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2015s-52

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Série Scientifique/Scientific Series

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Scientific Series

Montréal
Décembre/December 2015

CIRANO
Allier savoir et décision

Centre interuniversitaire de recherche en analyse des organisations

ISSN 2292-0838 (en ligne)

Compliance, Informality and Contributive Pensions[*]

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Résumé/abstract

We consider a political economy model in which agents have the possibility to hide part of their earnings in order to avoid taxation. Taxation is exclusively used to finance a pension system. If the pension system is implemented, agents in their old age receive a benefit which includes both a Bismarkian and a Beveridgian component. We show that in the absence of compliance costs, agents are indifferent to the tax rate level as in response, they can perfectly adapt their level of compliance. The public pension system is found to be at least partially contributory in order to increase compliance and thus to increase the tax base. When compliance costs are introduced, perfect substitutability between compliance and taxation breaks down. Depending on the relative returns from public pensions and private savings as well as on the elasticity of compliance to income, we obtain that the preferred tax rate should be increasing or decreasing in income. The majority voting tax rate is more likely to be positive when the median income is low and when the return from public pensions dominates that of private savings. The level of the Bismarkian pillar will now be chosen so as to account for increased political support, for increased direct redistribution toward the worst-off agent, and increased tax base.

Mots clés/keywords : Compliance costs, majority voting, public pensions, tax evasion

Codes JEL/JEL Codes : H55, I13, D91

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[*] We would like to thank Philippe De Donder for his comments and suggestions on this paper.
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1 Introduction

Tax evasion is endemic in many countries, in particular in developing countries, which do not collect even half of what they should if taxpayers complied with the written letter of the law (Moore and Mascagni, 2014). In these countries, enforcement mechanisms are weak, tax-collecting authorities are held in low esteem, and courts may not enforce the rules. Many developing countries still face tax shares of GDP below 15% which is considered as a reasonable threshold for ensuring government functioning. In recent years, domestic revenue mobilization in developing countries has gained increasing prominence in the policy debate. This is due to several factors, including the potential benefits of taxation for state building, long-term independence from foreign assistance and the continuing acute financial needs of developing countries.

Plenty of specific solutions to how boosting compliance have been offered in the literature and public authorities have tried many more. Increasing and better focused enforcement, improving the collection and management of information, reducing the costs of complying with the law, providing incentives for those who comply, and modifying tax bases and rates are some of the obvious examples. Another solution consists in sending messages to taxpayers so as to affect their behavior and increase compliance. These messages supposedly operate on the beliefs and moral values that people attach to paying taxes. Related to this issue, the recent paper by Castro and Scartascini (2013) explores the impact of the use of messages for affecting taxpayers' compliance by conducting a large-scale field experiment in Argentina. Another alternative solution is to focus on firms that choose to make use of the financial sector. As argued by Gordon and Li (2009) bank records allow governments to identify taxable entities and to measure the amount of their taxable activity. Finally, another possible solution consists in establishing a close link between the amount of taxes paid and the benefits obtained. After all, if that link were perfect there would not be any problem of compliance. The tax would play the same role as any market price. Naturally, the link is often not perfect as the government is expected to provide public goods and redistribute income. This is the avenue of research this paper explores.

Unlike most of the literature on taxation in developing countries that deals with capital taxation and foreign duties, we focus in this paper on income and payroll taxation.[1] In particular, we are interested in studying how a government that uses the pension system to redistribute towards the olds and the poors must design it when it faces evasion and political restrictions

[1] See for instance Martinez-Vasquez and Alm (2003).

on the tax rate. To study this issue, we use a model where the level of taxation is chosen by majority voting and where, in addition, non compliance is possible but costly. Since we focus on developing countries where tax enforcement mechanisms are very weak, our model is different from the traditional literature where the taxpayer faces a probability of detection and pays some penalty if evasion is detected (e.g. Allingham and Sandmo, 1972). Instead, individuals receive direct benefits from tax payments so that it may be in their interest to contribute. This can be done in several ways but in essence what one needs is a model where the proceeds of taxation are spent in private or semi-private goods which are not perfect substitutes to standard consumption. Examples of such goods are health care, education or housing, granted that these services are essential to the individuals' welfare. In this paper we choose the example of public pensions when claimed benefits depend on individual payments. Pension benefits are essential to the individuals and have poor substitutes in the private sector. The reason for this is the absence of private annuity markets and the inefficiency of the financial sector. In the introduction of a pension system we follow the model initiated by Casamatta et al. (2000a,b).

We consider a society where individuals live for two periods, a first-period of activity and a second-period of retirement. They only differ in income. In the first period, they choose how much income to report to fiscal authorities and pay a tax on this amount so as to finance the pension system. They further allocate their income net of tax between current consumption and saving. Depending on their income and thus on whether they are liquidity constrained, individuals save or not. This will also depend on the existence and the level of compliance costs incurred in the first period, that is at the time of paying taxes.[2] The period of retirement is uncertain as a fraction of individuals do not survive after the first period. In this second period, individuals' consumption is financed by the proceeds of saving and by the public pension. As in Casamatta et al. (2000a,b) this pension benefit depends on two components. The first one is proportional to their *reported* labor income; the factor of proportionality is called the Bismarkian factor. The second component, the Beveridgian paart, is common to all individuals and depends on average tax collection. We assume that agents vote over the level of the tax rate and that the Bismarkian factor is set constitutionally, prior to any vote over the tax rate, so as to maximize the utility of the poorest agent (Rawlsian Social Welfare Function).

To better understand the (main) model with compliance costs, we proceed by steps and first

[2]Our model can also be seen as a reduced form of the Allingham and Sandmo (1972) model with non probabilistic compliance and a convex cost function of the unreported income.

introduce simple models where 1) compliance is so costly that everyone chooses to comply and 2) there is no compliance cost.

We obtain the following results. Without compliance cost, agents prefer to comply (at least partially) with the tax system as soon as they obtain a higher marginal return from the pension system than from the private saving system (which is close to the case of developing countries where financial markets are thin and very inefficient). Interestingly, in such a case, choosing how much income to report or choosing the level of the tax rate are perfectly equivalent decisions so that the agents are indifferent as to the level of the tax rate chosen at the voting equilibrium. The reason is that they are perfectly able to adapt their level of compliance following a variation in the tax rate so as to keep unchanged the amount of resources they want to transfer for their old age. We find that it is always optimal to have a contributory pension system, as increasing the degree of contribution makes the agents willing to report more income and thus it increases the resources collected and distributed through the flat pension benefit part (which mainly benefits low-income agents). The reason for a contributive pension system is therefore different from that in Casamatta et al. (2000a). Here, increasing the Bismarkian factor increases the tax base.

Whenever we introduce a cost of compliance, we now find that agents will always choose to report at least some fraction of their income, independently of the returns from savings and from the pension system. Under some circumstances (for example when the return from pension contributions is higher than that of private savings or if labour income is high), it may even be the case that agents fully comply. Agents will now choose to save only if they are not liquidity constrained, that is for a sufficiently high-income level. Also, the introduction of a compliance cost breaks down perfect substitutability between the tax rate and the amount reported to fiscal authorities so that the agent's preferred tax rate now depends on his income, and on the relative returns from pension contributions and from savings. As a consequence, the variation of this tax rate with income depends on these relative returns as well as on the elasticity of compliance with respect to income. In addition, we find that if the marginal return from public pensions is higher than that of private savings and if the agent's income is relatively high, this agent votes for a tax rate that lies on the decreasing part of the Laffer curve. The intuition is simply that he would like to use the public system rather than private savings to transfer resources to the old age and to report even more than his full income. As he cannot do so, he votes for a tax rate that is greater than the top of the Laffer curve, even if it is at the expense of lower collected

resources and of a lower flat pension benefit.
At any rate, the majority voting tax rate corresponds to the preferred tax rate of the median agent and it is in general strictly positive. It is more likely to be the case whenever the median voter has a relatively small income and when the return from pensions is greater than that from savings. Therefore, in developing countries where such conditions are more likely to be satisfied, making agents vote over the existence of a public pension system would favor the emergence of such system and would push individuals to report at least a fraction (or even all) of their income, since the contributory part of the pension benefit guarantees that they receive some resources at a relatively high return in the second period.
Finally, we find that, even though at the constitutional stage, the objective of the government is to maximize the utility of the least-favored agents, the Bismarkian factor should in general be positive. This is so so as to both increase the support for the pension system, and increase the level of the flat pension benefit, through a larger tax base. Although it may seem obvious, this second reason is new in the literature.

This paper at least in its first part is about a particular type of voluntary tax. Voluntary taxation defined as a contribution paid without any type of formal enforcement is an old issue in economics. There are several types of potential voluntary taxes. The first one is the so-called private contribution to a public good, which has been largely studied in the literature, with the standard outcome of under provision and thus of insufficient taxation (see Bergstrom et al. 1986). Another type of voluntary taxation appears when a tax can easily be avoided. This is for instance, the case of estate taxation dubbed as "the tax for stupid" as it can be easily avoided and it is stupid not to do so. The problem with this type of tax is that it introduces horizontal inequity as people contribution depends on how they value citizenship. In fact, most taxes are to a certain extent voluntary. As noted by Slemrod (1998), for most taxes we face a puzzle: we need to explain why people pay taxes as, in the context of the standard economic model, people should not voluntarily comply. They thus seem to exhibit nothing short of "pathological honesty". In our paper, we do not have any public good *per se* and individuals are purely individualistic. In the absence of effective compliance control, the only reason they pay taxes is because it implies a benefit they could not obtain privately.

In addition, our paper can be related to the work on tax evasion such as the paper by Borck (2009), who studies a political economy model of income redistribution where agents have the possibility to evade income. He shows that in such a case, redistribution may go from the middle class to the rich and poor. It is different from our paper in several respects. First, in his paper, agents are risk neutral so that they either avoid all their income or none of it. Second, it assumes that agents get a uniform transfer from their contributions while we model a pension system that includes both a contributory and a redistributive part. Third, his model is one in which the only motive for taxation is redistribution and there is no consumption smoothing motive as in our model with pensions. Another paper by Borck (2004) shows how stricter enforcement policies on tax evasion may actually increase tax evasion. Again, his model is different from ours mainly because the tax paid serves to finance a lump sum transfer and second, because evasion entails a cost *only* if the agent is audited (in our model, the cost of evasion is non probabilistic).[3] Traxler (2012) also looks at the efficiency effect of tax avoidance when the rate is chosen by a majority of taxpayers. His main finding is that the traditional inefficiency carried by majority voting decreases with the extent of tax avoidance. Also, Alm et al. (1999) briefly discuss a political economy model with risk neutral agents where the presence of a social norm influences the tax compliance behaviour of agents and thus their voting behaviour over the increase in tax rate, in the amount of fine and in the probability of detection.[4] This model is different from ours at least in three dimensions: agents either fully comply or not at all, they receive a lump sum benefit in exchange of their previous contributions and, the heterogeneity in this society arises from individual differences in the psychological loss from not complying. Finally, the paper of Kopczuk (2001) studies the optimal income taxation scheme when agents have different tax avoidance behaviour, either because they have different preferences for avoidance or because of different avoidance cost functions. This is different from our paper first because it is normative and second, because in our model agents only differ with respect to income. To the best of our knowledge, our paper is one of the few political economy papers which look at the possibility of collecting taxes and at the existence of a pension system when individuals are free to hide part or the entirety of their earnings.

The rest of the paper is organized as follows. The next section presents the assumptions and the benchmark model in which every agents fully comply with the tax system. Section 3 presents a model where agents are free to report their income and where there is no compliance

[3] See also the survey by Borck (2007) on inequality, redistribution and taxation in political economy models.

[4] Their paper is mainly experimental. They show in particular that "when the group rejects any attempt to raise the level of enforcement, compliance always falls, often collapsing to zero". Also cheap talk helps increasing compliance and may change the result of the vote in favour of paying more taxes.

cost. In Section 4, such costs are introduced. A final section is devoted to concluding remarks.

2 The model

2.1 Assumptions

We assume a two-period model, with a mass one of individuals who face uncertain survival. Agents have different productivity, y which is continuously distributed over $[y_{min}, y_{max}]$, with median productivity below the average one, $y_m < \bar{y}$, and a density function denoted by $f(y)$. In the first period, agents are working and supply inelastically one unit of labour. They also consume and save on private markets. In the second period, say old-age, they are alive with a probability $0 \leq \pi \leq 1$. If alive, they are retired and consume a pension benefit and their savings.

Agents derive utility from consumption in each period. Without loss of generality, we assume that there are no pure time preferences. Expected utility is written as follows:

$$u(c) + \pi u(d).$$

Per period utility is such that $u'(.) > 0$, $u''(.) < 0$ and $u'(0) \to +\infty$. We also make the following assumption regarding the coefficients of relative risk aversion, $R_r(c) = -u''(c)c/u'(c)$:

$$\textbf{A1: } R_r(c) = R_r(d) \leq 1.$$

To pay for consumption individuals use their disposable income (labor income minus taxes) and their pension benefits. In the first period, agents pay a tax at rate τ on their *reported* income so as to finance the public pension program.[5] The proceeds of taxation are only used to fund the pension system. In the second period, individuals receive a pension benefit which is obtained from balancing the government's budget and it is equal to:

$$P(\tau, \alpha; y) = \frac{\tau}{\pi}\left(\alpha\tilde{y} + (1-\alpha)E(\tilde{y})\right)$$

where α is the Bismarckian factor and it is set at the constitutional level. The pension benefit is constituted of two parts as in Casamatta et al. (2000a). The first part $(\tau\alpha\tilde{y}/\pi)$ which we call the Bismarkian part, is directly related to the agent's contribution. The second Beveridgian part, $(\tau(1-\alpha)E(\tilde{y})/\pi)$ is flat and depends on average earnings in the economy. The variable $\tilde{y}$ denotes the income *reported* by an agent with income y and $E(\tilde{y})$ is the expected reported

[5] We therefore assume that the government only imperfectly observes the distribution of incomes in the society and/ or lacks coercive measures to make agents report their true income.

earnings in the economy. For simplicity, we assume that the reported income is simply equal to a fraction of the agent's true income, so that $\tilde{y} = \gamma y$ where $0 \leq \gamma \leq 1$ is the rate of compliance and it is privately chosen by the agent. As a consequence,

$$E(\tilde{y}) = E(\gamma(y)y) = \int_{y_{min}}^{y_{max}} \gamma(y) y f(y) dy.$$

For future reference, we also define here the flat pension benefit part as $b(\tau, \alpha)$ and it is equal to $(1-\alpha)\tau E(\tilde{y})/\pi$.

First and second-period consumptions are thus respectively equal to:

$$\begin{aligned} c &= y - \tau\tilde{y} - s = (1-\tau\gamma)y - s \\ d &= \frac{\varepsilon}{\pi}s + \frac{\tau}{\pi}\left(\alpha\gamma y + (1-\alpha)E(\tilde{y})\right) \end{aligned}$$

where s is the amount of savings which is assumed to be invested on the private annuity market. The agent can thus transfer resources across periods by both contributing to the pension system and investing in a private annuity market. For simplicity, we assume that the interest rate is null. The parameter $0 \leq \varepsilon \leq 1$ reflects both the weakness of the annuity market and the inefficiency of the financial institutions for individual savers.[6] Therefore the marginal return the agent obtains from buying private annuities is equal to ε/π while that of contributing to the pension system is α/π. We make no specific assumption regarding the relative levels of α and ε and will study the different cases depending on whether $\alpha \gtrless \varepsilon$.

The timing of the model is the following. First, the government sets how contributive the pension system should be, that is the level of the Bismarkian factor, α. Agents then vote over the level of the tax rate that will finance the pension system. Finally, given the tax rate chosen at the majority voting equilibrium, they decide how much to comply with the tax system, that is the fraction of income to report, and how much to save. As usual in this type of problem, we proceed backward. We first find the optimal individual decisions in terms of savings and of compliance for given (τ, α) parameters. We then find the majority-voting tax rate, given individual preferences over this tax rate and for a given α. We finally set α assuming that the government is Rawlsian and thus maximizes the utility of the worst-off agent.

[6] If there was no private annuity market but if the individual could save at the current interest rate (here assumed to be zero), ε would simply be equal to π and the return from saving would be 1.

2.2 The benchmark model

In the next sections of this paper, we will focus on two cases: one where complying is costless and one where not complying entails some (monetary) costs. To the contrary, in this section, we will start by looking at a situation where individual incomes are publicly observable, in other words, where full reporting is unavoidable. This can be seen as a case with extremely high compliance costs. In this simple model, an agent with income y solves the following problem:[7]

$$\max_{s,\tau} U(s;y) = u((1-\tau)y - s) + \pi u(\frac{\tau}{\pi}(\alpha y + (1-\alpha)\bar{y}) + \frac{\varepsilon s}{\pi}) \tag{1}$$

where implicitly $\gamma = 1$. From the first order conditions of this problem, we find that this agent either saves $s^*(\alpha; y)$ on private markets or votes for a positive tax rate $\tau^*(\alpha; y)$ depending on whether $\alpha \gtreqless \varepsilon$. If the marginal return from pensions is lower than that of private savings, $\alpha < \varepsilon$, agents rely exclusively on private savings and vote in favour of a zero tax rate. To the opposite, if $\alpha \geq \varepsilon$, agents prefer public pensions over private savings and $s^*(\alpha; y) = 0$. In the following, we concentrate on this latter case as this is the only one that guarantees the emergence of a pension system.

In this case, the agent's preferred tax rate $\tau^*(\alpha; y)$ satisfies $u'(c) = \alpha u'(d)$.[8] Using the implicit function theorem and assumption A1, we obtain that $d\tau^*(\alpha; y)/dy > 0$: agents with higher incomes are willing to transfer more resources to old-age than those with lower incomes so as to smooth their consumption. Also, under A1, namely under the assumption that c and d are complements, $d\tau^*(\alpha; y)/d\alpha > 0$ for every agent independently of y. In that case, low-income individuals benefit from a contributive scheme that guarantees them a sufficiently wide tax base while high-income agents always prefer a less redistributive pension system (i.e. a higher α).

Let us determine the voting equilibrium in this simple setting. Preferences are single-peaked so that the majority voting equilibrium corresponds to the preferred tax rate of the agent with median income, $\tau^V(\alpha) = \tau^*(\alpha; y_m) > 0$.

Finally, we determine the optimal level for the contribution rate, α^R. The social planner is assumed to be Rawlsian so that he seeks to maximize the utility of the worst-off agent, i.e. the

[7]Usually this problem is solved in two steps, by first finding the individual decisions for given pension parameters and second, by determining the agent's preferred tax rate given previous individual decisions. Since preferences are identical in these two decision problems, we can solve it simultaneously without changing the results.

[8]Under the assumption that $u'(0) \to \infty$, the preferred tax rate is always strictly positive.

one with minimum income y_{min}. It therefore satisfies the following first order condition:

$$\frac{\partial V(\alpha; y_{min})}{\partial \alpha} = [\alpha u'(d) - u'(c)] y_{min} \frac{d\tau^V(\alpha)}{\alpha} + (y_{min} - \bar{y})\tau^V(\alpha)u'(d) + (1-\alpha)\bar{y}u'(d)\frac{d\tau^V(\alpha)}{\alpha} \leq 0 \quad (2)$$

where $V(\alpha; y)$ is the indirect utility function of an individual with income y. Since the agent with minimum income would like a smaller tax rate than the tax rate chosen at the voting equilibrium, $\tau^*(\alpha; y_{min}) < \tau^V(\alpha) = \tau^*(\alpha; y_m)$, the first term, which is evaluated at $(\tau^V(\alpha), y_{min})$, in the expression above is negative. This is due to the fact that increasing α increases the majority voting tax rate and thus, constrain the agent with the lowest income to transfer too many resources to the old age. He thus would like a smaller Bismarkian factor. The second term is also negative and this is related to the fact that every agent with income below $\bar{y}$ obtains more from the pension system when it becomes more Beveridgian. Indeed, with the form of pension benefit $P(\tau, \alpha; y)$ considered here, any agent with $y < \bar{y}$ is a net beneficiary from the pension system as his expected benefit $\tau[\alpha y + (1-\alpha)\bar{y}]$ exceeds his total contributions, τy and increases when α decreases. The last term to the contrary is positive and is related to the support for the pension system. One needs to make it at least partially contributive and thus less redistributive than optimal so as to ensure higher political support.[9] Adding up these terms we find that, when the public scheme prevails ($\alpha \geq \varepsilon$), a positive Bismarckian factor, $\alpha^R \in [\varepsilon, 1]$ is desirable from the viewpoint of the worst off.

3 A compliance-choice model with no cost of compliance

3.1 Individual decisions

We now turn to a model where individuals decide both how much to save on private markets and how much income to report. This in turn determines the size of the pension benefit they receive in the second period and the amount of resources left for consumption in each period. We assume for the moment that not complying does not entail any (moral or financial) cost as well as no risk.

For given pension parameters (τ, α), the problem of an agent with income y consists in solving

$$\max_{s,\gamma} U(\gamma, s; y) = u((1-\gamma\tau)y - s) + \pi u(\frac{\tau}{\pi}(\alpha\gamma y + (1-\alpha)E(\tilde{y})) + \frac{\varepsilon s}{\pi}) \quad (3)$$

[9]This result is identical to that of Casamatta et al. (2000b).

First order conditions of this problem are:

$$\begin{aligned}\frac{\partial U}{\partial s} &= -u'(c)+\varepsilon u'(d)\leq 0 && (4)\\ \frac{\partial U}{\partial \gamma} &= \tau y[-u'(c)+\alpha u'(d)]\leq 0 && (5)\end{aligned}$$

These equations allow to obtain the optimal savings level and the proportion of income reported by individuals. These will depend on α, τ and y. Let these functions be $\gamma^*(\tau,\alpha;y)$ and $s^*(\tau,\alpha;y)$.

Whether agents choose to save on private markets or to comply (which is equivalent to preferring public pensions over private savings) only depends on the comparison between the marginal returns from savings (ε/π) and from pensions (α/π). According to the above first order conditions, they never choose both to report some income and to save; they either save on private markets or comply (at least partially) with the public pension system. This result is independent from their income and depends only on economic and institutional parameters. If $\alpha<\varepsilon$, all agents prefer private savings to public pensions, $\gamma^*(\tau,\alpha;y)=0\,\forall y$ and $s^*(\tau,\alpha;y)>0$ under the assumption that $u'(0)\to+\infty$. In such a case, the pension benefit $P(\tau,\alpha;y)$ is obviously nil since nobody ever comply. If on the contrary $\alpha\geq\varepsilon$, all agents prefer public pensions to private savings: $s^*(\tau,\alpha;y)=0$ and $\gamma^*(\tau,\alpha;y)\geq 0$. In the rest of this section, we will thus concentrate on the case where $\alpha\geq\varepsilon$ as it is a necessary condition for the emergence of a pension system. To anticipate on the results of the next section, as soon as we introduce compliance costs, this condition is not necessary anymore and a pension system can emerge even when $\alpha<\varepsilon$.

When $\alpha\geq\varepsilon$, some agents may still choose $\gamma^*(\tau,\alpha;y)=0$ and rely exclusively on the flat pension benefit $b(\tau,\alpha)=(1-\alpha)E(\tilde{y})\tau/\pi$ in their old age. This is the case for agents with income y such that

$$\frac{\partial U}{\partial \gamma}|_{\gamma=0}=\tau y[\alpha u'((1-\alpha)E(\tilde{y})\frac{\tau}{\pi})-u'(y)]<0.$$

From this, we can define a threshold productivity, $\hat{y}(\tau,\alpha)$ which satisfies

$$u'(\hat{y}(\tau,\alpha))=\alpha u'((1-\alpha)E(\tilde{y})\frac{\tau}{\pi}) \tag{6}$$

and such that, under $u''(.)<0$, every agents with $y\leq\hat{y}(\tau,\alpha)$ prefer $\gamma^*(\tau,\alpha;y)=0$ while those with $y>\hat{y}(\tau,\alpha)$ prefer a strictly positive $\gamma^*=\gamma^*(\tau,\alpha;y)$ satisfying

$$u'((1-\tau\gamma^*)y)=\alpha u'(\frac{\tau}{\pi}(\alpha\gamma^* y+(1-\alpha)E(\tilde{y}))). \tag{7}$$

Also, from now on and so as to ensure that $\gamma^*(\tau, \alpha; y) \leq 1$, we assume that for each agent, the tax rate chosen in equilibrium is at least equal to $\bar{\tau}(\alpha; y)$ defined by:[10]

$$u'((1-\bar{\tau})y) = \alpha u'(\frac{\bar{\tau}}{\pi}(\alpha y + (1-\alpha)E(\tilde{y}))). \tag{8}$$

This tax rate $\bar{\tau}(\alpha; y)$ then corresponds to the minimum tax rate level which generates full compliance for an agent with income y. Under A1, $\bar{\tau}(\alpha; y)$ is increasing in y so that in the rest of this section, we make the simplifying assumption that the tax rate chosen in equilibrium is at least equal to $\bar{\tau}(\alpha; y_{max})$ which ensures that $\gamma^*(\tau, \alpha; y) \leq 1 \, \forall y$.[11]

When $\tau > \bar{\tau}(\alpha; y_{max})$, agents choose to comply only if they expect to obtain a sufficiently high marginal return from public pensions. This depends both on their income and on the parameters of the pension system. If their income is small, below $\hat{y}(\tau, \alpha)$, marginal utility from consumption in the first period is high so that the marginal utility cost of reporting income in the first period is always higher than the marginal utility benefit they obtain from previous contributions in the second period. In the same way, if for instance, $\alpha = 0$, the system is fully redistributive so that increasing the rate of compliance, γ only entails a marginal utility cost in the first period but no additional benefit in the second one (agents cannot affect their pension benefit by reporting more). With a Beveridgian system, every agent then prefers to report nothing, $\gamma^*(\tau, 0; y) = 0 \, \forall y$. On the contrary, if $\alpha = 1$, the system is fully contributive and every agent chooses to report a fraction $\gamma^*(\tau, 1; y) = \frac{1}{\tau}\frac{\pi}{1+\pi} \, \forall y$, which corresponds to the level of compliance that perfectly smoothes consumption across periods.[12] We come back on the comparative statics of $\gamma^*(\tau, \alpha; y)$ at the end of this section.

We summarize our findings in the following proposition:

Proposition 1 *Assume a population of agents with income $y \in [y_{min}, y_{max}]$ who have the possibility to save on private markets and to choose the fraction of income they wish to report so as to finance a public pension system. In the absence of evasion costs,*

- *If $\alpha < \varepsilon$, every agents prefer private savings over the public pension system, so that $\gamma^*(\tau, \alpha; y) = 0$ and $s^*(\tau, \alpha; y) > 0 \, \forall y$.*

[10]This condition is obtained by evaluating equation (5) at $\gamma = 1$. For ease of notation, we have dropped the arguments of $\bar{\tau}$ in (8).

[11]Relaxing this assumption would add additional mathematical complexities to our model. In particular, agents may have different preferences for the tax rate but, more importantly, for some agents (with a high income) preferences may not be single-peaked so that the median voter theorem may not apply. As we will see, in the section with compliance costs, we will not face this problem and we will have instead conditions on income levels.

[12]Note that when $\alpha = 1$, if π=1, the "effective" tax rate $\gamma\tau$ is equal to 1/2, the top of the Laffer curve.

- *If $\alpha \geq \varepsilon$, no agent ever saves on private markets. Agents with income $y \leq \hat{y}(\tau, \alpha)$ do not report any income and rely exclusively on the Beveridgian part of the pension benefit. Agents with income $y > \hat{y}(\tau, \alpha)$ report at least a fraction $0 < \gamma^*(\tau, \alpha; y) \leq 1$ and receive a pension benefit which includes both a contributive and a redistributive part.*

Interestingly, in the case where $\alpha \geq \varepsilon$, we see that even if agents choose not to save on private markets, they are nonetheless not all going to comply and be willing to contribute to the public pension system. For any $0 \leq \alpha < 1$, at least part of the benefit they receive is a lump sum, which may be enough for them not to be willing to report any resources. On the contrary, if the system is fully contributive and there is no flat pension benefit, i.e. $\alpha = 1$, they will always report some income.

Before going further, let us come back to the definition of our problem and show that choosing γ for a given τ is equivalent to letting the agent choose his preferred *effective* tax rate level. In order to understand this, we make a change of variable and define $t \in [0, 1]$ as the effective rate of taxation: $t = \gamma\tau$. With $s = 0$, the agent's utility function (3) can be rewritten as:

$$U(t; y) = u((1-t)y) + \pi u(\frac{1}{\pi}(\alpha t y + (1-\alpha)E(ty)))$$

Maximizing $U(t; y)$ with respect to t yields the following first-order condition:

$$\frac{\partial U(t; y)}{\partial t} = y[-u'(c) + \alpha u'(d)] \leq 0 \tag{9}$$

This optimality condition, which gives the optimal level $t^*(\alpha; y)$, is the same as the optimality condition for $\gamma^*(\tau, \alpha; y)$. Also, the threshold income below which agents prefer a zero effective tax rate t, $\hat{y}(\alpha)$ defined by

$$u'(\hat{y}(\alpha)) = \alpha u'\big((1-\alpha)\frac{E(t^*(\alpha, y)y)}{\pi}\big) \tag{10}$$

with $E(t^*(\alpha, y)y) = \tau E(\tilde{y})$ under our change of variable, exactly corresponds to the threshold income below which agents decide not to report any income, $\hat{y}(\tau, \alpha)$ (see equation 6). Hence, the problem where an agent chooses his rate of compliance γ^* for a given τ is exactly equivalent to a problem where the agent chooses directly his preferred rate of contribution t^*, not having any choice on how much income to report.

Using A1, we also find that the effective preferred tax rate, $t^*(\alpha; y)$ is increasing in y so that the higher the agent's income, the higher the effective rate of taxation he is ready to bear.

Indeed, an agent with higher income would like to transfer more resources to the old age. Also, as it is clear from (9), $t^*(\alpha; y)$ is independent of τ. This means that for any tax rate $\tau > 0$, the agent will always pay his preferred amount of effective taxation $t^*(\alpha; y)$ by adapting exactly how much he reports, i.e. $\gamma^*(\tau, \alpha; y)$ to a variation in the level of the tax rate. As a consequence, the flat pension benefit, $b(\tau, \alpha) = (1 - \alpha)\tau E(\tilde{y})/\pi = (1 - \alpha)E(t^*(\alpha; y)y)/\pi$ is simply a residual, independent of the level of τ. This point will prove to be crucial for determining the voting equilibrium.

Let us finally make some comparative statics on $\gamma^*(\tau, \alpha; y)$. First, using equation (7), we show that the rate of compliance is increasing in income, $d\gamma^*(\tau, \alpha; y)/dy > 0$.[13] Indeed, as the agent becomes richer, both the (first-period) marginal cost of transferring resources to the old age and its (second-period) marginal benefit decrease; A1 guarantees however that the marginal cost decreases more rapidly than the marginal benefit so that the agent is willing to transfer more resources to the second period and $\gamma^*(\tau, \alpha; y)$ increases. Also, using the fact that $t^*(\alpha; y)$ is independent of τ, we can show that[14]

$$\frac{d\gamma^*(\tau, \alpha; y)}{d\tau} < 0.$$

Hence, a government who would be willing to increase compliance could do it through the pension system by decreasing the contribution rate (but the total resources collected would not vary, as shown above). Furthermore, we show in Appendix 6.1 that the variation of $\gamma^*(\tau, \alpha; y)$ with respect to α is ambiguous and that it depends both on the value of relative risk aversion and on the elasticity of the pension benefit to the Bismarkian factor, which itself depends on y. We can only show that, at the two extreme values of α, $\alpha = \{0; 1\}$, the preferred level of compliance, $\gamma^*(\tau, \alpha; y)$ is increasing in α. Also using $t = \gamma\tau$, we show that the reported tax base $E(\tilde{y})$ is always increasing with α. This will be useful when setting the constitutional value of α. Finally, using equations (6) or (10), we show that $d\hat{y}(\tau, \alpha)/d\tau = 0$. In words, an increase in the tax rate does not modify the threshold income after which agents start reporting income. This is a direct consequence of the flat pension benefit $b(\tau, \alpha)$ being independent of the tax rate. Besides, the effect of α on this threshold is ambiguous: $d\hat{y}(\tau, \alpha)/d\alpha \lesseqgtr 0$. Therefore, in

[13]We apply the implicit function theorem on (7).

[14]To see this, recall that

$$\frac{dt^*(\alpha; y)}{d\tau} = \gamma^*(\tau, \alpha; y) + \frac{d\gamma^*(\tau, \alpha; y)}{d\tau}\tau = 0$$

which implies that $d\gamma^*(\tau, \alpha; y)/d\tau < 0$.

the following, we will now denote the threshold below which agents do not report any income simply by $\hat{y}(\alpha)$ defined either by equation (6) or (10).

3.2 Preferences over the tax rate and the majority-voting equilibrium

We now study individual preferences over the tax rate and infer the majority-voting equilibrium. The indirect utility function of an agent with income y takes the following form:

$$V(\tau,\alpha;y)=u((1-\gamma^*(\tau,\alpha;y)\tau)y-s^*(\tau,\alpha;y))+\pi u(\frac{\tau}{\pi}(\alpha\gamma^*(\tau,\alpha;y)y+(1-\alpha)E(\tilde{y}))+\frac{\varepsilon}{\pi}s^*(\tau,\alpha;y))$$

where $\gamma^*(\tau,\alpha;y)\geq 0$ and $s^*(\tau,\alpha;y)=0$ when $\alpha\geq\varepsilon$.

The preferred tax rate is such that

$$\frac{\partial V(\tau,\alpha;y)}{\partial\tau} = \gamma^*(\tau,\alpha;y)y(\alpha u'(d)-u'(c))+(1-\alpha)u'(d)[E(\tilde{y})+\tau\frac{dE(\tilde{y})}{d\tau}]\leq 0 \qquad (11)$$

with $\gamma^*(\tau,\alpha;y)\geq 0$ for $y\geq\hat{y}(\alpha)$ and 0 otherwise. Note that the second term in the above expression is always nil which is a direct consequence of $b(\tau,\alpha)=(1-\alpha)\tau E(\gamma^*(\tau,\alpha;y)y)/\pi$ or equivalently of $b(\tau,\alpha)=(1-\alpha)E(t^*(\alpha;y)y)/\pi$ being independent of τ (see previous section).

The first term is also always nil. Indeed, if agents have $y\leq\hat{y}(\alpha)$, $\gamma^*(\tau,\alpha;y)=0$. These agents are then indifferent to the tax rate and $\tau^*(\alpha;y)\in[0,1]$. If now agents have $y>\hat{y}(\alpha)$, $\gamma^*(\tau,\alpha;y)>0$ satisfies (7) so that the first term in (11) also vanishes. However, the difference with agents having $y\leq\hat{y}(\alpha)$ is that agents with $y>\hat{y}(\alpha)$ are indifferent to the tax rate as long as $\tau^*(\alpha;y)$ is above $\bar{\tau}(\alpha;y)$.[15] In such a case, they can always choose $\gamma^*(\alpha;y)\in[0,1]$ so as to transfer exactly the amount of resources, $\tau^*(\alpha;y)\gamma^*(\tau,\alpha;y)>0$ they want for the next period. If on the contrary, $\tau^*(\alpha;y)$ could be below $\bar{\tau}(\alpha;y)$, these agents would not be able to transfer as much resources as they would be willing to since $\gamma^*(\tau,\alpha;y)$ is constrained to be below 1.

Therefore, whatever their income, agents are indifferent to the level of the tax rate as long as it lies within some boundaries. To understand this indifference, recall that $t^*(\alpha;y)$ is independent of τ so that the agent will always adapt $\gamma^*(\tau,\alpha;y)$ to the tax rate level so as to bear the preferred rate of effective taxation $t^*(\alpha;y)$, accounting for the constraint that $\gamma^*(\tau,\alpha;y)\leq 1$.

Let us now study the majority-voting equilibrium. When public pensions are more attractive than private saving markets, $\alpha\geq\varepsilon$, all agents are indifferent to the tax rate that serves to finance the pension system as long as it is above some threshold. There is thus unanimity in favor of

[15] To see this, evaluate (11) in $\tau=0$. It is infinitely positive under the assumption that $u'(0)\to\infty$. In that case, the FOC on γ, equation (5), is also infinitely positive.

any strictly positive tax rate such that $\tau^V \in]\bar{\tau}(\alpha; y_{max}), 1]$ and a pension system emerges. Once the majority voting tax rate is chosen, agents choose their rate of compliance, depending on their income level. Those at the bottom of the distribution with $y \leq \hat{y}(\alpha)$ choose to report nothing, $\gamma^*(\tau^V, \alpha; y) = t^*(\tau^V, \alpha; y) = 0$ as, for them, the marginal utility from first-period consumption is very high. They only get the Beveridgian part of the pension benefit in the second period. On the contrary, richer individuals, with $y > \hat{y}(\alpha)$, choose a positive level of compliance, $\gamma^*(\tau^V, \alpha; y) > 0$ or equivalently $t^*(\tau^V, \alpha; y) = \tau^V \gamma^*(\tau^V, \alpha; y) > 0$ which increases in their income so as to ensure $u'(c) = \alpha u'(d)$. Their compliance to the tax system will benefit the entire population, even agents who did not report any income through the distribution of the lump sum benefit.
Since agents are indifferent as to the level of taxation and can perfectly adapt the rate of compliance so as to obtain their most-preferred level of effective tax rate $t^*(\alpha; y)$, they always obtain, at the majority-voting equilibrium, the maximum utility level, which corresponds to the level obtained at their preferred allocation $(\gamma^*(\tau^*(\alpha; y), \alpha; y), \tau^*(\alpha; y))$.

Our results are summarized in the following proposition.

Proposition 2 *Whenever $\alpha \geq \varepsilon$ agents prefer public pensions over private savings. In such a case,*

- *agents with $y \leq \hat{y}(\alpha)$ are indifferent as to the tax rate level, $\tau^*(\alpha; y) \in [0, 1]$ while agents with $y > \hat{y}(\alpha)$ are also indifferent to the tax as long as $\tau^*(\alpha; y) \in [\bar{\tau}(\alpha; y), 1]$. Indifference is a direct consequence of them being able to choose $\gamma^*(\tau, \alpha; y)$ so as to keep the effective tax rate $t^*(\alpha; y) = \tau^*(\alpha; y)\gamma^*(\tau, \alpha; y)$ constant.*

- *At the majority voting equilibrium, there is unanimity in favor of $\tau^V \in [\bar{\tau}(\alpha; y_{max}), 1]$ and every agent obtains maximum utility.*

3.3 Constitutional choice of the level of the Bismarkian factor

As in Section 2, the social planner chooses α that maximizes the utility of the agent with income y_{min}. We assume for the moment that the level of the Bismarkian factor is greater than the return from private savings and check ex post that it is effectively the case. Using our previous results on individual optimal decisions, $\gamma^*(\tau, \alpha; y)$ (Section 3.1) and the level of the majority voting tax rate, τ^V (Section 3.2), the problem of the social planner consists in solving:

$$\max_{\alpha} V(\tau^V, \alpha; y_{min}) = u(y_{min}) + \pi u(\frac{\tau^V}{\pi}(1-\alpha)E(\tilde{y}))$$

where $\tau^V \in]\bar{\tau}(\alpha; y_{max}), 1]$ is independent of α and $E(\tilde{y}) = E(\gamma^*(\tau^V, \alpha, y)y)$. Note that since it is likely that $y_{min} < \hat{y}(\alpha)$, the agent with minimum income does not comply and thus, receives in the second period only the flat pension benefit, $b(\tau^V, \alpha)$. Differentiating this expression with respect to α, we obtain:

$$\frac{\partial V(\alpha; y_{min})}{\partial \alpha} = u'(d)\tau^V \left[(1-\alpha)\frac{dE(\tilde{y})}{d\alpha} - E(\tilde{y})\right] \leq 0 \tag{12}$$

First note that $\partial V(\alpha; y_{min})/\partial\alpha|_{\alpha=0} > 0$ and that $\partial V(\alpha; y_{min})/\partial\alpha|_{\alpha=1} < 0$ so that the optimal α is always interior.[16] Using the result shown in Appendix 6.1 that $dE(\tilde{y})/d\alpha > 0$, we find that the level of α^R is then implicitly defined by:

$$(1-\alpha^R)\frac{dE(\tilde{y})}{d\alpha} = E(\tilde{y}).$$

Contrary to Casamatta et al. (2000a, b) and to expression (2), equation (12) does not include the variation of the majority-voting tax rate with respect to α. This a direct consequence of the indifference of agents as to the level of τ^V; they will always modify $\gamma^*(\tau^V, \alpha; y)$ accordingly to keep $t^*(\alpha; y)$ constant. Therefore, having a positive α is not a way here to ensure a higher support from richer agents when the system is partly redistributive. Here, the only reason for a positive α is related to the distortion effect through $E(\tilde{y})$: a higher α increases the tax base $E(\tilde{y})$ and thus the flat pension benefit part which benefits mostly the least-favored. This effect is absent under full compliance and is new with regards to the existing literature on the political economy of pension systems.

Ex post, if the constitutional level of α is effectively greater than the return from private savings ε, a public pension system with parameters (τ^V, α^R) emerges. It provides maximum utility to agents with minimum income.[17] If, on the contrary, solving (12) yields that α^R should be smaller than ε, the social planner should arbitrarily choose $\alpha^R = \varepsilon$. In such a case, a pension system emerges and the agent with minimum wage is certainly better-off than with private savings. To see this clearly, assume that the agent at the bottom of the distribution

[16] In order to see that $\partial V(\alpha; y_{min})/\partial\alpha|_{\alpha=0} > 0$, recall that $\gamma^*(\tau, 0; y) = 0 \forall y$ so that $E(\tilde{y}) = 0$ in (12).

[17] Recall that agents are indifferent to the level of the tax rate and that, at the majority-voting equilibrium, they obtain maximum utility for a given α.

has $y_{min} = 0$. In such a case, he would not save and would have a utility level equal to $(1+\pi)u(0)$ while with a pension system, he would obtain $u(0) + \pi u(b(\tau^V, \alpha^R))$ and would be strictly better-off. This should also be the case if the worst-off agent has a strictly positive income; if his reported income is small relative to the average one $E(\tilde{y})$, he would get more from the Beveridgian part of the pension system than if he was saving on private markets, even if the marginal return from private savings is higher than that of the public pension.

Before turning to the section with evasion costs, let us finally briefly discuss the situation where private markets are more attractive than public pensions, $\alpha < \varepsilon$. In that case, agents prefer to rely on private savings rather than on public pensions so as to transfer income to the old-age period. They are thus indifferent to the existence of a public pension system and deciding on τ or α is not relevant as nobody will ever report resources to finance the pension system.

4 Introducing a cost of evasion

4.1 Individual decisions

Not complying with the tax system generally entails a cost to the agent. This cost can either be financial, when the agent for instance has to pay lawyers or financial experts to avoid taxation, or when it takes time (which the agent cannot use for remunerated activities) not to comply. It could also be psychological when the agent deviates from the moral norm of reporting honestly income.

To take this feature into account, we assume from now on that an agent with income y has to incur a cost of evasion which has the following form:[18]

$$C(\gamma, \tau; y) = \frac{1}{2}\tau y(1-\gamma)^2, \tag{13}$$

This cost is supported in the first period, at the time the agent makes the decision to evade income and it is increasing and convex in the fraction $(1-\gamma)$ of income evaded.[19] Note also that since γ is a function of (τ, α) and y as we show below, total cost of evasion will depend both directly and indirectly on y and τ but only indirectly on α. Given the form of this cost,

[18]Here, this cost is paid with certainty. This is different from assuming auditing, as in Borck (2004, 2009) and Traxler (2012) where agents face a probability of being caught and having to pay expensive fines. However, to keep the problem tractable, they assume quasi-linearity which makes their formulation and ours very close.

[19]If the cost is psychological, the above function represents the monetary-equivalent of the psychological cost.

one can rewrite first-period consumption as follows:

$$\begin{aligned} c &= y(1-\gamma\tau) - \frac{1}{2}\tau y(1-\gamma)^2 \\ &= y(1-\varphi(\gamma)\tau) \end{aligned}$$

where $\varphi(\gamma) = \frac{1}{2}(1+\gamma^2) > \gamma$ represents the overall cost of taxation to the individual and $\varphi'(\gamma) = \gamma > 0$, $\varphi''(\gamma) = 1$. For any $0 \leq \tau \leq 1$ and $0 \leq \gamma \leq 1$, c is always positive.

For given pension parameters (τ, α), problem (3) of an agent with income y is now modified so as to account for the cost of not complying with the tax system, as follows:

$$\max_{\gamma} u(y(1-\varphi(\gamma)\tau) - s) + \pi u(\frac{\tau}{\pi}[\alpha\gamma y + (1-\alpha)E(\tilde{y})] + \frac{\varepsilon s}{\pi}) \tag{14}$$

First order conditions with respect to s and γ are:

$$\frac{\partial U}{\partial s} = -u'(c) + \varepsilon u'(d) \leq 0 \tag{15}$$

$$\frac{\partial U}{\partial \gamma} = y\tau[-\gamma u'(c) + \alpha u'(d)] \leq 0. \tag{16}$$

While the first condition is identical to our original model, the second condition differs by a term γ which decreases the marginal utility cost of complying with the system. Evaluating (16) in $\gamma = 0$, we obtain that

$$\frac{\partial U}{\partial \gamma}|_{\gamma=0} = y\tau\alpha u'(d) > 0,$$

so that $\gamma^*(\tau, \alpha; y)$ will always be positive for strictly positive levels of (τ, α). Hence, introducing a convex cost of evasion like the one in equation (13) guarantees that agents always report some income. To the contrary, evaluating (15) in $s = 0$, we cannot rule out the possibility that for some levels of y, $s^*(\tau, \alpha; y) = 0$. Replacing (16) into (15), we find that this is the case for income levels such that

$$\frac{\alpha}{\gamma^*(\tau, \alpha; y)} > \varepsilon. \tag{17}$$

In words, the marginal return from public pensions (net of the evasion cost) has to be strictly greater than that of private savings. This relation will be used below.

Therefore, contrary to our previous section, it is now possible to have both partial compliance and positive savings, independently of the relation between α and ε. This is due first to the existence of compliance costs which force agents to report income and second, to the fact that the marginal return from compliance, α/γ, which has to be compared with that of savings ε, is now endogenous and depends on the individual's preferred level of compliance, $\gamma^*(\tau, \alpha; y)$. Since evasion is costly, every agent wants to report at least a fraction of his income but at the same time, increased compliance reduces the net marginal return he can obtain from the public system, making savings more attractive.

In what follows we characterize the individuals' optimal saving and preferred tax rate. These decisions depend both on their income and on whether the marginal return from saving is higher than that of pension. We will first characterize the income range for which individuals do not save. We show that there exist some income thresholds such that individuals with lower income have this behavior. We then characterize the behavior of individuals with income above and below these thresholds. In both cases we will have to consider separately the two possible relations between α and ε.

Definition of income thresholds.

Let us consider first the case where agents have an income level so that they do not save, i.e. $s^*(\tau, \alpha; y) = 0$ and (15) is strictly negative. In that situation, they report a positive fraction of their income, $0 < \gamma^*(\tau, \alpha; y) \leq \min\{\alpha/\varepsilon; 1\}$ which satisfies (16). Using the implicit function theorem and assumption A1, it is possible to show that in such a case, $\gamma^*(\tau, \alpha; y)$ is increasing in y. Hence, this situation where savings are nil corresponds to a situation where the agents' income is so low that compliance is low and that agents obtain a strictly greater marginal return from pensions (net of the cost of compliance), i.e. a greater marginal return from complying than from private savings: $\frac{\alpha}{\gamma^*(\tau,\alpha;y)} > \varepsilon$. However, as y increases, $\alpha/\gamma^*(\tau, \alpha; y)$ decreases until it becomes equal to ε which enables us to define income thresholds above which agents report a constant fraction of their income and eventually save. The definition of these thresholds yet crucially depends on the value of the ratio α/ε as the rate of compliance is constrained to be below one, as we shall explain now.

Whenever $\alpha < \varepsilon$, we denote $y_1(\tau, \alpha)$ the income level beyond which agents report a constant fraction of their income *and* start saving. It is implicitly defined by[20]

$$u'(y_1(1 - \varphi(\gamma^*(\tau, \alpha; y_1))\tau)) = \varepsilon u'(\frac{\tau}{\pi}(\alpha\gamma^*(\tau, \alpha; y_1)y_1 + (1 - \alpha)E(\tilde{y}))) \tag{18}$$

[20]For ease of notation, we drop the arguments of the function whenever it is clear.

with $\gamma^*(\tau, \alpha; y_1) = \alpha/\varepsilon < 1$ and $s^*(\tau, \alpha; y_1) = 0$. In that situation, every agent with $y \leq y_1(\tau, \alpha)$ reports an increasing amount of their income and do not save, while every agent with income above $y_1(\tau, \alpha)$ reports a constant fraction of their income, α/ε and makes positive savings. Note however that if the minimum income y_{min} in the society is sufficiently big, no agent is liquidity constrained (and do not save) so that $y_1(\tau, \alpha)$ is not relevant.[21]

When $\alpha \geq \varepsilon$, $\gamma^*(\tau, \alpha; y_1)$ would be higher than one so that we need to define instead two income thresholds, $y_2(\tau, \alpha)$ and $y_3(\tau, \alpha)$. The first one, $y_2(\tau, \alpha)$ corresponds to the income threshold at which the rate of compliance becomes just equal to 1, $\gamma^*(\tau, \alpha; y_2) = 1$ and it satisfies

$$u'(y_2(1-\tau)) = \alpha u'(\frac{\tau}{\pi}(\alpha y_2 + (1-\alpha)E(\tilde{y}))) \tag{19}$$

with $s^*(\tau, \alpha; y_2) = 0$ since at this income level, $u'(c) > \varepsilon u'(d)$. Note that $y_2(\tau, \alpha)$ is relevant only if the range of incomes in the population is not too big, i.e. y_{min} and $\bar{y}$ are not too different.[22] As y increases further, first-period marginal utility of consumption, $u'(c)$ decreases faster than second-period marginal utility $\varepsilon u'(d)$ under assumption A1 so that the two become equal at some income threshold $y_3(\tau, \alpha)$ defined by[23]

$$u'(y_3(1-\tau)) = \varepsilon u'(\frac{\tau}{\pi}(\alpha y_3 + (1-\alpha)E(\tilde{y}))). \tag{20}$$

with $s^*(\tau, \alpha; y_3) = 0$ and $\gamma^*(\tau, \alpha; y_3) = 1$. Beyond this threshold, agents who already fully comply effectively start saving.

Individual decisions when $y < y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$

In that situation, agents do not save and partially comply with $\gamma^*(\tau, \alpha; y)$ defined by (16). As we already mentioned, the rate of compliance is increasing in y. Moreover, in Appendix 6.2 we show that, for agents with income below $y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$ (i.e. for those who do not save), the variation of $\gamma^*(\tau, \alpha; y)$ with τ is negative as long as we are on the increasing part of the Laffer

[21] To see this, replace (16) in (15) and evaluate it in $s = 0$. It is always non negative if $\gamma^*(\tau, \alpha, y) > \alpha/\varepsilon$. Since $\gamma^*(\tau, \alpha, y)$ is monotonically increasing in y, this will be the case for sufficiently high levels of y.

[22] To see this, evaluate (16) at $\gamma = 1$ and $y = y_{min}$. If y_{min} is much lower than $\bar{y}$, this expression is more likely to be positive. To the opposite, if $y_{min} \to \bar{y}$, this expression can be positive or negative.

[23] Under assumption A1 and using the fact that $u'(c) > \varepsilon u'(d)$ for income levels between $y_2(\tau, \alpha)$ and $y_3(\tau, \alpha)$, one can prove that $-u'(c) + \varepsilon u'(d) < 0$ increases with y.

curve.[24] In such a case, when the tax rate increases, the marginal cost of complying increases while its marginal benefit decreases (both the contributive and the flat parts of the pension benefit are higher) so that agents with $y \leq y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$ comply less.

Individual decisions when $y > y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$

Let us now study the decisions of agents whose income is sufficiently big that they report a constant fraction of their income. As shown above, this fraction depends on whether $\alpha \gtreqless \varepsilon$. Whenever $\alpha < \varepsilon$, agents with income $y > y_1(\tau, \alpha)$ choose both to save and to partially comply. Setting (15) and (16) to zero, it is straightforward to see that, for them:

$$\gamma^*(\tau, \alpha; y) = \frac{\alpha}{\varepsilon}.$$

If the rate of return of public pensions is smaller than that of savings, agents always report a fraction of their income, because it is costly to evade, but not all. In that case, they still face compliance costs for an amount $C(\alpha/\varepsilon, \tau; y) = \tau y(1 - \alpha/\varepsilon)^2/2$ and they set the degree of compliance so as to equalize the marginal returns from compliance and from savings. In equilibrium, the rate of compliance is independent of y and of τ, but it is increasing in the Bismarkian factor as a higher degree of contributiveness generates a higher return from compliance to the public system. For any additional dollar above $\gamma^*(\tau, \alpha; y_1)y_1 = \alpha y_1/\varepsilon$ these agents would like to transfer to the second period, they will then use the private saving system (whose marginal return is now higher). The level of savings $s^*(\tau, \alpha; y)$ satisfies

$$u'(y(1 - \varphi(\frac{\alpha}{\varepsilon})\tau) - s^*) = \varepsilon u'(\frac{\tau}{\pi}(\frac{\alpha^2}{\varepsilon}y + (1 - \alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}s^*). \tag{21}$$

Using the implicit function theorem on this condition and the fact that $\gamma^*(\tau, \alpha; y)$ is independent of y, we obtain that the optimal level of savings, $s^*(\tau, \alpha; y)$ is monotonically increasing in y under assumption A1.

To the opposite, whenever the rate of return from public pensions is strictly greater than that of private savings ($\alpha \geq \varepsilon$), agents with income $y > y_2(\tau, \alpha)$ report all their income, $\gamma^*(\tau, \alpha; y) = 1$ and face no evasion cost, $C(1, \tau; y) = 0$. They would even like to report more than $\gamma^*(\tau, \alpha; y) = 1$

[24] An increase in τ then unambiguously leads to an increase in resources collected $\tau E(\tilde{y})$. At the preferred tax rate of agents with incomes below $y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$, this is effectively the case (see equation 24 hereafter). At the voting equilibrium, this is also generally the case; only if *both* $\alpha \geq \varepsilon$ and y_m is relatively high, this will not the case (see Section 4.2.2).

and $\alpha u'(d) > u'(c)$ in that situation. Regarding savings, agents with intermediate income $y_2(\tau, \alpha) < y \leq y_3(\tau, \alpha)$ are liquidity constrained and choose not to save. In that case, their income level is such that $\alpha u'(d) > u'(c) > \varepsilon u'(d)$. Finally, agents at the top of the income distribution, $y > y_3(\tau, \alpha)$ complement pension benefits received from full compliance with private savings. Their preferred level of savings, $s^*(\tau, \alpha; y)$ satisfies

$$u'(y(1-\tau) - s^*) = \varepsilon u'(\frac{\tau}{\pi}(\alpha y + (1-\alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}s^*) \tag{22}$$

and it increases in y. Therefore, when the marginal return from public pensions is higher than that of private savings, a high-income agent reports all his income and incurs no cost. The only way for a high-income agent to transfer more resources to the old-age than what is permitted through the pension system (the compliance rate cannot exceed one) is done through the private saving market, but this is at the expense of a smaller marginal return.

Our results are summarized in the following proposition.

Proposition 3 *Assume a population of agents with income $y \in [y_{min}, y_{max}]$ who have the possibility to save on private markets and to choose the fraction of income they wish to report so as to finance a public pension system. If evasion is costly,*

- *When $\alpha < \varepsilon$, there exists an income threshold $y_1(\tau, \alpha)$ such that*
  - *agents with income $y \leq y_1(\tau, \alpha)$ do not save, $s^*(\tau, \alpha; y) = 0$ and they report a positive fraction of income, $0 < \gamma^*(\tau, \alpha; y) \leq 1$ which satisfies (16). This fraction is increasing in y.*
  - *Agents with $y > y_1(\tau, \alpha)$ report a constant fraction of their income, $\gamma^*(\tau, \alpha; y) = \alpha/\varepsilon$ and save a positive amount. For these individuals, saving is defined by (21) and it is increasing in y.*
- *When $\alpha \geq \varepsilon$, there exist two income thresholds $y_2(\tau, \alpha)$ and $y_3(\tau, \alpha)$ such that:*
  - *agents with income $y \leq y_2(\tau, \alpha)$ do not save, $s^*(\tau, \alpha; y) = 0$ and they report an increasing fraction of income, $0 < \gamma^*(\tau, \alpha; y) \leq 1$ which satisfies (16). This fraction is increasing in y.*
  - *agents for whom y satisfies $y_2(\tau, \alpha) < y \leq y_3(\tau, \alpha)$ fully comply but do not save.*

– *Agents with $y > y_3(\tau, \alpha)$ fully comply and save on private markets ($s^*(\tau, \alpha; y) \geq 0$). Saving is defined by (22) and it is increasing in income.*

Let us finally make two remarks. First, using the facts that $\gamma^*(\tau, \alpha; y_1) = \alpha/\varepsilon$ and $\gamma^*(\tau, \alpha; y_2) = \gamma^*(\tau, \alpha; y_3) = 1$, we show that under A1, $y_1(\tau, \alpha)$, $y_2(\tau, \alpha)$ and $y_3(\tau, \alpha)$ are increasing in τ. Therefore, agents start reporting a constant fraction of their income and saving at higher levels of income for higher levels of τ.[25]

Second, for $y \leq y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$, the variation of $\gamma^*(\tau, \alpha; y)$ with α is ambiguous and depends on whether $\gamma^*(\tau, \alpha; y)y \gtreqless E(\tilde{y})$ and thus, on the specific value of y. We can only show that $dE(\tilde{y})/d\alpha > 0$ (see Appendix 6.2). This last result will prove to be useful when determining the optimal level of α (Section 4.3).

Having determined the agents' private decisions, we now turn to studying their preferred tax rate and the majority-voting equilibrium.

4.2 Preference for the tax rate and the majority-voting equilibrium.

4.2.1 Individuals' preferred tax rate

The indirect utility function of an agent with income y writes:

$$V(\tau, \alpha; y) = u(y(1 - \varphi(\gamma^*(\tau, \alpha; y))\tau) - s^*(\tau, \alpha; y)) + \pi u\big(\frac{\tau}{\pi}[\alpha\gamma^*(\tau, \alpha; y)y + (1 - \alpha)E(\tilde{y})] + \frac{\varepsilon s^*(\tau, \alpha; y)}{\pi}\big)$$

where $\gamma^*(\tau, \alpha; y) > 0 \, \forall y$ and $s^*(\tau, \alpha; y) \geq 0$ have been characterized in the previous section. Differentiating $V(\tau, \alpha; y)$ with respect to τ, we obtain

$$\begin{aligned} \frac{\partial V(\tau, \alpha; y)}{\partial \tau} &= -u'(c)y\varphi(\gamma^*(\tau, \alpha; y)) \\ &+ u'(d)\big[\alpha\gamma^*(\tau, \alpha; y)y + (1 - \alpha)E(\tilde{y}) + \tau(1 - \alpha)\frac{dE(\tilde{y})}{d\tau}\big] \leq 0 \end{aligned} \tag{23}$$

where we made use of the envelop theorem for $\gamma^*(\tau, \alpha; y)$ and for $s^*(\tau, \alpha; y)$.

Case where $\alpha < \varepsilon$

Agents with income $y \leq y_1(\tau, \alpha)$ choose $s^*(\tau, \alpha; y) = 0$ and $\gamma^*(\tau, \alpha; y) > 0$ defined by $\gamma^*(\tau, \alpha; y)u'(c) = \alpha u'(d)$. Evaluating the above FOC in $\tau = 0$, it is straightforward to see that under the assumption that $u'(0) \to +\infty$, the preferred tax rate level is always interior for these agents. This is not surprising as for them the pension system constitutes the only way to transfer income toward

[25] The variation with α is ambiguous, independently of whether $\alpha \gtreqless \varepsilon$.

the old age. Replacing for (16) in (23), we obtain after some rearrangements that the preferred tax rate satisfies

$$\frac{\partial V(\tau,\alpha;y)}{\partial \tau} = u'(d)\big[\alpha y\big(\gamma^*(\tau,\alpha;y) - \frac{\varphi(\gamma^*(\tau,\alpha;y))}{\gamma^*(\tau,\alpha;y)}\big) + (1-\alpha)(E(\tilde{y}) + \tau\frac{dE(\tilde{y})}{d\tau})\big] \leq 0 \qquad (24)$$

where $\gamma^*(\tau,\alpha;y) - \varphi(\gamma^*(\tau,\alpha;y))/\gamma^*(\tau,\alpha;y) = (\gamma^*(\tau,\alpha;y)^2 - 1)/2\gamma^*(\tau,\alpha;y) < 0$. The above expression sums up the marginal utility benefits and costs of increasing the tax rate for an agent with income $y \leq y_1(\tau,\alpha)$. On the one hand, increasing the tax rate makes the agent obtain more from the pension benefit in the second period, both through the Bismarkian part $(\alpha\gamma^*(\tau,\alpha;y)y)$ of the pension benefit and through the Beveridgian part $((1-\alpha)(E(\tilde{y})+\tau\frac{dE(\tilde{y})}{d\tau}))$ which is positive on the increasing part of the Laffer curve. On the other hand, increasing the tax rate in the first period is costly as it decreases consumption; this includes both the cost of taxation itself and the cost of non-compliance which is an increasing function of τ. This marginal cost evaluated in second-period utility terms is equal to $u'(d)\alpha y\varphi(\gamma^*(\tau,\alpha;y))/\gamma^*(\tau,\alpha;y)$ where we made use of (16).

Differentiating the above expression with respect to y, we obtain after some rearrangements that

$$\frac{\partial^2 V(\tau,\alpha;y)}{\partial \tau \partial y} = \frac{\alpha u'(d)}{2\gamma^*(\tau,\alpha;y)}\big[\gamma^*(\tau,\alpha;y)^2 - 1 + \nu_{\gamma^*,y}(\gamma^*(\tau,\alpha;y)^2+1)\big]$$

with $\nu_{\gamma^*,y} = d\gamma^*(\tau,\alpha;y)/dy \times y/\gamma^*(\tau,\alpha;y) > 0$, the elasticity of compliance with respect to income. If the value of this elasticity is relatively small, below $(1-\gamma^*(\tau,\alpha;y)^2)/(1+\gamma^*(\tau,\alpha;y)^2)$, we find, using the implicit function theorem, that $d\tau^*(\alpha;y)/dy < 0$ while, if the value of the elasticity is high, it should be positive. Hence, the variation of the preferred value of the tax rate of agents with income $y \leq y_1(\tau,\alpha)$ crucially depends on how responsive the compliance rate is to income variation. To understand this, note that in (24),

$$\alpha y\big(\gamma^*(\tau,\alpha;y) - \frac{\varphi(\gamma^*(\tau,\alpha;y))}{\gamma^*(\tau,\alpha;y))}\big)$$

represents the net marginal benefit obtained from direct contributions (evaluated in the second period) and this is negative, given the existence of compliance costs. Whenever the elasticity of compliance with respect to income is small, a higher income increases relatively more the marginal cost of taxation (second term) than its marginal benefit (first term), therefore leading to a smaller preferred tax rate. To the opposite, if the elasticity of compliance is relatively high, an increase in income increases relatively more the marginal benefit of taxation than its marginal cost and the preferred tax rate thus increases in y. This condition on the size of $\nu_{\gamma^*,y}$ will prove to be crucial for the existence of a voting equilibrium. We come back to this point in the next section.

When agents have an income $y > y_1(\tau, \alpha)$, they report a constant fraction of their income $\gamma^*(\tau, \alpha; y) = \alpha/\varepsilon$ and make positive savings which satisfy condition (21). Replacing for this condition into (23), we obtain:

$$\frac{\partial V(\tau, \alpha; y)}{\partial \tau} = u'(d)\big[\alpha y(\gamma^*(\tau, \alpha; y) - \frac{\varepsilon}{\alpha}\varphi(\gamma^*(\tau, \alpha; y))) + (1-\alpha)(E(\tilde{y}) + \tau\frac{dE(\tilde{y})}{d\tau})\big] \leq 0 \quad (25)$$

Again, the above expression represents the marginal benefits and costs of increasing the tax rate τ for an agent with income $y > y_1(\tau, \alpha)$. The difference with the previous case is in the evaluation of the cost of evasion in the second period, which is now equal to $\varepsilon\varphi(\gamma^*(\tau, \alpha; y))y$ as (15) is now binding. Replacing for $\gamma^*(\tau, \alpha; y) = \alpha/\varepsilon$,

$$\frac{\partial V(\tau, \alpha; y)}{\partial \tau} = u'(d)\big[\frac{y}{2\varepsilon}(\alpha^2 - \varepsilon^2) + (1-\alpha)(E(\tilde{y}) + \tau\frac{dE(\tilde{y})}{d\tau})\big] \leq 0 \quad (26)$$

where the first term is negative. Also, since this expression is strictly decreasing in y, we find that agents with a sufficiently high income, beyond some threshold $\dot{y}(\alpha)$ defined by[26]

$$\dot{y}(\alpha) = \frac{2\varepsilon}{\varepsilon^2 - \alpha^2}(1-\alpha)E(\gamma^*(0, \alpha, y)y), \quad (27)$$

prefer a zero tax rate. This is directly related to their opportunity to save on private markets so as to obtain income for their old age. To the contrary, agents with income $y \leq \dot{y}(\alpha)$ choose an interior tax rate such that (26) holds with equality, and making use of the implicit function theorem, we have that $d\tau^*(\alpha; y)/dy < 0 \, \forall y \leq \dot{y}(\alpha)$. To understand this, note that as before, agents are willing to equalize the marginal benefits of increasing taxation with its marginal cost. But, when y increases, the first-period marginal overall cost of taxation, $\varepsilon\varphi(\gamma^*(\tau, \alpha; y))$ increases relatively more than its marginal benefit $\alpha\gamma^*(\tau, \alpha; y)$ (through the contributory part of the pension benefit) so that agents with higher income prefer smaller tax rates. Beyond $\dot{y}(\alpha)$, they are always net contributors to the pension system so that they even prefer no taxation at all and to rely exclusively on savings as in such a case the marginal cost of increasing the tax rate always strictly outweighs its marginal benefit (and (26) is strictly negative $\forall\tau$).

Let us remark that the threshold $\dot{y}(\alpha)$ is relevant only if $y_1(\tau, \alpha) \leq \dot{y}(\alpha)$ and if this is the case, the preference for the tax rate with respect to y is continuous in $y_1(\tau, \alpha)$.[27]

[26] This threshold is obtained by evaluating (26) in $\tau = 0$.

[27] To see this, note that at $y_1(\tau, \alpha)$, both (15) and (16) hold with an equality, so that at $y_1(\tau, \alpha)$, (24) and (25) are the same.

Case where $\alpha \geq \varepsilon$

Agents with income below $y_2(\tau, \alpha)$ prefer a tax rate level that satisfies (24) and it is decreasing or increasing in y depending on the value of the elasticity of compliance with respect to income, as shown above.

Agents with intermediate income, $y_2(\tau, \alpha) < y \leq y_3(\tau, \alpha)$ report all their income, $\gamma^*(\tau, \alpha; y) = 1$ but do not save on private markets. Since savings are nil, their preferred tax rate is always strictly positive, under $u'(0) \to +\infty$. Replacing for $\gamma^*(\tau, \alpha; y) = 1$, (23) simplifies to[28]

$$\frac{\partial V(\tau, \alpha; y)}{\partial \tau} = y[u'(d)\alpha - u'(c)] + u'(d)(1-\alpha)(E(\tilde{y}) + \tau \frac{dE(\tilde{y})}{d\tau}) \leq 0 \tag{28}$$

with $\alpha u'(d) > u'(c)$. This condition holds with equality at $\tau^*(\alpha; y)$. Using the implicit function theorem and assumption A1, we find that agents with higher incomes most-prefer higher values of the tax rate. The intuition for this result is the following. Whenever $\alpha \geq \varepsilon$, agents with an income above $y_2(\tau, \alpha)$ most prefer a higher-than-one value of $\gamma^*(\tau, \alpha; y)$ and would like to transfer more resources to the future than what they actually can (they already fully comply and cannot report more than their true income level). The only way to do so is by choosing a higher tax rate.[29] Since agents with a higher income would like to transfer more to the old age, they prefer a higher tax rate than those with a smaller income.

Finally agents with income above $y_3(\tau, \alpha)$ fully comply with the tax system and also choose to save on private markets. Replacing for $\gamma^*(\tau, \alpha; y) = 1$ and for the FOC on savings in (23), we obtain

$$\frac{\partial V(\tau, \alpha; y)}{\partial \tau} \quad = \quad u'(d)y[\alpha - \varepsilon] + u'(d)(1-\alpha)\big[E(\tilde{y}) + \tau \frac{dE(\tilde{y})}{d\tau}\big] \leq 0 \tag{29}$$

This condition holds with equality at $\tau^*(\alpha; y)$. Using the implicit function theorem, we obtain that the preferred tax rate of agents with income $y > y_3(\tau, \alpha)$ is increasing in income. The interpretation of this result is identical to that for agents with intermediate incomes. Even though these agents now save on private markets, the return from the pension system is greater than that from savings so that they would rather use public pensions than private savings to transfer income to the old age. Since they already fully comply, the only way to do so is through a higher tax rate.

[28]Recall that in this case, the FOC on saving is not binding and that the FOC for γ cannot be used either since we have constrained $\gamma^*(\tau, \alpha; y)$ to be equal to 1.

[29]Recall from the previous section that γ and τ are substitutes. Here, because of compliance costs, they are only *imperfect* substitutes.

Let us finally note that, interestingly, when $\alpha \geq \varepsilon$, every agent with $y > y_2(\tau, \alpha)$ most-prefers a value of the tax rate that lies on the decreasing part of the Laffer curve (see conditions (28) and (29)). Although this may seem surprising at first glance, this is in fact a direct consequence of the rate of compliance being constrained to be at most equal to 1. In such a case, agents would like to report even more than their true income amount. The only way to get more resources in the future is therefore to vote for a high tax rate, even if it is at the expense of a smaller tax base and thus, of a smaller Beveridgian part.

4.2.2 Majority-voting equilibrium

One of the difficulties in characterizing the voting equilibrium resides in the fact that whether agents partially or fully comply with the tax system and whether they save or not (i.e. depending on whether $y \gtrless y_1(\tau, \alpha), y_2(\tau, \alpha)$ or $y_3(\tau, \alpha)$) shape their preference for the tax rate differently so that one cannot use directly the single-crossing condition established by Gans and Smart (1996).

We show (see the Appendix) that for each category of agents, the marginal rates of substitution between τ and $b(\tau, \alpha)$ vary monotonically in the same direction and that this variation is continuous in $y_1(\tau, \alpha)$ when $\alpha < \varepsilon$ (resp. $y_2(\tau, \alpha)$ and $y_3(\tau, \alpha)$ when $\alpha \geq \varepsilon$). However, one sufficient condition for all marginal rates of substitution to vary in the same direction is that for low-income agents (i.e. with income below $y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$), the elasticity of compliance with respect to income, $\nu_{\gamma^*,y}$ is small and below $(1 - \gamma^*(\tau, \alpha; y)^2)/(1 + \gamma^*(\tau, \alpha; y)^2)$ whenever $\alpha < \varepsilon$, while for $\alpha \geq \varepsilon$, $\nu_{\gamma^*,y}$ should be above $(1 - \gamma^*(\tau, \alpha; y)^2)/(1 + \gamma^*(\tau, \alpha; y)^2)$. Intuitively, this means that whenever it is more interesting to rely on the public than on the private sector ($\alpha \geq \varepsilon$), agents should comply relatively more as their income increases than when $\alpha < \varepsilon$; this seems reasonable.

Only in these cases, the marginal rates of substitution between τ and $b(\tau, \alpha)$ are all monotonically increasing (resp. decreasing) in y over the whole $[y_{min}, y_{max}]$ interval when $\alpha < \varepsilon$ (resp. $\geq$). Preferences therefore satisfy the single-crossing property and there exists a unique Condorcet winner, $0 \leq \tau^V(\alpha) < 1$ which corresponds to the preferred tax rate of the agent with median income.

The value of $\tau^V(\alpha)$ is likely to depend on whether $\alpha \gtrless \varepsilon$. If $\alpha < \varepsilon$, the agent's preferred tax rate is everywhere decreasing in income as the elasticity of compliance with respect to income for low-income agents is assumed to be below $(1-\gamma^*(\tau, \alpha; y)^2)/(1+\gamma^*(\tau, \alpha; y)^2)$ in that case (for the single-crossing condition to be satisfied). If $y_m \leq \max\{y_1(\tau,\alpha), \dot{y}(\alpha)\}$, $\tau^V(\alpha) > 0$ and defined by either (24) or (26) evaluated in y_m, depending on the precise ranking between $y_1(\tau,\alpha), \dot{y}(\alpha)$ and y_m.[30] Note that a positive $\tau^V(\alpha)$ is more likely whenever $y_1(\tau,\alpha) < \dot{y}(\alpha)$ and that it always lies on the increasing part of the Laffer curve, as it is clear from equations (24) and (26).

To the contrary, if $\alpha \geq \varepsilon$, the preferred tax rate is everywhere increasing in y as we have assumed that the elasticity of compliance with respect to income for low-income agents is above $(1-\gamma^*(\tau,\alpha;y)^2)/(1+\gamma^*(\tau,\alpha;y)^2)$ in that case. The majority voting tax rate $\tau^V(\alpha)$ is then always strictly positive and defined either by (24), (28) or (29) depending on the relative position of y_m with respect to $y_2(\tau,\alpha)$ and $y_3(\tau,\alpha)$. Interestingly, if $y_m > y_2(\tau,\alpha)$, the majority voting tax rate always lies on the decreasing part of the Laffer curve. A majority of agents will therefore be in favour of pushing the majority tax rate above the level that maximizes the tax base, simply because it enables them to transfer resources to the old-age at a higher return, through the contributory pension benefit part, than with private savings, even if it is at the expense of a smaller tax base and of a lower flat pension benefit.

Our results are summarized in the following proposition:

Proposition 4 *Assume a population of agents with income $y \in [y_{min}, y_{max}]$ who have the possibility to save on private markets and to choose the fraction of income they wish to report so as to finance a public pension system. The majority voting equilibrium tax rate, $\tau^V(\alpha)$ is such that*

1. *If $\alpha < \varepsilon$:*
   - *(a) If $y_m \leq \max\{y_1(\tau,\alpha), \dot{y}(\alpha)\}$, $\tau^V(\alpha) > 0$ and*
     - *If $y_m \leq y_1(\tau,\alpha)$, $\tau^V(\alpha)$ is defined by (24) evaluated in y_m,*
     - *If $y_1(\tau,\alpha) < y_m \leq \dot{y}(\tau,\alpha)$, $\tau^V(\alpha)$ is defined by (26) evaluated in y_m.*
   - *(b) If $y_m > \max\{y_1(\tau,\alpha), \dot{y}(\tau,\alpha)\}$, $\tau^V(\alpha) = 0$.*
2. *If $\alpha \geq \varepsilon$, $\tau^V(\alpha) > 0$ and*
   - *(a) If $y_m \leq y_2(\tau,\alpha)$, $\tau^V(\alpha)$ is defined by (24) evaluated in y_m,*
   - *(b) If $y_2(\tau,\alpha) < y_m \leq y_3(\tau,\alpha)$, $\tau^V(\alpha)$ is defined by (28) evaluated in y_m,*
   - *(c) If $y_m > y_3(\tau,\alpha)$, $\tau^V(\alpha)$ is defined by (29) evaluated in y_m.*

[30] Recall that the threshold $\dot{y}(\alpha)$ is relevant only if $y_1(\tau,\alpha) < \dot{y}(\alpha)$.

This proposition shows how crucial the levels of the marginal returns from pensions and from savings are for the existence of a pension system. Whenever $\alpha < \varepsilon$, it is possible that no pension system emerges (if y_m is quite high) while for $\alpha \geq \varepsilon$, there is always a majority in favour of a pension system. The obvious reason is that in this latter case, the pension system is more attractive than private savings. Low-income agents choose not to save because they cannot afford to and rely exclusively on more profitable public pensions. Intermediate- and high-income agents choose a high tax rate since the return from pensions is more attractive than that from private savings. They choose therefore to report all their income and do not bear any compliance cost. Depending on their income level, they will also eventually save on private markets (the resources they can obtain at old age through the pension benefit are bounded so that the only way for them to obtain more resources is to invest on the less efficient private saving market). To the opposite, when $\alpha < \varepsilon$, for high-income agents, the marginal costs of taxation can be much higher than its marginal benefits, leading them to prefer no pension system at all and to rely on more attractive private savings (even though they always report a positive fraction of income since evasion is costly).

Note that in developing countries, y_m is likely to be small and α is likely to be greater than ε (i.e. the private sector is highly inefficient). Hence, for these countries, making agents vote over the existence of a public pension system would favor the emergence of such system and would push individuals to report a fraction of their income (or even all of it), since the contributory part of the pension benefit guarantees that they will receive some resources at a relatively high return in the second period.

In the next section, whenever $\alpha < \varepsilon$, we will only consider the cases where $\tau^V(\alpha) > 0$ as this seems reasonable to assume that y_m is relatively low and below $y_1(\tau, \alpha)$ or $\dot{y}(\alpha)$.

4.3 Constitutional choice of the level of the Bismarkian factor

In this section, we assume that y_{min} is small and below $y_1(\tau, \alpha)$ or $y_2(\tau, \alpha)$ so that this agent does not save on private markets.[31] The problem of the Rawlsian social planner then consists in solving:

$$\begin{aligned} \max_{0 \leq \alpha \leq 1} V(\tau^V(\alpha), \alpha; y_{min}) &= u(y_{min}(1 - \varphi(\gamma^*(\tau^V(\alpha), \alpha; y_{min}))\tau^V)) \\ &+ \pi u(\frac{\tau^V(\alpha)}{\pi}(\alpha\gamma^*(\tau^V(\alpha), \alpha; y_{min})y_{min} + (1-\alpha)E(\tilde{y}))) \end{aligned}$$

[31] Assuming that $y_{min} = 0$ simplifies some of the expressions below but does not allow to have clearer results.

where $\gamma^*(\tau^V(\alpha), \alpha; y_{min})$ is defined by (16) and $\tau^V(\alpha)$ is defined in Proposition 4. The first order condition of this problem is:

$$\frac{dV(\tau^V(\alpha), \alpha; y_{min})}{d\alpha} = \frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \alpha} + \frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \tau} \frac{d\tau^V(\alpha)}{d\alpha} \leq 0 \qquad (30)$$

where

$$\frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \alpha} = u'(d)\tau^V(\alpha)\big[\gamma^*(\tau^V(\alpha), \alpha; y_{min})y_{min} - E(\tilde{y}) + (1-\alpha)\frac{dE(\tilde{y})}{d\alpha}\big]$$

represents the direct impact of increasing the Bismarkian factor on the utility of the poorest agent. If the degree of compliance was exogenous (i.e. all agents reported a fixed fraction of their income as in the benchmark model), $dE(\tilde{y})/d\alpha = 0$ and $\partial V(\tau^V(\alpha), \alpha; y_{min})/\partial \alpha$ would unambiguously be negative as the tax base of the poorest agent is smaller than the average one ($y_{min} < \bar{y}$). This term accounts for the fact that increasing the degree of contributiveness implies less redistribution which is to the detriment of low-income agents. This should also be the case here with endogenous rate of compliance as $\gamma^*(\tau, \alpha; y)$ is an increasing function of y and thus $\gamma^*(\tau^V, \alpha; y_{min})y_{min} \leq E(\tilde{y})$. This first (redistributive) term therefore pushes toward a lower level of α. However, when the degree of compliance is endogenous, we have an additional term which accounts for the effect of α on the size of the tax base, $E(\tilde{y})$. We have shown (see end of Section 4.1 and Appendix 6.2) that increasing α increases $E(\tilde{y})$ and, thus the uniform benefit agents receive. This pushes toward a higher α. This effect is not present in standard political economy models of pension systems (for instance, in models a la Casamatta et al. 2000a,b) since agents have no other choice than reporting their true income. Therefore, the overall sign of $\partial V(\tau^V(\alpha), \alpha; y_{min})/\partial \alpha$ is undetermined.

The second term in (30) accounts for the indirect impact of α on the utility of the poorest agent, through the majority-voting tax rate. This term was absent from the government problem when there was no cost of evasion, simply because the majority-voting tax rate was independent of α. This term represents, here, how the level of the Bismarkian factor affects the support for the pension system and in turn, the utility of the worst-off agent at the voting equilibrium. The expression $\partial V(\tau^V(\alpha), \alpha; y_{min})/\partial \tau$ is equal to (24) evaluated at $(\tau^V(\alpha), y_{min})$:

$$u'(d)\big[\alpha y_{min}(\gamma^*(\tau^V(\alpha), \alpha; y_{min}) - \frac{\varphi(\gamma^*(\tau^V(\alpha), \alpha; y_{min}))}{\varphi'(\gamma^*(\tau^V(\alpha), \alpha; y_{min}))}) + (1-\alpha)(E(\tilde{y}) + \tau^V(\alpha)\frac{dE(\tilde{y})}{d\tau})\big].$$

As shown in Proposition 4, the sign of the above expression depends on whether $\alpha \gtrless \varepsilon$. Under $\alpha < \varepsilon$, the preferred tax rate is decreasing in income so that the worst-off agent prefers a higher tax rate than the median and, this expression evaluated in $y_{min}(< y_m)$ should therefore be positive. To the opposite, under $\alpha \geq \varepsilon$, it is negative since the preferred tax rate is increasing in y.

Finally, finding the sign of $d\tau^V(\alpha)/d\alpha$ proves rather difficult as one would need to fully differentiate (24), (26), (28) or (29) depending on the equilibrium considered with respect to α and to know the sign of $d^2E(\tilde{y})/d\tau d\alpha$.

To sum up, the above analysis shows that there is a priori no reason to believe that the pension system chosen at the constitutional level should be fully contributive ($\alpha^R = 1$) or fully redistributive ($\alpha^R = 0$). It seems reasonable that, with compliance costs, $0 < \alpha^R < 1$ depending on the size of the different effects described above. At least, it is possible to show that $\alpha^R > 0$ so that the pension system will never be fully Beveridgian.[32]

In order to have some additional analytical results, we study here an extreme case where we assume that all agents report a constant fraction of income, either $\gamma^*(\tau, \alpha; y) = \alpha/\varepsilon$ or 1 for all y. In other words, we assume that the thresholds $y_1(\tau, \alpha)$ (when $\alpha < \varepsilon$) or $y_2(\tau, \alpha)$ (when $\alpha \geq \varepsilon$) are not relevant. In the former case, this would happen if y_{min} is sufficiently big while in the latter case, this would happen if the range of incomes in the population is large (see Section 4.1).

Let us first study the case where $\alpha < \varepsilon$. All agents have $\gamma^*(\tau, \alpha; y) = \alpha/\varepsilon$ and thus, the tax base $E(\tilde{y}) = \alpha\bar{y}/\varepsilon$ is independent of τ. Using condition (26), we find that the agents' preferred tax rate is such that

$$\begin{aligned} \tau^*(\alpha) &= 1 \text{ if } y < \frac{2(1-\alpha)\alpha\bar{y}}{\varepsilon^2 - \alpha^2} \\ &= 0 \text{ otherwise.} \end{aligned}$$

Assuming that y_m is below $\dot{y}(\alpha) = 2(1-\alpha)\alpha\bar{y}/(\varepsilon^2 - \alpha^2)$, the majority voting tax rate should be equal to unity.[33] Note that since agents do not report all their income, first-period consumption is strictly positive: $c = y(1 - \alpha/\varepsilon)$. Replacing for $\tau^V = 1$ and for $\gamma^*(\tau^V, \alpha; y_{min}) = \alpha/\varepsilon$ in (30),

[32] To see this, suppose instead that $\alpha = 0$. In such a case, $\gamma^*(\tau, \alpha; y) = 0\,\forall y$ so that regardless of the tax rate, the government does not collect any revenue and is not able to provide any pension benefit. There is thus no redistribution. Introducing instead $\alpha > 0$ would be welfare improving as in such a case, it would enable redistribution and agents with $y = y_{min}$ would necessarily be better-off than when $\alpha = 0$.

[33] For $\tau^V = 0$, no pension system emerges and agents are indifferent as to the level of the Bismarkian factor (expression (30) reduces to 0).

we obtain after some simplifications that

$$\begin{aligned}\frac{dV(\tau^V,\alpha;y_{min})}{d\alpha} &= \frac{\partial V(\tau^V(\alpha),\alpha;y_{min})}{\partial\alpha} \\ &= \frac{u'(d)}{\varepsilon}[\alpha y_{min}+\bar{y}(1-2\alpha)]\leq 0\end{aligned} \tag{31}$$

and thus, that the optimal level of the Bismarkian factor should be interior and equal to

$$\alpha^R=\min\{\frac{\bar{y}}{2\bar{y}-y_{min}};\varepsilon\},$$

with $\bar{y}/(2\bar{y}-y_{min})<1$. The intuition for this result is that whenever $\alpha^R<\varepsilon$, the tax base is equal to $E(\tilde{y})=\alpha\bar{y}/\varepsilon$ so that one needs the Bismarkian factor to be strictly positive so as to ensure that agents effectively comply. There is thus a trade-off between making the system more beneficial to the worst-off agent (through a lower level of α) and making the system more profitable (through a higher α which increases the amount reported to fiscal authorities and thus the flat part of the pension benefit).

Let us then turn to the case where $\alpha\geq\varepsilon$. In that situation, all agents report all their income and $E(\tilde{y})=\bar{y}$ is now independent of α. Using either (28) or (29) depending on the relative position of y_m with respect to $y_2(\tau,\alpha)$ and $y_3(\tau,\alpha)$, we find that the median voter always prefers the maximum tax rate possible and that it is independent of the level of α.[34] This result is directly related to the fact that $\alpha\geq\varepsilon$ and that the tax base does not depend on τ through the level of compliance. Replacing for τ^V and $\gamma^*(\tau,\alpha;y_{min})$ in (30), we obtain after some rearrangements that

$$\frac{dV(\tau^V(\alpha),\alpha;y_{min})}{d\alpha}=u'(d)\tau^V[y_{min}-\bar{y}]<0$$

so that α^R should be minimum. Indeed, contrary to the previous situation, the rate of compliance does not depend on the value of α^R: agents fully comply simply because the rate of return from the pension system is greater than that from private savings and because they are not liquidity constrained. The government can thus simply set $\alpha^R=\varepsilon$ to maximize the utility of the worst-off without creating any distortion on the fiscal resources collected. This result is however specific to this case as in our general model, we have shown that the tax base $E(\tilde{y})$ increases in α.

Note finally that in these two latter cases, the majority-voting tax rate is independent of α. The choice of α therefore does not affect the political support for the pension system, which allows us to have clear cut results.

[34] The tax rate is likely to be smaller than one as otherwise, first-period consumption would be null.

5 Conclusion

In standard models of income taxation, tax authorities observe the individuals' wage rate or at least the amount of their earnings. If tax evasion is possible, then an audit technology allows them to recoup a large chunk of tax revenue. Common knowledge of earnings and effective tax enforcement are in general characteristics of advanced economies. In many less advanced economies, informality and self-employment make it more difficult to enforce a workable tax system, which explains the low level of tax revenue observed there, particularly when the tax base is individual income. In this paper, we show that even under those circumstances public resources mobilization is possible and politically sustainable if taxes are at least partially related to benefits and if the government supplies services for which there are no good substitutes in private markets. This is precisely the case of retirement saving whose market returns are often not attractive because of the absence of annuity markets and of the high loading costs of the financial institutions.

In this paper, we study the political sustainability of a pension system that has three main features: it is financed by a payroll tax whose base is not observable; it provides benefits that are partially contributive and the marginal return of those social contributions can be higher than the financial rate of interest. It appears that in the absence of compliance costs, individuals are indifferent to the tax rate chosen as they can perfectly adapt their level of compliance. The public pension system is found to be at least partially contributory in order to increase compliance and thus to increase the tax base. When compliance costs are introduced, perfect substitutability between compliance and taxation breaks down. Depending on the relative returns from public pensions and private savings as well as on the elasticity of compliance to income, we obtain that the preferred tax rate should be increasing or decreasing in income. The majority voting tax rate is more likely to be positive when the median income is low and when the return from public pensions dominates that of private savings. The share of the contributive pension pillar will now be chosen so as to take into account increased political support, increased direct redistribution toward the worst-off, and increased tax base through higher compliance. While reasonable, this latter reason is however new in the political economy of pension systems literature.

The policy implication of this paper is rather obvious. If one wants to design a sustainable tax system in developing countries it is important to at least partially link individual contributions to individual benefits.

References

[1] Alm J., G. H. McClelland and W. D. Schulze, 1999, Changing the social norm of tax compliance by voting, *Kyklos*, 52(2), 141-171.

[2] Allingham M. G. and A, Sandmo, 1972, Income tax evasion: a theoretical analysis, *Journal of Public Economics*, 1, 323-338.

[3] Bergstrom, T., L. Blume, and H. Varian, 1986, On the Private Provision of Public Goods, *Journal of Public Economics*, 29, 125-149.

[4] Borck R., 2004, Stricter enforcement may increase tax evasion, *European Journal of Political Economy*, 20, 725-737.

[5] Borck R., 2007, Voting, inequality and redistribution, *Journal of Economic Surveys*, 21(1), 90-109.

[6] Borck R., 2009, Voting on redistribution with tax evasion, *Social Choice and Welfare,* 32, 439-454.

[7] Casamatta G., H. Cremer, and P. Pestieau, 2000a, Political sustainability and the design of social insurance, *Journal of Public Economics*, 75, 341-364.

[8] Casamatta G., H. Cremer, and P. Pestieau, 2000b, The political economy of social security, *The Scandinavian Journal of Economics*, 102(3), 503-522.

[9] Castro, L., and C. Scartascini, 2013, Tax compliance and enforcement in the Pampas. Evidence from a field experiment. IDB Working Paper No. 472. Washington, DC: Inter-American Development.

[10] Gans, J. S. and M. Smart, 1996, Majority-voting with single-crossing preferences, *Journal of Public Economics*, 59, 219-237.

[11] Gordon R. and W. Li, 2011, Tax structure in developing countries: many puzzles and a possible explanation, *Journal of Public Economics*, 93(7-8), 855-866.

[12] Kopczuk, W., 2001, Redistribution when avoidance behaviour is heterogenous, *Journal of Public Economics*, 81, 51-71.

[13] Martinez-Vasquez, J. and J. Alm, (eds) (2003), *Public Finance in Developing countries and transitional countries: essays in honour of Richard Bird*, Cheltenham,UK; Edward Elgar Publishers.

[14] Moore, M. and G. Mascagni, 2014, Tax revenue mobilisation in developing countries: issues and challenges, European Parliament Policy Department Studies Paper.

[15] Slemrod, J., 1998, On voluntary compliance, voluntary taxes, and social capital, *National Tax Journal*, 51, 485-494.

[16] Traxler, C., 2012, Majority voting and the welfare implications of tax avoidance, *Journal of Public Economics*, 96(1), 1-9.

6 Appendix

6.1 Variation of $\gamma^*(\tau, \alpha; y)$ with α without evasion costs.

Fully differentiating (5) with respect to α and rearranging terms, we obtain that

$$\frac{d\gamma^*(\tau, \alpha; y)}{d\alpha} = -\frac{u'(d) + (\alpha\tau/\pi)u''(d)[\gamma y - E(\tilde{y}) + \frac{dE(\tilde{y})}{d\alpha}(1-\alpha)]}{SOC_\gamma}$$

with $SOC_\gamma < 0$. Since the sign of the expression inside brackets is ambiguous, $\frac{d\gamma^*(\tau,\alpha;y)}{d\alpha}$ may be positive or negative depending on y. Equivalently, since $d = P(\tau, \alpha; y)$ when $\alpha > \varepsilon$, one can rewrite the above expression as follows:

$$\frac{d\gamma^*(\tau, \alpha; y)}{d\alpha} = -\frac{u'(d)}{SOC_\gamma}[1 - R_r(d)\frac{\partial P(\tau, \alpha; y)}{\partial \alpha}\frac{\alpha}{P(\tau, \alpha; y)}]$$

with $\partial P(\tau, \alpha; y)/\partial\alpha = (\tau/\pi)[\gamma y - E(\tilde{y}) + \frac{dE(\tilde{y})}{d\alpha}(1-\alpha)] \lesseqgtr 0$. Evaluating $d\gamma^*(\tau, \alpha; y)/d\alpha$ at $\alpha = 0$, it is straightforward to see that it is positive. Evaluating it at $\alpha = 1$, we obtain that:

$$\begin{aligned}\frac{d\gamma^*(\tau, 1; y)}{d\alpha} &= -\frac{u'(d) + (\tau/\pi)u''(d)[\gamma(\tau, 1; y)y - E(\tilde{y})]}{SOC_\gamma} \\ &= -\frac{u'(d)}{SOC_\gamma}[1 - R_r(d)\frac{\tau/\pi(\gamma(\tau, 1; y)y - E(\tilde{y}))}{d}] > 0\end{aligned}$$

with $d = \tau\gamma(\tau, 1; y)y/\pi$. This is independent of whether $\gamma(\tau, 1; y)y - E(\tilde{y}) \gtrless 0$.

Let us also show that one always have $dE(\tilde{y})/d\alpha > 0$ and $d\gamma^*(\tau, \alpha; y)/d\alpha \gtrless 0$. To do so, we apply the implicit function theorem on FOC (9) which yields:

$$\begin{aligned}\frac{\partial^2 U}{\partial t \partial \alpha} &= u'(d) + \frac{\alpha}{\pi}[t^*(\alpha; y)y - E(t^*(\alpha; y)y) + (1-\alpha)\frac{dE(t^*(\alpha; y)y)}{d\alpha}] \\ &= u'(d)\big[1 - R_r(d)\frac{\frac{1}{\pi}(\alpha(t^*(\alpha; y)y - E(t^*(\alpha; y)y)) + (1-\alpha)\alpha\frac{dE(t^*(\alpha;y)y)}{d\alpha}}{d}\big]\end{aligned}$$

with $d = \frac{1}{\pi}(\alpha t^*(\alpha; y)y + (1-\alpha)E(t^*(\alpha; y)y))$. We show this result by contradiction. Assume that $dE(t^*(\alpha; y)y)/d\alpha < 0$. If $t^*(\alpha; y)y > E(t^*(\alpha; y)y)$, the fraction inside brackets is smaller than 1, which yields $\partial^2 U/\partial t\partial\alpha > 0$. If $t^*(\alpha; y)y < E(t^*(\alpha; y)y)$, the expression inside brackets is positive and $\partial^2 U/\partial t\partial\alpha > 0$. Thus $dt^*(\alpha; y)/d\alpha > 0 \forall y$. This leads to $dE(t^*(\alpha; y)y)/d\alpha > 0$, a contradiction. Hence, the only possible solution is $dE(t^*(\alpha; y)y)/d\alpha \geq 0$ and $dt^*(\alpha; y)/d\alpha \gtrless 0$ depending on the size and the sign of the fraction inside brackets. This implies that $dE(\tilde{y})/d\alpha \geq 0$ and $d\gamma^*(\tau, \alpha; y)/d\alpha \gtrless 0$.

6.2 Variation of $\gamma^*(\tau,\alpha;y)$ with τ and α with evasion costs.

When $y \leq y_1(\tau,\alpha)$ or $y_2(\tau,\alpha)$, $s^*(\tau,\alpha;y)=0$ and $\gamma^*(\tau,\alpha;y)>0$ satisfies (16). Differentiating this condition with respect to y, we obtain

$$\frac{\partial^2 U}{\partial\gamma\partial\tau} = u''(c)y\varphi(\gamma)\varphi'(\gamma) + \frac{\alpha^2}{\pi}\gamma y u''(d) + \frac{\alpha(1-\alpha)}{\pi}u''(d)[E(\tilde{y}) + \tau\frac{dE(\tilde{y})}{d\tau}].$$

Assuming that we are on the increasing part of the Laffer curve, the last expression inside brackets is positive, so that the above expression is negative. Using the implicit function theorem, we conclude that $d\gamma^*(\tau,\alpha;y)/d\tau < 0$.

Let us now make some comparative statics on $\gamma^*(\tau,\alpha;y)$ with respect to α. Differentiating (16) with respect to α, we obtain

$$\begin{aligned}\frac{\partial^2 U}{\partial\gamma\partial\alpha} &= u'(d) + \alpha u''(d)\frac{\tau}{\pi}[\gamma^*(\tau,\alpha;y)y - E(\tilde{y}) + (1-\alpha)\frac{dE(\tilde{y})}{d\alpha}] \\ &= u'(d)[1 - R_r(d)\frac{dP(\tau,\alpha;y)}{d\alpha}\frac{\alpha}{P(\tau,\alpha;y)}]\end{aligned}$$

with $d = P(\tau,\alpha;y)$ when $y \leq y_1(\tau,\alpha)$ or $y_2(\tau,\alpha)$. Note also that

$$\frac{dP(\tau,\alpha;y)}{d\alpha}\frac{\alpha}{P(\tau,\alpha;y)} = \frac{\frac{\tau}{\pi}\alpha(\gamma^*(\tau,\alpha;y)y - E(\tilde{y}) + (1-\alpha)dE(\tilde{y})/d\alpha)}{\frac{\tau}{\pi}(\alpha\gamma^*(\tau,\alpha;y)y + (1-\alpha)E(\tilde{y}))}.$$

Assume first that $dE(\tilde{y})/d\alpha < 0$. In such a case, if $\gamma^*(\tau,\alpha;y)y > E(\tilde{y})$ for some y, the above expression is smaller than 1 and $\frac{\partial^2 U}{\partial\gamma\partial\alpha} > 0$. If $\gamma^*(\tau,\alpha;y)y \leq E(\tilde{y})$ for some other y, the above expression is negative and $\frac{\partial^2 U}{\partial\gamma\partial\alpha} > 0$. This implies that $d\gamma^*(\tau,\alpha;y)/d\alpha > 0 \forall y$ and thus that $dE(\tilde{y})/d\alpha > 0$, a contradiction. Hence the only possible solution is such that $dE(\tilde{y})/d\alpha > 0$ and using the implicit function theorem, we find that $d\gamma^*(\tau,\alpha;y)/d\alpha \gtrless 0$ depending on whether $\gamma^*(\tau,\alpha;y)y \gtrless E(\tilde{y})$.

6.3 Single crossing condition with evasion costs

Let us define the intermediate indirect utility function of an agent with income y as

$$V(\tau,\alpha;y) = u(y(1-\varphi(\gamma^*(\tau,\alpha;y))\tau) - s^*(\tau,\alpha;y)) + \pi u(\frac{\varepsilon s^*(\tau,\alpha;y)}{\pi} + \frac{\tau}{\pi}\alpha\gamma^*(\tau,\alpha;y)y + b)$$

where $b(\tau,\alpha) = \frac{\tau}{\pi}(1-\alpha)E(\tilde{y})$. The marginal rate of substitution between τ and $b(\tau,\alpha)$ has the following general form

$$MRS_{\tau,b} = \frac{\partial V/\partial\tau}{\partial V/\partial b} = -\frac{-u'(c)\varphi(\gamma^*(\tau,\alpha;y))y + \alpha\gamma^*(\tau,\alpha;y)yu'(d)}{\pi u'(d)} \ \forall y.$$

When $\alpha < \varepsilon$, for agents $y \leq y_1(\tau, \alpha)$, it simplifies to

$$MRS_{\tau,b}^{y \leq y_1} = -\frac{\alpha}{2\pi} y \frac{\gamma^*(\tau, \alpha; y)^2 - 1}{\gamma^*(\tau, \alpha; y)} > 0$$

where we replaced for (16) and the functional form of $\varphi(\gamma)$. Therefore,

$$\frac{\partial MRS_{\tau,b}^{y \leq y_1}}{\partial y} = -\frac{\alpha}{2\pi\gamma^*(\tau, \alpha; y)} \left[\gamma^*(\tau, \alpha; y)^2 - 1 + (\gamma^*(\tau, \alpha; y)^2 + 1)\nu_{\gamma^*,y}\right] \tag{32}$$

where $\nu_{\gamma^*,y} = \frac{d\gamma^*(\tau,\alpha;y)}{dy} \frac{y}{\gamma^*(\tau,\alpha;y)} > 0$. Hence, a sufficient condition for the $MRS_{\tau,b}$ of every agent with income $y \leq y_1(\tau, \alpha)$ to be monotonically increasing (resp. decreasing) in y is that $\nu_{\gamma^*,y}$ is small (resp. big) and below (resp. above) $(1 - \gamma^*(\tau, \alpha; y)^2)/(1 + \gamma^*(\tau, \alpha; y)^2)$.

For agents with $y > y_1(\tau, \alpha)$, $\gamma^*(\tau, \alpha; y) = \alpha/\epsilon$ and $s^*(\tau, \alpha; y) \geq 0$ satisfies $u'(c) = \varepsilon u'(d)$. In that case,

$$MRS_{\tau,b}^{y > y_1} = -y \frac{\alpha^2 - \varepsilon^2}{2\pi\varepsilon} > 0$$

and it monotonically increases in y.

Noticing that at $y_1(\tau, \alpha)$, $MRS^{y \leq y_1} = MRS^{y > y_1}$, marginal rates of substitution are then monotonically increasing over the whole interval $[y_{min}, y_{max}]$, under the condition that $\nu_{\gamma^*,y}$ is small.

Let us now derive the $MRS_{\tau,b}$ when $\alpha \geq \varepsilon$. For any agent with $y \leq y_2(\tau, \alpha)$,

$$MRS_{\tau,b}^{y \leq y_2} = -\frac{\alpha}{2\pi} y \frac{\gamma^*(\tau, \alpha; y)^2 - 1}{\gamma^*(\tau, \alpha; y)} > 0$$

so that, as above, it is decreasing in y for $\nu_{\gamma^*,y}$ above $(1 - \gamma^*(\tau, \alpha; y)^2)/(1 + \gamma^*(\tau, \alpha; y)^2)$.

For agents with $y_2 < y \leq y_3(\tau, \alpha)$, $\gamma^*(\tau, \alpha) = 1$ and $\alpha u'(d) > u'(c) > \varepsilon u'(d)$ so that

$$MRS_{\tau,b}^{y_2 < y \leq y_3(\tau,\alpha)} = -\frac{y}{\pi u'(d)} [\alpha u'(d) - u'(c)] < 0.$$

Differentiating this expression with respect to y, we obtain after some rearrangements that

$$\frac{\partial MRS_{\tau,b}}{\partial y} = -\frac{1}{\pi u'(d)} [\alpha u'(d)(1 - R_r(d)\frac{\tau\alpha y/\pi}{d}) - u'(c)(1 - R_r(c))] + \frac{u''(d)}{\pi u'(d)^2} \frac{\tau}{\pi} \alpha y(\alpha u'(d) - u'(c)).$$

Under A1, this is always negative.

Finally for agents with $y > y_3(\tau, \alpha)$, $\gamma^*(\tau, \alpha; y) = 1$ and $u'(c) = \varepsilon u'(d)$ so that $MRS_{\tau,b}$ simplifies to

$$MRS_{\tau,b}^{y > y_3(\tau,\alpha)} = -\frac{y(\varepsilon - \alpha)}{\pi} < 0$$

which is monotonically decreasing in y.

Using the same reasoning as in the previous case, we have that at $y_2(\tau,\alpha)$, $MRS_{\tau,b}^{y\leq y_2} = MRS_{\tau,b}^{y_2<y\leq y_3(\tau,\alpha)} = 0$ and, at $y_3(\tau,\alpha)$, $MRS_{\tau,b}^{y_2<y\leq y_3(\tau,\alpha)} = MRS_{\tau,b}^{y>y_3(\tau,\alpha)}$. Hence, whenever $\alpha \geq \varepsilon$, if $\nu_{\gamma^*,y}$ is greater than $(1-\gamma^*(\tau,\alpha;y)^2)/(1+\gamma^*(\tau,\alpha;y)^2)$, all the $MRS_{\tau,b}$ are monotonically decreasing in y over the whole interval $[y_{min}, y_{max}]$.

We can therefore use the single crossing condition established by Gans and Smart (1996) and the median voter theorem. Preferences are single crossing so that there exists a unique Condorcet winner which corresponds to the preferred tax rate of the agent with median income.

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