measure over a VaR measure. However, other papers have built tail or downside risk VaR-based measures and shown that they were superior to other risk measures (see in particular Bali et al. (2009)). We will then reproduce our results for VaR measures and compare them to our original results with expected shortfall. Finally, our benchmark measure has been obtained from a set of 5 principal components of the 25 size and book-to-market portfolios. We already checked that the sorting portfolio results survived when we extracted the principal components from the whole cross-section of individual stocks. Another important question concerns the choice of portfolios. Will the measure keep similar properties when constructed from industry portfolios, or from the financial sector only, for example? This will be our third line of investigation for measuring the robustness of our methodology for measuring tail risk.

5.1 Robustness to the parameters of the measure

In Table 11, we report the returns of the Long-Short portfolios associated with different discrepancy parameters (first four columns). We can see that the patterns and magnitudes of returns are very similar to the benchmark results. The statistical significance of the last set of control variables, which includes volatility, disappears for 3 out of 4. The discrepancy closest to ours in the negatives, $\gamma = -1$, provides therefore the most similar results. Adding principal components (10 instead of 5) does not improve results, nor increasing nor reducing the number of days over which we compute the expected shortfall. Again, we seem to lose precision in estimating the risk premiums, especially after controlling for volatility. Finally, setting the threshold at 5% instead of 10% does not have much of an effect. We have also computed the sensitivity of our prediction results to these different values of the parameters.

For both market returns prediction and macroeconomic indicators forecast we arrived at very similar results for all configurations and all variables. We can then confidently conclude that the results are very robust to the parameters used to construct our tail risk measure. One could add that they are too robust in the sense that the risk neutralization could be done indifferently with any gamma value. In this regard it is important to emphasize that our performance criterion is a simple average where the timing of the prediction is not taken into account. If we consider that errors are more costly in bad times, then more important differences will appear between the various discrepancy gamma parameters. Risk neutralization will matter more.

5.2 Measuring Tail Risk from Industry and Sector Portfolios

We chose to build our benchmark tail risk measure on the first five principal components of 25 size and book-to-market portfolios. In this section we want to illustrate that our results in terms of risk premia and predictability are not changed significantly if we take other reference portfolios. We have seen in section 3.3 that risk premia on the high-minus-low portfolios were similar when the five principal components were computed from the whole set of securities available in the cross-section. Here we choose three sets of portfolios: a set of 10 industry portfolios, an aggregate portfolio of the financial sector, and an aggregate portfolio of the real sector. Results are reported in Table 12. Apart from the sorting portfolios columns where alphas are reported for the high-minus-low portfolio we also feature market returns predictions, in a long sample (similar to Kelly and Jiang (2014), starting in the 60s) and a short one (as in Bollerslev et al. (2015), starting in the 90s). For the long sample we also show market prediction results controlling for dividend yield and stock variance as predictors.

In the first panel featuring 10 industry portfolios, risk premiums are remarkably similar in terms of magnitudes and statistical significance to our benchmark estimates in Table 4. Note that the sign of the one-year holding period when controlling for all factors including volatility is now negative, although not significant. For the prediction of market returns, the statistical significance patterns are very similar, where the short-term prevails, but the regression coefficients are lower in general.

In the financial sector panel, risk premia are generally higher in absolute value, especially for the long holding period, but the statistical siagnificance remains quite comparable. The predictive power of market returns disappears in the short sample (after the 90s) which may be explained by the turmoil in financial markets during this period. We also lose some significance and impact when controlling for stock variance. This is to be expected since tail risk measured from financial returns is harder to disentangle from the high stock variance episodes. This interpretation is supported by the evidence in the last panel when we consider tail risk measured from the real sector alone.

The risk premia are again very close to the benchmark values in Table 4 with a slightly higher level of precision in the estimates. The predictive power is also estimated with much more precision, especially when controlling for other predictors.

We did not include results on economic predictability because they were very similar to the benchmark results with the size and book-to-market portfolios.

We can conclude that the risk-neutralized excess expected shortfall is a very robust measure of tail risk once we aggregate the individual stock returns into portfolios irrespective of the nature of the portfolios. Risk premia are of the same magnitude and estimated roughly with the same degree of precision. The financial sector tail risk measure tends to accentuate the premia but the pattern remains the same.

5.3 Measuring Tail Risk from Risk-Neutral Value-at-Risk

We have argued that expected shortfall should be preferred to value-at-risk on theoretical grounds, but some researchers have produced some empirical results that support the opposite. Therefore we construct a tail risk measure based on the risk-neutral VaR that we used in computing our excess expected shortfall for the base case with the 5 principal components of the 25 size and book-to-market portfolios. We report some selected representative results in Table 13. Let us start with our benchmark risk premium measure for the high minus low portfolio in the last columns of the table. We can see right away that the only significant figures are the averages when we do not control for other factors. Although the signs are preserved compared to our base-case results in Table 4 when we control for other factors, the magnitudes are different and the statistical significance is either marginal or disappears.

The other columns illustrate also the poor performance of the risk-neutral VaR tail risk. Market returns are not predicted well, neither in a long sample nor in a short one. No coefficient is nearly significant. The same is true for the prediction of the Kelly and Jiang (2014) tail risk or the option-based Bollerslev et al. (2015) measure. These series were predicted very well by the Hellinger excess expected shortfall measure, especially the Kelly Jiang measure. This lack of predictive power extends to all series, except the macroeconomic uncertainty index and the NBER indicator of recessions. Finally, impulse response functions for employment and industrial production (not reported for space considerations) are essentially zero.

6 Conclusion

We propose a new measure of tail risk based on the risk-neutralized returns of a few principal components of the cross-section of stock returns. We have shown that the results are robust whether we compute these principal components from a set of aggregate portfolios (Fama-French size and book-to-market portfolios or industry portfolios) or from the whole cross-section of CRSP returns. This measure has two main advantages: it accounts for changes in economic conditions through the risk-neutralization nonparametric procedure and it is based only on stock returns. Not using option prices is useful to extend the sample considerably but opens also the possibility to apply the measure in markets where liquid option markets do not exist. Moreover a similar measure can be computed for many other asset markets.

The series computed from 1926 to the present captures all main events that have increased uncertainty in the economy, whether they find their source in the financial market or in the political arena. It is therefore a tail risk measure that goes beyond financial systemic risk. We have investigated thoroughly its relation with other tail risk measures. For the option-based measures, our empirical comparison revealed that the implications of both measures were very similar in terms of risk premia for short and long holding periods. However we have shown that the nature of our series is very different from the tail-risk measure proposed by Kelly and Jiang (2014). The latter is very persistent and does not capture the big uncertainty events. In terms of forecasting power of market returns our series has a good performance in the short-run while the

Kelly and Jiang (2014) tail risk predicts well six to twelve months ahead.

Our extensive investigation of economic predictability has confirmed the usefulness of our measure for capturing future conditions in employment, output or more comprehensive measures of real activity. A detailed comparison with the Bloom (2009) indicator of economic uncertainty based on volatility has shown that our tail risk measure adds new information and implies different impulse response functions for employment and industrial production. Finally, our measure predicts remarkably well the NBER recessions up to twelve months ahead and the data-rich aggregate financial and macroeconomic indicators of economic uncertainty produced recently by Jurado et al. (2015). Our limited-information measure appears to be a very useful indicator to test theories of business cycles with economic shocks or to estimate or calibrate equilibrium asset pricing models with disasters, ambiguity aversion or disappointment aversion, all implying an important role for tail events.

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Table 1: **Principal Component Variance**

Principal Component	1	2	3	4	5
Variance (Cumulative)	0.62	0.76	0.83	0.86	0.89

This table present the first five principal components cumulative variance. The principal component analysis was performed for the whole sample (07/1926-04/2014). Each month we compute the five principal components using the daily returns of the last 30 days for the 25 size and book-to-market Fama-French portfolios, and the corresponding cumulative explained variances. The reported figures are the averages of the monthly figures over the whole sample.

Table 2: **Correlations with Other Tail Risk Measures and Financial and Macroeconomic Indicators**

	Hellinger	S&P 500	CRSP	Bloom	KJ	BTX	VIX	Macro
Hellinger	1.0000							
S&P 500	-0.3210	1.0000						
CRSP	-0.2433	0.9842	1.0000					
Bloom	0.4572	-0.1333	-0.1498	1.0000				
KJ	-0.0723	0.0854	0.0802	-0.0202	1.0000			
BTX	0.4303	-0.1293	-0.1341	0.3726	-0.2289	1.0000		
VIX	0.5581	-0.3723	-0.3709	0.9288	-0.3820	0.6625	1.0000	
Macro	0.4684	-0.0578	-0.0243	0.5809	-0.2210	0.4395	0.5548	1.0000
CATFIN	0.4507	-0.4146	-0.4385	0.3811	-0.0819	0.5206	0.6463	0.5422

This table present the correlation coefficients between Hellinger Tail Risk and other tail measures. CRSP denotes value weighted CRSP stock index (07/1926 - 04/2014), Bloom refers to Bloom (2009) volatility factor (07/1962 - 06/2008), KJ refers to Kelly and Jiang (2014) tail risk index (01/1963 - 12/2010), BTX refers to Bollerslev, Todorov and Xu (2014) tail risk index (01/19963 - 08/2013), VIX denotes the CBOE volatility index (01/1990 - 04/2014), Macro refers to Bali, Brown and Caglayan (2014) macroeconomic uncertainty index (01/1994 - 12/2013) and CAFTIN denotes Allen, Bali and Tang (2012) systemic risk measure (01/1973 - 12/2012).

Table 3: **Option vs. Hellinger Sorted Portfolios**

	Option Implied Tail Risk										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	4.63	2.13	2.48	1.26	1.78	1.17	0.85	0.97	0.81	0.90	-3.73
	(2.57)	(2.29)	(2.49)	(1.54)	(1.86)	(1.55)	(1.33)	(1.64)	(1.23)	(1.72)	(-2.35)
FF3	3.51	1.47	1.74	0.50	1.15	0.55	0.31	0.40	0.36	0.56	-2.96
	(2.22)	(1.53)	(1.76)	(0.62)	(1.14)	(0.72)	(0.44)	(0.71)	(0.52)	(1.00)	(-2.15)
FF3+MOM	3.72	1.32	1.54	0.47	0.93	0.48	0.30	0.45	0.31	0.64	-3.08
	(2.02)	(1.41)	(1.59)	(0.60)	(1.01)	(0.65)	(0.45)	(0.81)	(0.48)	(1.19)	(-1.84)
FF3+MOM+LIQ	4.43	1.37	1.59	0.41	1.08	0.60	0.37	0.41	0.50	0.63	-3.80
	(1.72)	(1.26)	(1.42)	(0.46)	(0.91)	(0.68)	(0.49)	(0.67)	(0.62)	(1.03)	(-1.60)
FF3+MOM+LIQ+VOL	-0.89	0.92	0.85	-0.19	-0.37	-0.22	0.71	0.13	0.03	1.43	2.32
	(-0.33)	(0.51)	(0.49)	(-0.12)	(-0.25)	(-0.16)	(0.59)	(0.12)	(0.03)	(1.56)	(1.02)

	Hellinger Tail Risk										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	4.48	2.71	2.20	1.22	1.88	0.84	0.73	0.63	0.88	1.42	-3.06
	(2.52)	(2.44)	(2.27)	(1.53)	(1.96)	(1.25)	(1.20)	(1.12)	(1.52)	(2.18)	(-2.03)
FF3	3.37	1.92	1.48	0.49	1.27	0.30	0.22	0.21	0.32	0.97	-2.40
	(2.17)	(1.72)	(1.52)	(0.62)	(1.29)	(0.43)	(0.36)	(0.36)	(0.54)	(1.35)	(-1.87)
FF3+MOM	3.67	1.53	1.28	0.37	1.15	0.26	0.19	0.17	0.48	1.05	-2.63
	(2.04)	(1.44)	(1.36)	(0.48)	(1.28)	(0.38)	(0.33)	(0.30)	(0.82)	(1.51)	(-1.67)
FF3+MOM+LIQ	4.39	1.74	1.34	0.36	1.35	0.19	0.15	0.19	0.44	1.25	-3.14
	(1.73)	(1.34)	(1.21)	(0.42)	(1.17)	(0.25)	(0.23)	(0.29)	(0.66)	(1.58)	(-1.36)
FF3+MOM+LIQ+VOL	-0.01	0.26	0.90	-0.09	0.13	0.02	-0.41	-0.02	0.15	1.48	1.49
	(-0.00)	(0.15)	(0.51)	(-0.07)	(0.09)	(0.01)	(-0.37)	(-0.02)	(0.14)	(1.16)	(0.72)

This table features the returns attached to decile portfolios of all CRSP stocks with code 10-11 sorted according to their tail risk hedging capacity. In Panel A, the tail risk factor is computed according to the nonparametric methodology of Ait-Sahalia and Lo (2000) and uses option prices. In Panel B, the tail risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios according to the minimum discrepancy procedure to risk-adjust returns. For each month in our sample from January 1996 to April 2014, we sort the stocks and track their returns one month post-formation. In the first line we report the average portfolio returns. In the following lines we report the $\alpha's$ of regressions where we control for the factors indicated in the first column. Newey-West t-statistics computed with one lag are reported between parentheses.

Table 4: **Hellinger Sorted Portfolios - 25 Size and Book-to-Market Portfolios**

	Panel A: One Month Holding Period										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	2.34	1.38	1.04	0.74	0.88	0.56	0.48	0.38	0.53	0.82	-1.52
	(4.18)	(3.57)	(3.13)	(2.51)	(2.83)	(2.16)	(1.99)	(1.74)	(2.39)	(3.30)	(-3.41)
FF3	1.21	0.31	0.05	-0.21	-0.08	-0.28	-0.31	-0.35	-0.18	0.06	-1.15
	(3.20)	(1.85)	(0.39)	(-2.13)	(-0.66)	(-2.90)	(-3.35)	(-4.34)	(-1.57)	(0.46)	(-3.09)
FF3+MOM	1.24	0.32	0.07	-0.19	-0.05	-0.27	-0.30	-0.33	-0.17	0.05	-1.20
	(3.17)	(1.96)	(0.51)	(-1.96)	(-0.41)	(-2.75)	(-3.22)	(-4.05)	(-1.53)	(0.34)	(-3.10)
FF3+MOM+LIQ	1.35	0.32	0.07	-0.18	-0.05	-0.27	-0.28	-0.32	-0.16	0.06	-1.28
	(2.90)	(2.03)	(0.54)	(-1.87)	(-0.41)	(-2.72)	(-3.04)	(-3.88)	(-1.36)	(0.48)	(-2.79)
FF3+MOM+LIQ+VOL	0.87	-0.02	-0.05	-0.31	-0.14	-0.21	-0.34	-0.31	-0.08	0.04	-0.83
	(2.02)	(-0.13)	(-0.32)	(-2.62)	(-1.03)	(-1.98)	(-3.00)	(-3.20)	(-0.53)	(0.25)	(-1.80)

	Panel B: One Year Holding Period										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	20.83	11.19	9.33	6.09	6.06	4.55	3.57	3.04	4.60	6.99	-13.84
	(3.49)	(3.04)	(2.82)	(2.29)	(2.43)	(1.87)	(1.55)	(1.44)	(2.04)	(2.57)	(-3.10)
FF3	0.77	-3.51	-4.00	-5.62	-4.91	-5.90	-6.61	-6.06	-4.82	-2.40	-3.17
	(0.25)	(-2.18)	(-2.61)	(-4.64)	(-4.09)	(-4.32)	(-5.05)	(-5.14)	(-3.79)	(-1.70)	(-1.16)
FF3+MOM	2.60	-2.32	-3.34	-5.15	-4.89	-5.40	-6.10	-5.34	-4.11	-2.39	-4.99
	(0.93)	(-1.53)	(-2.26)	(-4.68)	(-4.42)	(-4.27)	(-4.99)	(-4.87)	(-3.23)	(-1.64)	(-2.01)
FF3+MOM+LIQ	4.04	-2.26	-3.39	-5.23	-5.04	-5.25	-5.94	-5.08	-3.52	-0.94	-4.98
	(1.49)	(-1.81)	(-2.36)	(-5.34)	(-4.60)	(-4.04)	(-4.58)	(-4.35)	(-2.56)	(-0.53)	(-2.05)
FF3+MOM+LIQ+VOL	-15.34	-10.25	-10.61	-10.54	-11.14	-9.02	-8.52	-6.77	-8.35	-3.79	11.56
	(-1.38)	(-1.89)	(-2.64)	(-3.26)	(-4.60)	(-3.82)	(-3.81)	(-3.58)	(-4.41)	(-1.33)	(1.03)

This table presents the results for the sorting portfolio procedure where CRSP stocks with code 10-11 are sorted into 10 decile portfolios according to their tail risk hedging capacity. The tail risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios. The returns are adjusted according to the minimum discrepancy procedure with a gamma of -0.5 (Hellinger measure). For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the $\alpha's$ of regressions where we control for the factors indicated in the first column. Newey-West t-statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Table 5: **Hellinger Sorted Portfolios - All securities**

	Panel A: One Month Holding Period										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	2.24	1.12	1.02	0.93	0.62	0.66	0.54	0.60	0.45	0.97	-1.28
	(4.09)	(3.21)	(3.31)	(3.08)	(2.35)	(2.39)	(2.25)	(2.21)	(2.09)	(3.38)	(-2.98)
FF3	1.13	0.08	0.04	0.02	-0.24	-0.20	-0.25	-0.24	-0.25	0.14	-0.98
	(3.02)	(0.62)	(0.36)	(0.14)	(-2.31)	(-1.97)	(-2.85)	(-2.28)	(-2.83)	(0.92)	(-2.59)
FF3+MOM	1.16	0.11	0.07	0.03	-0.23	-0.18	-0.22	-0.22	-0.26	0.11	-1.05
	(3.01)	(0.78)	(0.61)	(0.26)	(-2.16)	(-1.73)	(-2.58)	(-1.99)	(-2.96)	(0.73)	(-2.69)
FF3+MOM+LIQ	1.25	0.11	0.08	0.04	-0.23	-0.18	-0.22	-0.19	-0.25	0.12	-1.13
	(2.72)	(0.81)	(0.69)	(0.33)	(-2.30)	(-1.71)	(-2.57)	(-1.58)	(-2.84)	(0.77)	(-2.42)
FF3+MOM+LIQ+VOL	0.90	-0.04	-0.13	-0.17	-0.32	-0.26	-0.21	-0.16	-0.29	0.13	-0.77
	(2.21)	(-0.25)	(-0.86)	(-1.09)	(-2.73)	(-2.58)	(-2.01)	(-1.32)	(-2.78)	(0.65)	(-1.73)

	Panel B: One Year Holding Period										
Portfolio	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High - Low
Average Return	18.93	10.33	8.18	6.26	5.64	4.64	4.56	4.38	4.26	9.05	-9.88
	(3.70)	(2.98)	(2.73)	(2.28)	(2.05)	(1.85)	(1.90)	(1.85)	(1.94)	(2.87)	(-2.92)
FF3	1.90	-4.33	-4.70	-5.82	-5.68	-5.80	-5.63	-5.82	-4.86	-2.30	-4.20
	(0.64)	(-2.88)	(-3.52)	(-4.68)	(-3.46)	(-4.17)	(-4.11)	(-4.51)	(-3.54)	(-1.33)	(-1.31)
FF3+MOM	1.54	-3.26	-3.94	-5.34	-4.46	-5.13	-5.04	-5.17	-3.99	-1.67	-3.21
	(0.67)	(-2.41)	(-3.22)	(-4.59)	(-2.86)	(-3.69)	(-3.70)	(-4.13)	(-3.00)	(-0.90)	(-1.32)
FF3+MOM+LIQ	2.91	-3.13	-3.61	-4.68	-4.71	-5.49	-5.15	-4.89	-3.66	-0.20	-3.10
	(1.18)	(-2.49)	(-2.69)	(-3.92)	(-2.89)	(-4.12)	(-3.73)	(-3.95)	(-2.81)	(-0.10)	(-1.16)
FF3+MOM+LIQ+VOL	-11.34	-10.15	-9.56	-10.18	-11.58	-8.21	-8.04	-9.81	-9.95	-5.50	5.85
	(-1.61)	(-2.41)	(-3.04)	(-3.43)	(-3.42)	(-3.33)	(-3.36)	(-4.23)	(-4.77)	(-1.30)	(1.08)

This table presents the results for the sorting portfolio procedure where CRSP stocks with code 10-11 are sorted into 10 decile portfolios according to their tail risk hedging capacity. The tail risk factor is computed from the returns of the first five principal components extracted from the whole set of securities available at any point in time. The returns are adjusted according to the minimum discrepancy procedure with a gamma of -0.5 (Hellinger measure). For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the $\alpha's$ of regressions where we control for the factors indicated in the first column. Newey-West t-statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Table 6: **Tail Risk Measure under Objective Excess Shortfall - Sorted Portfolios**

	Panel A: One Month Holding Period										
Portfolio	Low	2	3	4	5	6	7	8	9	High	High - Low
Average Return	1.84	1.37	1.11	0.77	0.59	0.61	0.65	0.49	0.54	1.17	-0.67
	(4.26)	(2.91)	(3.25)	(2.59)	(2.28)	(2.46)	(2.67)	(2.05)	(2.26)	(3.85)	(-2.50)
FF3	0.74	0.36	0.12	-0.16	-0.25	-0.18	-0.13	-0.30	-0.25	0.28	-0.46
	(3.53)	(1.06)	(0.80)	(-1.42)	(-2.68)	(-1.73)	(-1.35)	(-3.71)	(-2.58)	(1.79)	(-1.95)
FF3+MOM	0.74	0.38	0.13	-0.15	-0.24	-0.17	-0.10	-0.28	-0.23	0.29	-0.46
	(3.61)	(1.09)	(0.85)	(-1.32)	(-2.46)	(-1.53)	(-1.06)	(-3.46)	(-2.40)	(1.83)	(-1.99)
FF3+MOM+LIQ	0.75	0.46	0.14	-0.14	-0.22	-0.15	-0.12	-0.27	-0.23	0.33	-0.42
	(3.65)	(1.08)	(0.92)	(-1.21)	(-2.30)	(-1.27)	(-1.36)	(-3.36)	(-2.36)	(2.02)	(-1.81)
FF3+MOM+LIQ+VOL	0.30	0.06	0.04	-0.25	-0.26	-0.21	-0.16	-0.34	-0.17	0.43	0.12
	(1.30)	(0.18)	(0.28)	(-2.15)	(-2.13)	(-1.33)	(-1.30)	(-3.63)	(-1.31)	(2.17)	(0.46)

	Panel B: One Year Holding Period										
Portfolio	Low	2	3	4	5	6	7	8	9	High	High - Low
Average Return	17.97	10.17	8.49	5.99	4.56	4.89	4.10	4.11	5.46	10.50	-7.47
	(3.47)	(3.00)	(2.83)	(2.18)	(1.88)	(1.93)	(1.84)	(1.89)	(2.24)	(3.20)	(-2.38)
FF3	-0.51	-3.39	-4.01	-5.87	-5.98	-5.52	-5.68	-5.61	-4.77	-1.70	-1.19
	(-0.18)	(-2.02)	(-2.49)	(-4.46)	(-4.45)	(-4.04)	(-5.07)	(-5.60)	(-4.37)	(-1.05)	(-0.45)
FF3+MOM	-0.10	-2.21	-3.35	-5.39	-5.45	-5.09	-5.03	-5.07	-4.48	-0.28	-0.17
	(-0.04)	(-1.40)	(-2.24)	(-4.28)	(-4.08)	(-4.16)	(-4.87)	(-5.51)	(-4.18)	(-0.18)	(-0.09)
FF3+MOM+LIQ	0.62	-1.69	-3.00	-5.35	-5.67	-4.70	-5.30	-5.07	-3.84	1.41	0.79
	(0.27)	(-1.02)	(-1.88)	(-4.16)	(-4.11)	(-3.43)	(-5.56)	(-5.48)	(-3.49)	(0.85)	(0.42)
FF3+MOM+LIQ+VOL	-18.74	-13.20	-10.47	-11.27	-8.82	-7.98	-7.41	-8.03	-6.27	-2.15	16.59
	(-1.93)	(-2.19)	(-2.55)	(-3.56)	(-3.95)	(-2.87)	(-4.02)	(-4.66)	(-3.59)	(-0.88)	(1.75)

This table presents the results for the sorting portfolio procedure where CRSP stocks with code 10-11 are sorted into 10 decile portfolios according to their hedging capacity with respect to the ratio between risk neutral and objective tail risk measures. The tail risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios. The objective tail risk expected shortfall is calculated assuming a homogeneous probability density function. For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the $\alpha's$ of regressions where we control for the factors indicated in the first column. Newey-West t-statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Table 7: **Predicting Market Returns**

Panel A: Hellinger Tail Risk

Forecast	TR	t	TR	t	TR	t	TR	t	TR	t
1	0.10	(2.90)	0.10	(1.36)	0.10	(2.43)	0.03	(0.52)	0.08	(1.81)
2	0.10	(2.28)	0.11	(1.35)	0.12	(2.45)	0.08	(1.06)	0.10	(1.77)
3	0.10	(2.51)	0.17	(1.97)	0.15	(3.09)	0.12	(1.53)	0.13	(2.35)
4	0.08	(1.90)	0.21	(2.21)	0.15	(2.93)	0.15	(1.78)	0.12	(2.06)
5	0.06	(1.50)	0.23	(2.20)	0.14	(2.62)	0.17	(1.94)	0.11	(1.80)
6	0.03	(0.59)	0.21	(1.69)	0.12	(1.97)	0.15	(1.44)	0.08	(1.21)
7	0.03	(0.57)	0.22	(1.71)	0.12	(1.97)	0.17	(1.52)	0.09	(1.27)
8	0.01	(0.21)	0.21	(1.47)	0.11	(1.64)	0.14	(1.15)	0.08	(1.05)
9	-0.02	(-0.33)	0.18	(1.14)	0.09	(1.20)	0.11	(0.81)	0.06	(0.67)
10	-0.03	(-0.49)	0.20	(1.21)	0.09	(1.08)	0.12	(0.87)	0.06	(0.63)
11	-0.03	(-0.48)	0.20	(1.24)	0.09	(1.03)	0.12	(0.87)	0.05	(0.58)
12	-0.02	(-0.42)	0.19	(1.24)	0.08	(1.00)	0.11	(0.81)	0.05	(0.55)

Panel B: Other Tail Risk Measures

Forecast	Bloom	t	BTX	t	KJ	t	VIX	t	CATFIN	t
1	0.08	(1.98)	0.11	(1.13)	0.05	(1.28)	0.07	(1.00)	0.04	(0.73)
2	0.11	(2.11)	0.15	(1.21)	0.09	(1.67)	0.11	(1.15)	0.04	(0.58)
3	0.10	(1.74)	0.19	(1.95)	0.10	(1.65)	0.15	(1.63)	0.09	(1.08)
4	0.09	(1.37)	0.26	(3.33)	0.12	(1.77)	0.17	(2.01)	0.07	(0.84)
5	0.08	(1.15)	0.30	(3.61)	0.15	(1.89)	0.21	(2.45)	0.08	(0.98)
6	0.08	(0.96)	0.29	(2.87)	0.17	(2.04)	0.22	(2.52)	0.06	(0.75)
7	0.07	(0.77)	0.31	(2.48)	0.19	(2.13)	0.23	(2.31)	0.06	(0.90)
8	0.04	(0.41)	0.30	(2.10)	0.21	(2.22)	0.22	(2.04)	0.06	(0.85)
9	0.03	(0.32)	0.30	(2.11)	0.22	(2.27)	0.20	(1.78)	0.05	(0.65)
10	0.02	(0.20)	0.30	(2.05)	0.23	(2.32)	0.20	(1.67)	0.06	(0.79)
11	0.01	(0.08)	0.32	(2.19)	0.25	(2.49)	0.20	(1.58)	0.06	(0.76)
12	0.01	(0.05)	0.34	(2.29)	0.27	(2.69)	0.21	(1.53)	0.07	(0.88)

This table presents results for CRSP value weighted market index returns prediction regressions: $r_{[t,t+k]} = \alpha + \beta TR_{t-1} + r_{[t-k,t]}$. We consider the following financial "tail risk" measures (sample in parenthesis) as explanatory variables: Hellinger Tail Risk, Bloom (2009) volatility (07/1962 - 06/2008), Kelly and Jiang (2014) tail risk measure (01/1963 - 12/2010), Allen, Bale and Tang (2012) systemic risk measure (01/1973 - 12/2012), the VIX index (01/1990 - 04/2014) and Bollerslev, Todorov and Xu (2015) tail risk measure (01/1996 - 08/2013). In Panel A we run the same regression for the Hellinger tail risk measure for various samples corresponding to the respective measures considered in Panel B. t-statistics are calculated using Newey West variance matrix with 2 - 24 lags for 1 - 12 forecasting horizon.

Table 8: **Welsh and Goyal (2008) Regressions**

Panel A: Hellinger Tail Risk

Forecast	TR (BM)	t	TR (DP)	t	TR (EPR)	t	TR (DPR)	t	TR (DY)	t	TR (SV)	t	TR (DYS)	t
1	0.03	(0.58)	0.02	(0.42)	0.04	(0.77)	0.03	(0.59)	0.03	(0.59)	0.12	(2.19)	0.01	(0.18)
2	0.09	(1.62)	0.08	(1.36)	0.11	(1.96)	0.09	(1.61)	0.09	(1.63)	0.16	(3.21)	0.06	(1.14)
3	0.14	(2.62)	0.11	(2.17)	0.15	(2.96)	0.14	(2.65)	0.14	(2.70)	0.19	(3.19)	0.10	(1.92)
4	0.14	(2.59)	0.11	(2.01)	0.15	(2.86)	0.14	(2.60)	0.14	(2.68)	0.17	(2.82)	0.09	(1.67)
5	0.12	(2.55)	0.09	(1.82)	0.13	(2.82)	0.12	(2.59)	0.13	(2.73)	0.14	(2.31)	0.06	(1.26)
6	0.11	(2.42)	0.08	(1.61)	0.12	(2.65)	0.11	(2.50)	0.12	(2.65)	0.12	(2.00)	0.05	(1.04)
7	0.10	(2.01)	0.07	(1.38)	0.11	(2.25)	0.10	(2.13)	0.11	(2.28)	0.09	(1.44)	0.04	(0.85)
8	0.10	(1.98)	0.04	(1.32)	0.11	(2.21)	0.10	(2.10)	0.11	(2.25)	0.08	(1.31)	0.04	(0.83)
9	0.10	(1.74)	0.06	(1.20)	0.11	(2.00)	0.10	(1.86)	0.11	(2.00)	0.07	(1.11)	0.03	(0.70)
10	0.08	(1.35)	0.05	(0.82)	0.10	(1.60)	0.09	(1.46)	0.09	(1.60)	0.05	(0.78)	0.02	(0.33)
11	0.08	(1.18)	0.04	(0.66)	0.09	(1.42)	0.08	(1.28)	0.09	(1.42)	0.04	(0.58)	0.01	(0.18)
12	0.08	(1.13)	0.04	(0.58)	0.09	(1.39)	0.08	(1.22)	0.09	(1.39)	0.03	(0.48)	0.01	(0.11)

Panel A (Cont'd.)

Forecast	BM	t	DP	t	EPR	t	DPR	t	DY	t	SV	t	DYS	t
1	0.06	(1.30)	0.02	(0.35)	0.07	(1.39)	0.09	(2.16)	0.09	(2.17)	-0.18	(-3.79)	0.06	(1.04)
2	0.08	(1.45)	0.03	(0.45)	0.10	(1.40)	0.13	(2.40)	0.13	(2.41)	-0.15	(-2.22)	0.07	(0.90)
3	0.11	(1.61)	0.04	(0.69)	0.12	(1.44)	0.16	(2.59)	0.16	(2.58)	-0.13	(-1.45)	0.09	(1.03)
4	0.13	(1.59)	0.06	(0.92)	0.13	(1.32)	0.19	(2.54)	0.19	(2.55)	-0.10	(-0.95)	0.13	(1.42)
5	0.14	(1.56)	0.08	(1.10)	0.14	(1.24)	0.21	(2.45)	0.21	(2.51)	-0.08	(-0.92)	0.16	(1.75)
6	0.16	(1.55)	0.09	(1.16)	0.15	(1.25)	0.22	(2.39)	0.23	(2.44)	-0.05	(-0.74)	0.17	(1.80)
7	0.17	(1.56)	0.09	(1.22)	0.16	(1.30)	0.24	(2.37)	0.25	(2.40)	-0.02	(-0.36)	0.18	(1.89)
8	0.18	(1.56)	0.09	(1.21)	0.17	(1.39)	0.26	(2.35)	0.26	(2.38)	0.00	(-0.05)	0.18	(1.89)
9	0.19	(1.56)	0.09	(1.20)	0.18	(1.50)	0.27	(2.36)	0.27	(2.38)	0.01	(0.19)	0.19	(1.97)
10	0.19	(1.56)	0.10	(1.19)	0.19	(1.58)	0.28	(2.37)	0.28	(2.39)	0.01	(0.25)	0.20	(2.10)
11	0.20	(1.56)	0.10	(1.09)	0.21	(1.70)	0.29	(2.38)	0.29	(2.40)	0.02	(0.49)	0.21	(2.12)
12	0.20	(1.53)	0.10	(1.03)	0.22	(1.79)	0.30	(2.38)	0.31	(2.42)	0.03	(0.68)	0.20	(2.05)

Welch and Goyal (2008) Regressions (Cont'd)

Panel B: Control Variables														
Forecast	TR (DRS)	t	TR (Infl.)	t	TR (LTY)	t	TR (LTRR)	t	TR (TS)	t	TR (TBR)	t	TR (NEE)	t
1	0.02	(0.37)	0.02	(0.34)	0.03	(0.52)	0.02	(0.30)	0.02	(0.34)	0.03	(0.50)	0.02	(0.44)
2	0.08	(1.37)	0.08	(1.51)	0.09	(1.52)	0.08	(1.32)	0.07	(1.27)	0.09	(1.54)	0.08	(1.50)
3	0.12	(2.28)	0.12	(2.38)	0.13	(2.45)	0.12	(2.25)	0.11	(2.17)	0.13	(2.48)	0.12	(2.50)
4	0.12	(2.24)	0.12	(2.24)	0.13	(2.41)	0.12	(2.21)	0.11	(2.13)	0.13	(2.44)	0.12	(2.50)
5	0.10	(2.14)	0.10	(2.04)	0.11	(2.30)	0.10	(2.07)	0.09	(1.99)	0.11	(2.32)	0.10	(2.19)
6	0.09	(1.99)	0.08	(1.86)	0.10	(2.11)	0.09	(1.90)	0.08	(1.76)	0.10	(2.11)	0.09	(1.83)
7	0.08	(1.65)	0.07	(1.51)	0.09	(1.73)	0.08	(1.54)	0.07	(1.42)	0.09	(1.72)	0.08	(1.47)
8	0.08	(1.64)	0.07	(1.44)	0.09	(1.69)	0.08	(1.54)	0.07	(1.34)	0.09	(1.67)	0.08	(1.42)
9	0.08	(1.43)	0.07	(1.27)	0.09	(1.45)	0.07	(1.32)	0.06	(1.16)	0.09	(1.46)	0.07	(1.22)
10	0.06	(1.04)	0.05	(0.89)	0.07	(1.09)	0.06	(0.94)	0.04	(0.79)	0.07	(1.10)	0.06	(0.89)
11	0.06	(0.85)	0.05	(0.73)	0.07	(0.93)	0.05	(0.75)	0.04	(0.61)	0.07	(0.93)	0.05	(0.72)
12	0.05	(0.78)	0.04	(0.63)	0.06	(0.87)	0.04	(0.67)	0.03	(0.52)	0.06	(0.88)	0.05	(0.65)

Panel B (Cont'd.														
Forecast	DRS	t	Infl.	t	LTY	t	LTRR	t	TS	t	TBR	t	NEE	t
1	-0.05	(-1.18)	-0.04	(-0.88)	0.03	(0.81)	0.09	(2.10)	0.05	(1.27)	0.01	(0.28)	-0.01	(-0.25)
2	-0.04	(-0.77)	0.00	(-0.05)	0.06	(1.06)	0.09	(2.00)	0.06	(1.04)	0.03	(0.56)	-0.01	(-0.08)
3	-0.01	(-0.28)	-0.01	(-0.11)	0.08	(1.15)	0.07	(1.49)	0.05	(0.82)	0.04	(0.64)	0.01	(0.07)
4	-0.02	(-0.36)	-0.02	(-0.31)	0.08	(1.11)	0.09	(1.84)	0.06	(0.84)	0.05	(0.59)	0.01	(0.08)
5	-0.04	(-0.76)	-0.04	(-0.63)	0.10	(1.14)	0.13	(3.08)	0.08	(0.97)	0.05	(0.61)	0.00	(-0.01)
6	-0.04	(-0.67)	-0.05	(-0.66)	0.12	(1.28)	0.14	(3.22)	0.09	(1.04)	0.07	(0.69)	-0.01	(-0.07)
7	-0.03	(-0.62)	-0.06	(-0.84)	0.14	(1.46)	0.15	(3.41)	0.10	(1.12)	0.08	(0.78)	-0.01	(-0.08)
8	-0.01	(-0.18)	-0.07	(-0.96)	0.16	(1.68)	0.14	(3.33)	0.12	(1.19)	0.09	(0.87)	-0.01	(-0.08)
9	0.01	(0.15)	-0.07	(-0.93)	0.18	(1.87)	0.13	(3.22)	0.13	(1.33)	0.10	(0.94)	-0.02	(-0.12)
10	0.03	(0.57)	-0.07	(-0.94)	0.20	(2.05)	0.11	(2.88)	0.14	(1.46)	0.10	(1.01)	-0.02	(-0.14)
11	0.04	(0.66)	-0.07	(-0.89)	0.21	(2.26)	0.11	(2.99)	0.15	(1.54)	0.11	(1.16)	-0.02	(-0.13)
12	0.04	(0.71)	-0.08	(-1.01)	0.23	(2.49)	0.12	(3.55)	0.16	(1.63)	0.12	(1.32)	-0.03	(-0.14)

This table presents results for CRSP value-weighted market index returns prediction regressions: $r_{[t,t+k]} = \alpha + \beta TR_t + r_{[t-k,t]} + GW_t + u_t$ where TR indicates the Hellinger Tail Risk. Additional controls (GW) indicate the Goyal and Welch (2008) surveyed predictors: Book-to- Market (BM), Dividend Payout (DP), Earnings Price Ratio (EPR), Dividend Price Ratio (DPR), Dividend Yield (DY), Stock Variance (SV), Default Yield Spread (DYS), Default Return Spread (DRS), Inflation (Infl.), Long-Term Yield (LTY), Long-Term Rate of Returns (LTRR), Term Spread (TS), Treasury Bill Rate (TBR) and Net Equity Expansion (NEE). In both panels A and B, we present the estimated coefficients for the Hellinger Tail Risk in the upper part and the estimated coefficient for the additional control variables in the lower part. t-statistics are calculated using Newey-West variance matrix with 2 - 24 lags for 1 - 12 forecasting horizon. Sample: July 1962 to December 2012.

Table 9: **Macroeconomic Activity Prediction Regressions**

Panel A												
Forecast	ADS	t	R^2	KCFED	t	R^2	NBER	t	R^2	CFNAI	t	R^2
1	-0.05	(-2.70)	0.84	0.01	(-0.27)	0.92	0.69	-3.17	0.78	-0.10	(-2.80)	0.53
2	-0.09	(-3.09)	0.52	-0.03	(-0.55)	0.81	0.52	-2.50	0.62	-0.12	(-3.37)	0.46
3	-0.08	(-2.40)	0.39	-0.09	(-1.05)	0.74	0.58	-3.27	0.50	-0.11	(-2.97)	0.36
4	-0.05	(-1.57)	0.30	-0.16	(-1.39)	0.65	0.54	-3.45	0.40	-0.08	(-1.89)	0.27
5	-0.04	(-1.27)	0.22	-0.17	(-1.40)	0.56	0.45	-3.24	0.30	-0.06	(-1.68)	0.19
6	-0.03	(-0.76)	0.17	-0.20	(-1.46)	0.49	0.43	-3.27	0.22	-0.07	(-1.89)	0.16
7	-0.02	(-0.45)	0.14	-0.21	(-1.36)	0.41	0.45	-3.65	0.15	-0.03	(-0.87)	0.12
8	-0.03	(-0.65)	0.12	-0.18	(-1.23)	0.33	0.42	-3.56	0.10	-0.03	(-0.70)	0.10
9	-0.03	(-0.57)	0.10	-0.16	(-1.15)	0.29	0.38	-3.35	0.07	-0.07	(-1.66)	0.09
10	-0.02	(-0.39)	0.08	-0.18	(-1.28)	0.26	0.40	-3.57	0.05	-0.05	(-1.26)	0.06
11	-0.04	(-0.79)	0.06	-0.12	(-1.03)	0.22	0.43	-3.80	0.03	-0.07	(-1.37)	0.05
12	-0.05	(-0.96)	0.04	-0.07	(-0.70)	0.19	0.42	-3.74	0.03	-0.10	(-1.69)	0.04
Panel B												
Forecast	Fin. Uncert.	t	R^2	Macro Uncert.	t	R^2	St. Louis	t	R^2	EPU	t	R^2
1	-0.22	(-2.44)	0.32	-0.10	(-2.35)	0.42	-0.17	(-1.81)	0.15	0.08	(4.18)	0.75
2	-0.26	(-4.17)	0.14	-0.16	(-2.59)	0.14	-0.25	(-2.21)	0.10	0.09	(3.80)	0.61
3	-0.20	(-4.47)	0.07	-0.14	(-2.17)	0.07	-0.24	(-2.51)	0.08	0.11	(3.15)	0.56
4	-0.16	(-3.40)	0.05	-0.17	(-2.16)	0.04	-0.25	(-2.47)	0.09	0.03	(0.82)	0.52
5	-0.14	(-2.66)	0.04	-0.13	(-1.66)	0.03	-0.17	(-1.63)	0.06	0.05	(1.05)	0.48
6	-0.12	(-2.63)	0.03	-0.12	(-1.83)	0.03	-0.14	(-1.79)	0.06	0.04	(0.90)	0.44
7	-0.13	(-2.66)	0.03	-0.13	(-2.16)	0.03	-0.16	(-1.80)	0.06	0.09	(2.24)	0.43
8	-0.16	(-3.38)	0.04	-0.11	(-1.90)	0.03	-0.04	(-0.55)	0.05	0.04	(1.05)	0.42
9	-0.11	(-2.83)	0.02	-0.11	(-2.31)	0.04	-0.08	(-1.07)	0.05	0.05	(0.86)	0.41
10	-0.10	(-3.05)	0.02	-0.09	(-1.90)	0.03	-0.14	(-2.23)	0.05	0.06	(1.05)	0.40
11	-0.15	(-3.06)	0.03	-0.08	(-2.18)	0.03	-0.02	(-0.35)	0.03	0.11	(1.75)	0.38
12	-0.07	(-1.81)	0.02	-0.05	(-1.52)	0.03	0.02	(0.59)	0.04	0.13	(2.43)	0.34

This table present the result for the prediction regressions for a variety of macroeconomic indicators (sample in parenthesis). ADS represents the Aruoba, Diebold and Scotti macroeconomic activity indicator (02/1960 - 04/2014), KCFED represents the Kansas City FED macroeconomic indicator index (01/1990 - 04/2014), NBER represents a recession period dummy (07/1926 - 04/2014), CFNAI represents the Chicago FED National Activity Index (02/1967 - 04/2014), Fin and Macro Uncert. indicates Jurando, Ludvigson and Ng (2015) financial and macroeconomic uncertainty index respectively (07/1960 - 04/2014), St. Louis stands for the St. Louis FED Financial Stress Index (01/1994 - 04/2014) and EPU represents the Economic Policy Uncertainty Index of Backer, Bloom and Davis (2015) (01/1985 - 04/2014). All regressions control for 12 lags of the endogenous variable. All t statistics are calculated using Newey and West matrix with 24 lags. For the NBER variable we include the Probit regression results.

Table 10: **Prediction of Other Tail Risk Measures**

Forecast	Macro	t	R^2	BTX	t	R^2	KJ	t	R^2	CATFIN	t	R^2
1.00	0.04	(-1.88)	0.96	0.32	(-2.14)	0.48	0.02	(-1.17)	0.88	-0.01	(-0.33)	0.54
2.00	0.11	(-1.80)	0.92	0.18	(-2.94)	0.30	0.03	(-1.33)	0.82	-0.05	(-1.87)	0.43
3.00	0.18	(-2.18)	0.88	0.15	(-1.18)	0.15	0.04	(-2.05)	0.80	-0.07	(-1.62)	0.39
4.00	0.18	(-2.40)	0.83	0.08	(-1.90)	0.11	0.05	(-2.11)	0.77	-0.06	(-1.41)	0.34
5.00	0.18	(-2.83)	0.77	0.14	(-1.34)	0.10	0.05	(-1.82)	0.74	-0.14	(-3.13)	0.34
6.00	0.17	(-3.11)	0.71	0.08	(-1.46)	0.06	0.08	(-2.69)	0.72	-0.12	(-2.06)	0.30
7.00	0.17	(-3.41)	0.65	0.20	(-2.85)	0.08	0.09	(-2.67)	0.69	-0.09	(-1.36)	0.28
8.00	0.15	(-3.15)	0.59	0.09	(-1.14)	0.04	0.08	(-2.45)	0.66	-0.06	(-0.91)	0.27
9.00	0.15	(-2.98)	0.53	0.11	(-1.56)	0.05	0.08	(-2.30)	0.65	-0.04	(-0.75)	0.24
10.00	0.14	(-2.62)	0.46	0.13	(-1.84)	0.05	0.09	(-2.29)	0.62	-0.03	(-0.63)	0.22
11.00	0.14	(-2.77)	0.39	0.08	(-0.99)	0.04	0.13	(-2.88)	0.61	-0.03	(-0.50)	0.20
12.00	0.15	(-2.82)	0.34	0.15	(-1.65)	0.05	0.15	(-3.38)	0.59	-0.04	(-0.85)	0.17

This table present the result for the prediction regressions for a variety of Tail Risk indexes usign the Hellinger Tail Risk as explanatory variable. KJ refers to Kelly and Jiang (2014) tail risk index (01/1963 - 12/2010), BTX refers to Bollerslev, Todorov and Xu (2014) tail risk index (01/19963 - 08/2013), Macro refers to Bali, Brown and Caglayan (2014) macroeconomic uncertainty index (01/1994 - 12/2013) and CAFTIN denotes Allen, Bali and Tang (2012) systemic risk measure (01/1973 - 12/2012). All regressions control for 12 lags of the endogenous variable. Bali, Brown and Caglayan (2014) is I(1) so we perform the regression for first differences. All t statistics are calculates using Newey and West matrix with 24 lags.

Table 11: **Robustness: Sorted Portfolio**

	Panel A: One Month Holding Period							
	$\gamma=-3$	$\gamma=-1$	$\gamma=0$	$\gamma=1$	10 PC	45 D.	20 D.	5%
Average Return	-1.55	-1.62	-1.42	-1.37	-1.28	-1.68	-1.29	-1.52
	-3.46	-3.61	-3.32	-3.16	-2.86	-3.68	-2.92	-3.34
FF3	-1.17	-1.24	-1.10	-1.05	-0.91	-1.19	-0.79	-0.99
	-3.08	-3.35	-3.01	-2.73	-2.37	-3.02	-2.20	-2.61
FF3+MOM	-1.22	-1.29	-1.15	-1.07	-0.93	-1.19	-0.86	-1.03
	-3.12	-3.36	-3.02	-2.71	-2.38	-2.96	-2.32	-2.63
FF3+MOM+LIQ	-1.29	-1.39	-1.21	-1.13	-1.02	-1.29	-0.98	-1.13
	-2.82	-3.04	-2.67	-2.41	-2.20	-2.72	-2.18	-2.44
FF3+MOM+LIQ+VOL	-0.83	-0.96	-0.74	-0.60	-0.30	-0.53	-0.57	-0.68
	-1.79	-2.09	-1.62	-1.36	-0.70	-1.23	-1.26	-1.63
	Panel B: One Year Holding Period							
Average Return	-14.67	-14.60	-12.27	-12.43	-9.42	-15.57	-11.32	-14.71
	-3.06	-3.23	-2.88	-3.02	-2.41	-3.54	-2.51	-3.63
FF3	-3.07	-3.78	-2.17	-2.19	0.56	-3.63	-0.29	-4.18
	-1.01	-1.38	-0.82	-0.84	0.22	-1.33	-0.10	-1.55
FF3+MOM	-5.50	-5.68	-4.08	-3.30	-1.61	-4.54	-2.42	-4.88
	-2.02	-2.29	-1.71	-1.40	-0.68	-1.87	-0.97	-1.94
FF3+MOM+LIQ	-6.10	-5.82	-3.76	-3.09	-2.05	-4.82	-2.67	-6.76
	-2.24	-2.40	-1.65	-1.29	-0.90	-1.88	-1.18	-2.79
FF3+MOM+LIQ+VOL	9.60	10.06	13.37	13.27	8.67	8.63	17.32	8.13
	0.80	0.91	1.20	1.16	0.92	0.67	1.38	0.84

This table presents the results for the sorting portfolio procedure where CRSP stocks with code 10-11 are sorted into 10 decile portfolios according to their hedging capacity with respect to tail risk. The columns indicate differences between the tail risk measure considered and the baseline case ($\gamma=-0.5$, 30 days returns, 5 principal components and 10% VaR threshold). PC denotes the number of principal components used, D the number of days and 5% the VaR threshold. For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the $\alpha's$ of regressions where we control for the factors indicated in the first column. We only report the values for the High minus Low portfolios. Newey-West t-statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Table 12: **Robustness: Industry Portfolios**

	10 Industry Portfolios										
	Return Prediction				Tail Risk Prediction				Sorting Portfolios		
Period	TR - KJ	t	TR - BTX	t	Div. Yld.	t	Variance	t		α	t
1	0.07	(1.83)	0.01	(0.20)	0.02	(0.46)	0.09	(1.90)		One Month	
2	0.10	(2.79)	0.09	(1.75)	0.08	(1.72)	0.11	(2.50)	Average	-1.44	(-3.30)
3	0.13	(3.82)	0.14	(2.29)	0.12	(3.19)	0.15	(3.05)	FF3	-1.13	(-2.99)
4	0.11	(3.02)	0.13	(2.01)	0.15	(3.30)	0.15	(3.29)	FF3+MOM	-1.17	(-3.02)
5	0.09	(2.37)	0.11	(1.97)	0.12	(2.50)	0.11	(2.40)	FF3+MOM+LIQ	-1.22	(-2.65)
6	0.10	(2.31)	0.11	(1.72)	0.11	(2.22)	0.08	(1.84)	FF3+MOM+LIQ+VOL	-0.79	(-1.76)
7	0.09	(2.07)	0.11	(1.54)	0.12	(2.49)	0.09	(1.91)		One Year	
8	0.05	(1.20)	0.04	(0.54)	0.12	(2.52)	0.07	(1.37)	Average	-12.05	(-3.36)
9	0.04	(0.75)	0.02	(0.20)	0.09	(2.13)	0.02	(0.46)	FF3	-4.28	(-1.90)
10	0.04	(0.76)	0.03	(0.33)	0.08	(1.78)	0.01	(0.15)	FF3+MOM	-4.67	(-2.31)
11	0.05	(0.82)	0.04	(0.56)	0.08	(1.91)	0.01	(0.22)	FF3+MOM+LIQ	-5.72	(-2.62)
12	0.03	(0.54)	0.04	(0.54)	0.09	(1.90)	0.01	(0.25)	FF3+MOM+LIQ+VOL	-3.15	(-0.57)

Robustness: Industry Portfolios (Cont'd)

	Financial Sector Tail Risk										
1	0.11	(2.22)	0.13	(1.55)	0.00	(-0.06)	0.07	(1.01)		One Month	
2	0.14	(2.52)	0.15	(1.53)	0.07	(0.99)	0.12	(2.01)	Average	-1.90	(-3.90)
3	0.16	(2.76)	0.17	(1.76)	0.15	(2.27)	0.18	(3.31)	FF3	-1.34	(-3.29)
4	0.11	(1.84)	0.14	(1.32)	0.13	(2.19)	0.14	(2.46)	FF3+MOM	-1.40	(-3.34)
5	0.13	(2.10)	0.20	(1.92)	0.09	(1.54)	0.07	(0.99)	FF3+MOM+LIQ	-1.52	(-3.14)
6	0.12	(1.90)	0.20	(1.75)	0.11	(2.05)	0.08	(1.29)	FF3+MOM+LIQ+VOL	-0.81	(-1.96)
7	0.12	(1.89)	0.22	(1.84)	0.12	(2.37)	0.08	(1.28)		One Year	
8	0.11	(1.60)	0.21	(1.64)	0.11	(2.31)	0.06	(0.95)	Average	-18.18	(-3.91)
9	0.09	(1.15)	0.20	(1.39)	0.11	(2.08)	0.05	(0.77)	FF3	-7.45	(-2.19)
10	0.09	(1.07)	0.19	(1.27)	0.10	(1.63)	0.03	(0.42)	FF3+MOM	-9.39	(-3.02)
11	0.08	(0.94)	0.17	(1.12)	0.10	(1.55)	0.02	(0.31)	FF3+MOM+LIQ	-10.67	(-3.32)
12	0.07	(0.87)	0.17	(1.16)	0.09	(1.37)	0.00	(0.06)	FF3+MOM+LIQ+VOL	9.87	(0.87)

Robustness: Industry Portfolios (Cont'd)

	Real Sector Tail Risk										
1	0.11	(3.39)	0.08	(1.30)	0.06	(1.35)	0.15	(3.60)		One Month	
2	0.14	(3.94)	0.11	(1.60)	0.13	(2.90)	0.19	(4.85)	Average	-1.57	(-3.73)
3	0.16	(4.17)	0.16	(2.29)	0.18	(3.91)	0.23	(5.24)	FF3	-1.16	(-3.27)
4	0.13	(3.61)	0.15	(2.27)	0.17	(3.41)	0.20	(4.44)	FF3+MOM	-1.21	(-3.30)
5	0.13	(3.40)	0.17	(3.14)	0.13	(2.62)	0.14	(3.31)	FF3+MOM+LIQ	-1.31	(-2.99)
6	0.11	(2.87)	0.15	(2.53)	0.13	(2.41)	0.13	(3.09)	FF3+MOM+LIQ+VOL	-0.90	(-2.19)
7	0.11	(2.91)	0.15	(2.34)	0.12	(2.36)	0.12	(2.82)		One Year	
8	0.08	(2.21)	0.10	(1.45)	0.12	(2.45)	0.11	(2.52)	Average	-14.15	(-3.54)
9	0.06	(1.47)	0.07	(0.98)	0.11	(2.19)	0.08	(1.94)	FF3	-4.16	(-1.72)
10	0.06	(1.42)	0.08	(1.10)	0.08	(1.86)	0.05	(1.28)	FF3+MOM	-4.66	(-2.12)
11	0.04	(1.06)	0.06	(0.88)	0.08	(1.94)	0.05	(1.39)	FF3+MOM+LIQ	-5.61	(-2.51)
12	0.04	(1.04)	0.07	(0.91)	0.07	(1.65)	0.04	(0.87)	FF3+MOM+LIQ+VOL	7.62	(0.80)

This table present assorted robustness tests for a tail risk measure constructed using Industry Portfolios. Columns 2-5 present the prediction regressions for market returns for two possible samples, the first one that matches Kelly and Jiang (2015, KJ) and the second that matched Bollerslev, Todorov and XU (2014, BTX) samples. Columns 6 to 9 present the respective KJ and BTX prediction regressions. Columns 10 to 12 present the results for the sorting portfolios procedure. All t-statistics are calculated using Newey-West standard errors.

Table 13: **Robustness: VaR Tail Risk**

	Return Prediction				Tail Risk Prediction				Sorting Portfolios		
Period	TR - KJ	t	TR - BTX	t	KJ	t	BTX	t		α	t
1	0.05	(0.76)	-0.01	(-0.12)	-0.01	(-0.32)	0.27	(1.36)		One Month	
2	0.06	(0.80)	-0.02	(-0.17)	0.01	(0.26)	0.12	(0.69)	Average	-1.17	(-2.58)
3	0.08	(1.08)	0.03	(0.24)	0.02	(0.76)	0.22	(0.91)	FF3	-0.86	(-1.96)
4	0.09	(1.26)	0.08	(0.69)	0.03	(0.99)	0.11	(0.75)	FF3+MOM	-0.84	(-1.89)
5	0.09	(1.29)	0.11	(1.10)	0.04	(0.97)	0.15	(0.84)	FF3+MOM+LIQ	-0.84	(-1.65)
6	0.09	(1.18)	0.12	(1.23)	0.05	(1.30)	0.14	(0.77)	FF3+MOM+LIQ+VOL	0.06	(0.13)
7	0.09	(1.10)	0.15	(1.19)	0.07	(1.51)	0.15	(0.75)		One Year	
8	0.07	(0.79)	0.15	(1.02)	0.08	(1.64)	0.17	(0.67)	Average	-10.97	(-3.24)
9	0.07	(0.73)	0.13	(0.90)	0.09	(1.83)	0.13	(0.70)	FF3	-2.16	(-0.77)
10	0.07	(0.75)	0.15	(0.98)	0.10	(1.78)	0.17	(0.87)	FF3+MOM	-2.10	(-0.95)
11	0.06	(0.62)	0.19	(1.08)	0.14	(2.45)	0.16	(0.85)	FF3+MOM+LIQ	-1.83	(-0.75)
12	0.05	(0.50)	0.18	(1.02)	0.16	(2.86)	0.17	(1.20)	FF3+MOM+LIQ+VOL	18.05	(1.63)

This table present assorted robustness tests for a Risk Neutral Value at Risk measure. Columns 2-5 present the prediction regressions for market returns for two possible samples, the first one that matches Kelly and Jiang (2015, KJ) and the second that matched Bollerslev, Todorov and XU (2014, BTX) samples. Columns 6 to 9 present the respective KJ and BTX prediction regressions. Columns 10 to 12 present the results for the sorting portfolios procedure. All t-statistics are calculated using Newey-West standard errors.

Risk Neutral Probabilities

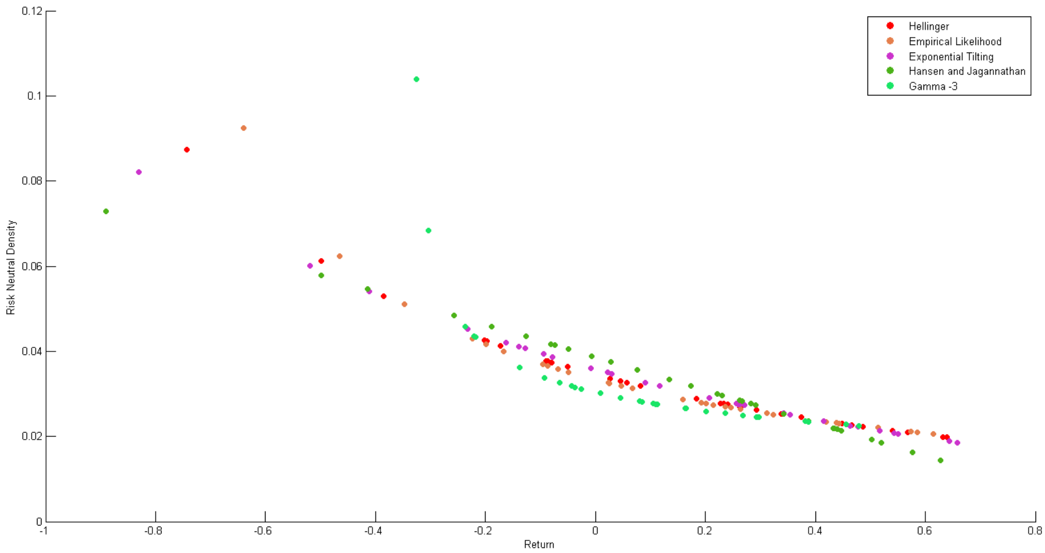

Figure 1: This figure presents the implied risk neutral probabilities for a five asset economy and a variety of $\gamma's$.

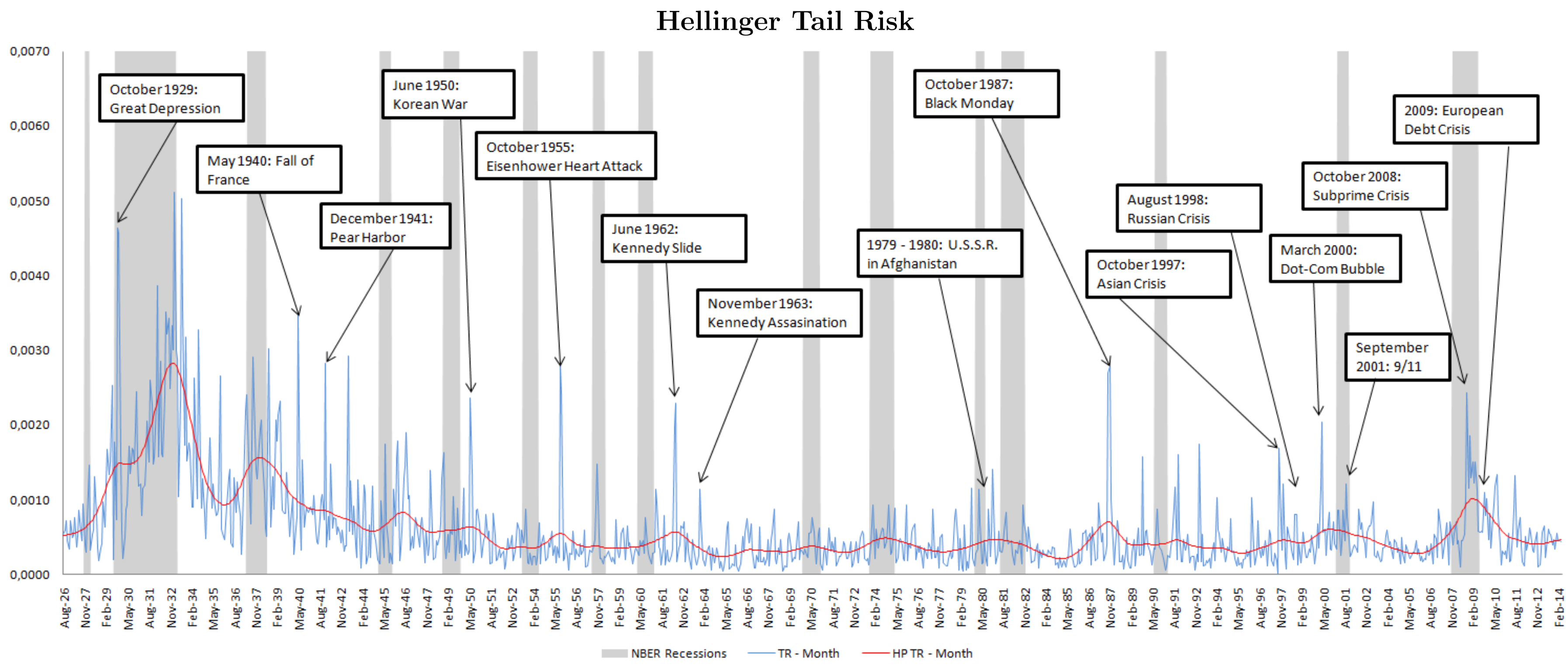

Figure 2: This figure plots the tail risk calculated using five principal components extracted from the 25 Fama and French size and book-to-market portfolios for the Hellinger estimated risk neutral density ($\gamma = -0.5$). Principal components were extracted for the last day of each month in our sample using 30 days of past returns on the portfolios.

Discrepancy Bounds and Disaster Model

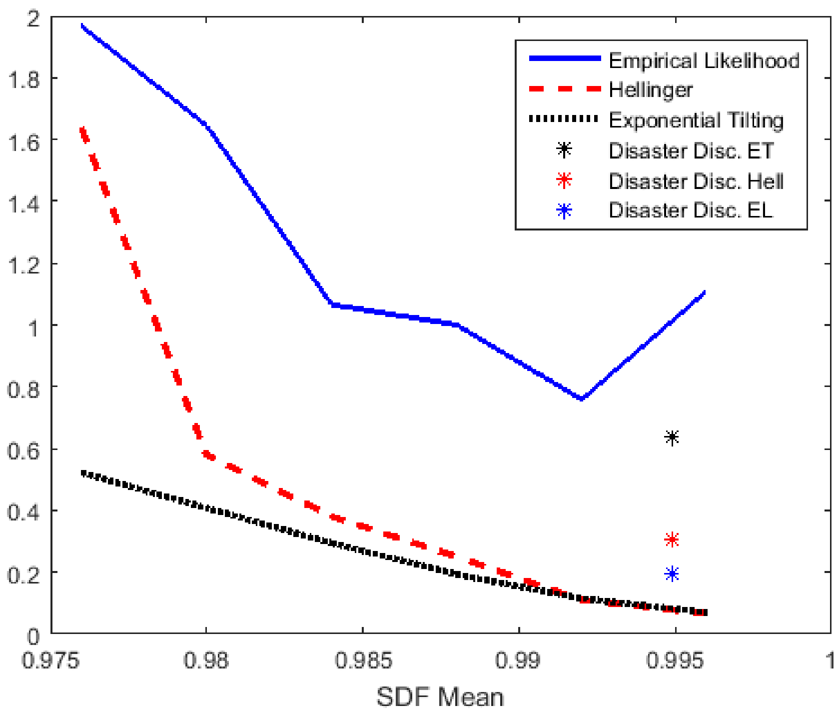

Figure 3: This figure plots .

Hellinger and Option-Based Tail Risk Measures

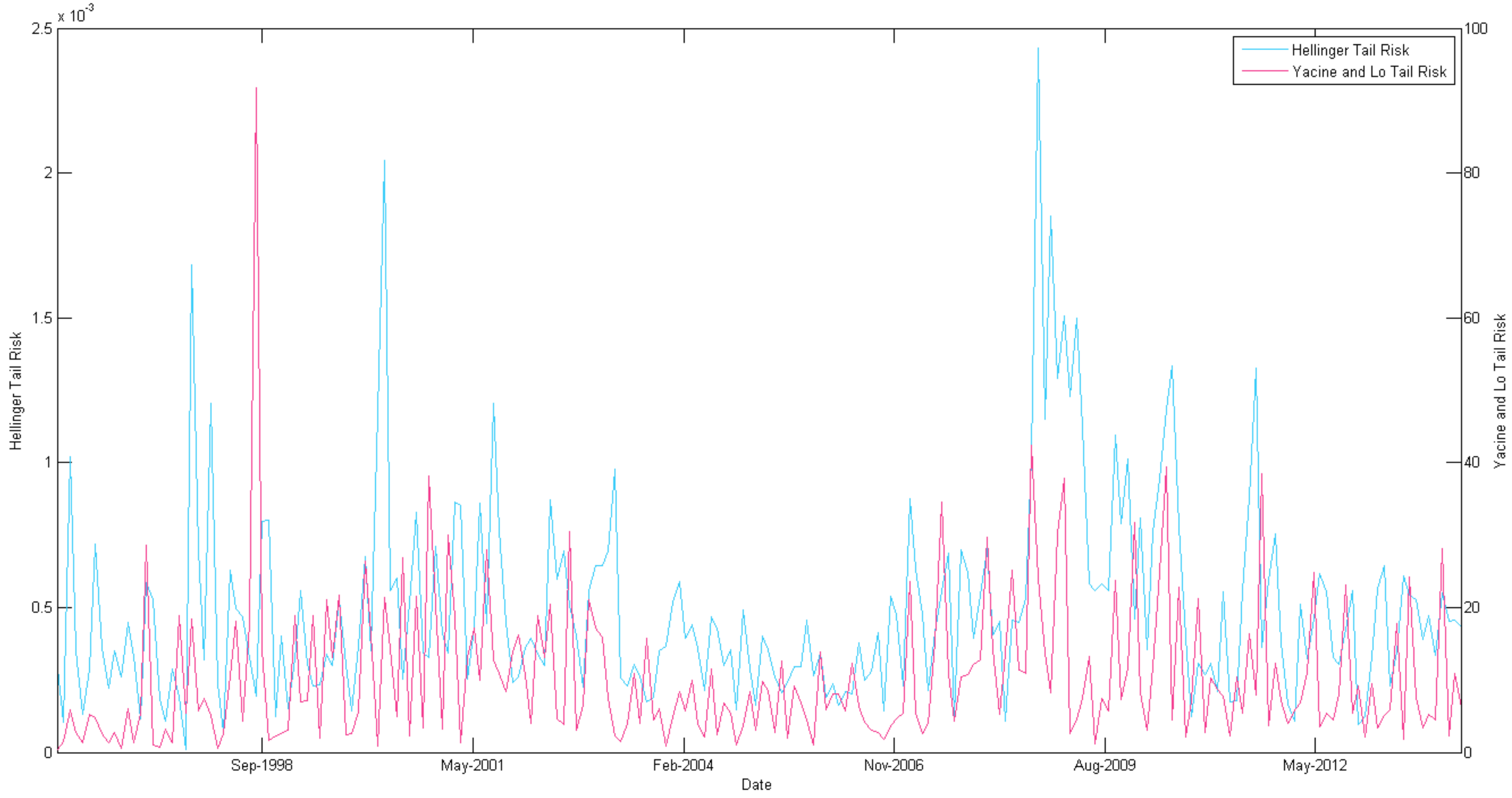

Figure 4: This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the 25 Fama and French size and book to market portfolios with the Hellinger estimated risk neutral density ($\gamma = -0.5$). The red line features the corresponding tail risk measure based on a state-price density obtained with the Yacine and Lo (1998,2000) methodology applied to a panel of S&P 500 index options. The sample period ranges from January 1996 to April 2014.

Hellinger and Kelly-Jiang Tail Risk Measures

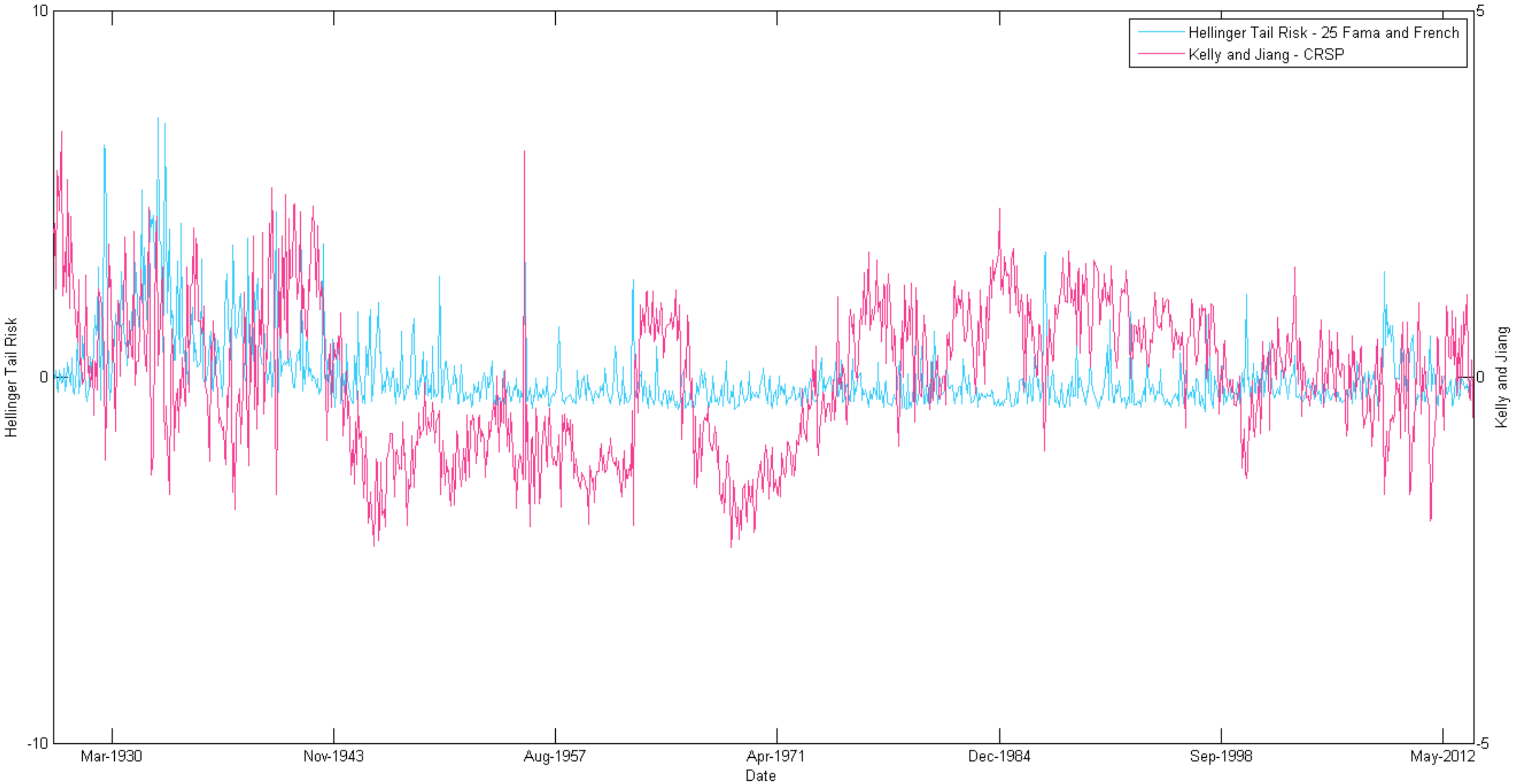

Figure 5: This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the 25 Fama and French size and book to market portfolios with the Hellinger estimated risk neutral density ($\gamma = -0.5$). The red line features the tail risk measure proposed by Kelly and Jiang (2013) based on a Hill estimator computed from the whole cross-section of CRSP securities available at each time period. The sample period ranges from January 1996 to December 2012.

Hellinger and Kelly-Jiang Tail Risk Measures

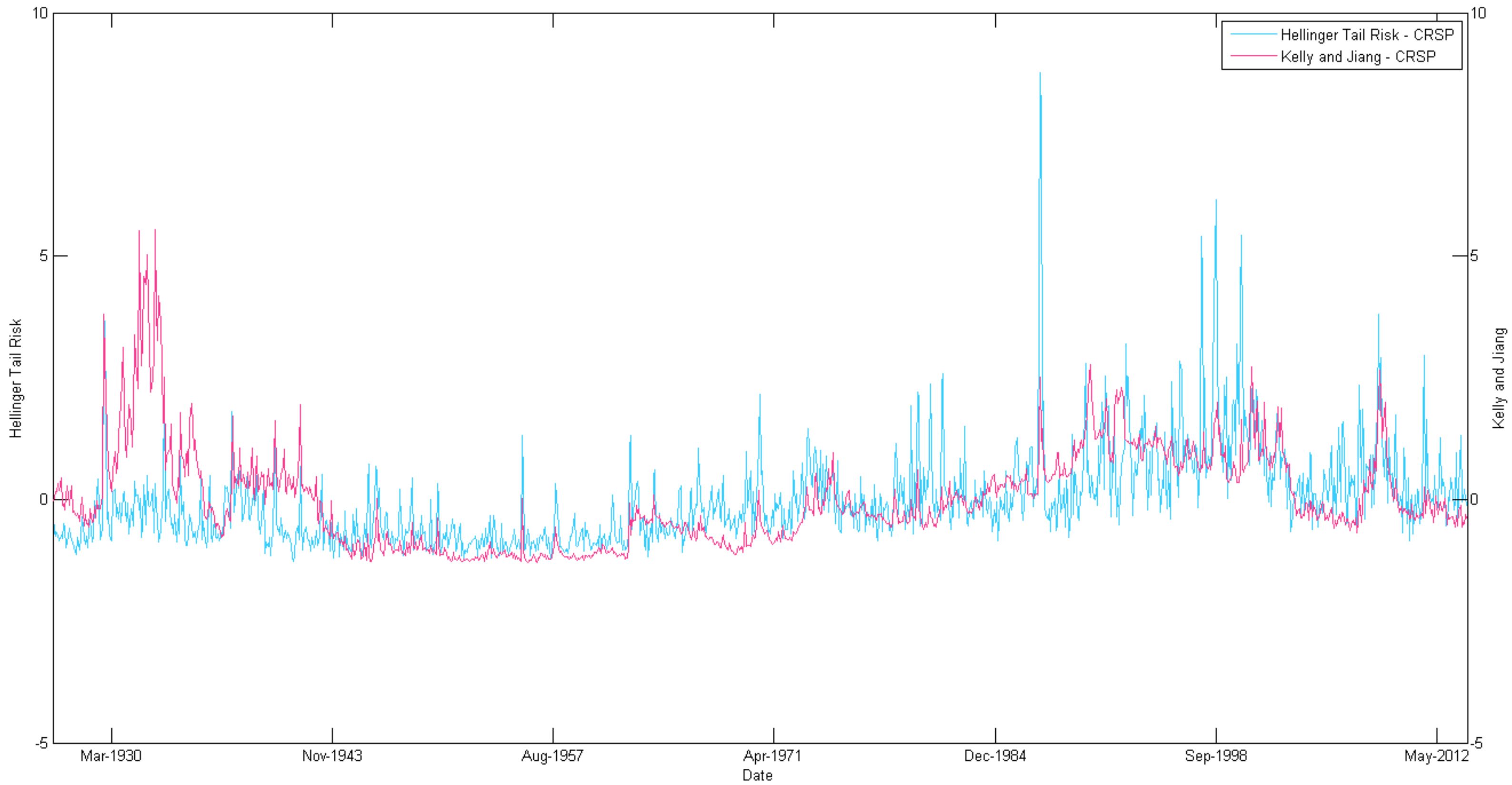

Figure 6: This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the whole cross-section of CRSP securities available at each time period with the Hellinger estimated risk neutral density ($\gamma = -0.5$). The red line features the tail risk measure proposed by Kelly and Jiang (2013) based on the Hill estimator computed from the whole cross-section of CRSP securities available at each time period, based on the Hill formula in (19). The sample period ranges from January 1996 to December 2012.

Impulse Response Functions: Tail Risk Shock

Figure 7: This figure presents the impulse response functions for Employment and Industrial Production following a shock in the Hellinger Tail Risk. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail Risk series (still controlling for the market variance).

Impulse Response Functions: Volatility Shock

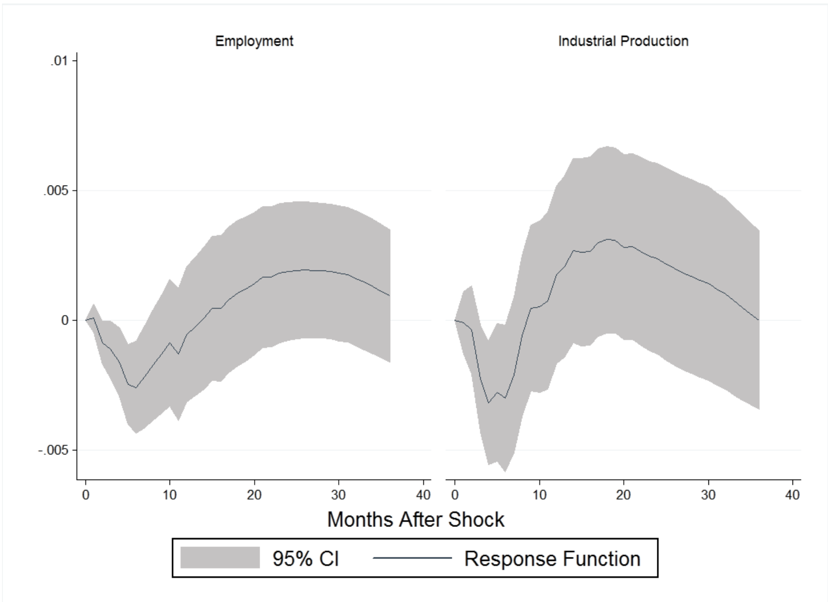

Figure 8: This figure presents the impulse response functions for Employment and Industrial Production following a shock in the market variance. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail Risk series.

Impulse Response Functions: Tail Risk Shock

Figure 9: This figure presents the impulse response functions for Employment and Industrial Production following a shock in the Hellinger Tail Risk. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail Risk series (still controlling for the big volatility events indicator).

Impulse Response Functions: Volatility Shock

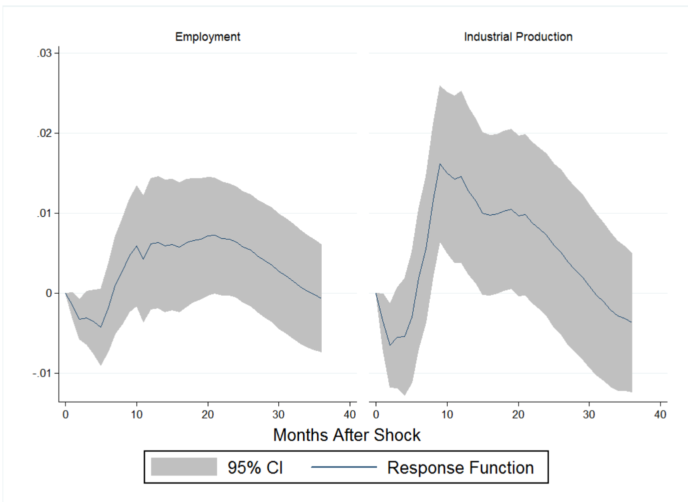

Figure 10: This figure presents the impulse response functions for Employment and Industrial Production following a shock in the big volatility event indicator. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail Risk series.

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