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Abstract

Growth forecasts are the foundation of fiscal planning. Risk management in fiscal planning therefore requires an appreciation of the uncertainty associated with the underlying growth forecasts. This paper estimates such uncertainty by examining medium-term government and private-sector forecasts for Québec as well as private sector forecasts for Canada. It shows the distribution of forecast errors for both real and potential output forecasts by forecast horizon. It also examines a variety of decompositions to better understand key sources of forecast uncertainty.

The results indicate that forecast uncertainty increases linearly with forecast horizon. Five-year ahead forecast errors for real output of ±5% are common, while those for potential output are roughly half the size, suggesting that the cumulative impact of cyclical factors play an important role. Of the two forecasts for Québec, the private sector forecast showed larger mean forecast errors while the government forecast had somewhat higher mean-squared forecast errors. The latter also tended to have offsetting mean errors in its forecasts of output gaps and trend productivity growth. Productivity growth together with labour force participation rates were a key contributor to most forecast errors while population growth tended to play a significant secondary role at longer horizons and variations in employments were generally minor.

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Executive Summary

This paper estimates the degree of uncertainty surrounding growth forecasts for the Québec economy as an aid to risk management in fiscal planning.

Formal estimates of the growth uncertainty facing fiscal planners are growing in popularity and have recently been institutionalized in some countries. While growth forecasts have been extensively studied, the existing empirical literature on forecast performance and uncertainty has several deficiencies from the perspective of assessing fiscal risks.

- It tends to examines forecast horizons that are too short relative to a fiscal planning horizon.
- It focuses on choosing the best forecasting model rather than assessing forecast uncertainty.
- The object of interest is quarterly or annual growth rather than the cumulative forecast error.
- Systematic analysis of forecasts for the Québec economy has been lacking.

Despite these drawbacks, earlier studies across a variety of economies are broadly in agreement on a few points

- Little if any of the future variability of real output growth is forecastable.
- Confidence intervals for real output growth two years ahead are broad and resemble unconditional distributions of growth rates.
- Government fiscal forecasts are most typically over-optimistic.

This paper examines medium-term forecasts for real GDP from the Ministère des Finances et de l'Économie du Québec (MFQ) as well as the Québec and national forecasts produced by the Conference Board of Canada (CBC), using their historical forecasts to gauge forecast uncertainty. Several decompositions of the forecast errors were also considered to identify key sources of forecast uncertainty. The available data series are too short relative to the forecast horizons considered to allow for reliable formal statistical inference; we therefore rely instead on descriptive and graphical analysis.

Forecast uncertainty for GDP tended to increase linearly with the forecast horizon, consistent with a standard error of roughly 5% at a 5-year horizon. Estimates for potential output were roughly half that amount. Typical distributional assumptions imply that forecast errors will commonly fall outside the range of ± 1 RMSFE and occasionally outside ±2 RMSFE.[1] For risk management purposes, this would imply planning for a range of real output growth outcomes that vary by 10%-20% at the five-year horizon, or potential output growth that varies by half those amounts.

[1] For example, a standard normal distribution implies that errors will be more than 1 standard deviation from the mean 31.7% of the time and more than 2 standard deviations away 4.6% of the time.

The CBC output growth forecasts for Québec tended to be excessively optimistic as were both forecasts for Québec potential output growth. The MFQ forecasts were influenced by overly optimistic forecasts of trend labour productivity growth which tended to be offset by mean errors in output gap forecasts. Errors in forecasting labour productivity growth were important contributors to overall forecast errors for most of the forecasts examined, although their role was often intertwined with that of errors in forecasting labour force participation rates. Population growth forecast errors also contributed at longer horizons, although this was less of a factor for the MFQ forecasts. Output gap forecasts played a key role in accounting for overall growth forecast errors at the shortest horizons but their importance quickly faded for longer horizons.

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Introduction

Growth forecasts form the foundation for fiscal planning. Risk management in fiscal planning therefore requires an appreciation of the uncertainty associated with the underlying growth forecasts. Formal assessments of such uncertainty have recently been institutionalized in some countries. In the U.K., the Office for Budget Responsibility (OBR) was formed in May 2010 to make independent assessments of the public finances and the economy. Its duties require it to assess "whether the Government's fiscal policy is consistent with a greater than fifty per cent chance" of achieving its stated target.[2] In Canada, the position of Parliamentary Budget Officer was established in 2008 to provide independent analysis to Parliament on the state of the nation's finances, the government's estimates and trends in the Canadian economy. Assessments of growth uncertainty have been a standard feature of these semi-annual reports.[3]

This paper estimates the degree of uncertainty surrounding growth forecasts for the Québec economy. As explained below, our estimates use a methodology similar to the studies cited above, which examine growth forecasts and their forecast errors at various horizons. While straightforward, we are not aware of previously published studies that have used this approach for the economy of Québec. We decompose the forecast errors into various economic components to better understand the sources of growth uncertainty.

We also compare results for Québec to those for Canada, where somewhat similar analyses of historical forecast errors already exist.[4] Our primary interest is in understanding the uncertainty surrounding projections of overall economic growth, as measured by real GDP. However, it may be argued that the growth of potential GDP would be a more appropriate yardstick than that of real GDP if fiscal planners are primarily concerned with the control of "structural" deficits. Structural deficits adjust government revenues and expenditures for the transitory effects of the business cycle. By focusing on structural deficits rather than actual deficits, a fiscal planning framework tries to avoid forcing governments to adopt pro-cyclical fiscal policy.[5]

Structural deficits are commonly estimated by both governments and international agencies (including the IMF, the OECD and the EC.) One important source of divergence in these estimates stems from differences in their estimates of potential output. This paper uses a

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[2] H.M. Treasury, *Charter for Budget Responsibliity*, April 2011, p. 12.

[3] For a recent example, see Figure 2-10 in Parliamentary Budget Officer (2012).

[4] For example, Barnett et al. (2009) examine shocks and revisions to the Bank of Canada's quarterly forecasts over the 1993-2005 period, with particular attention to the behaviour of forecasts for real GDP, potential output and inflation. PEAP-CIRANO (2005), O'Neill (2005) and Mühleisen et al. (2005) have examined the Canadian Ministry of Finance's Survey of Private Forecasters for evidence of bias, focusing on one- and two-year ahead forecasts.

[5] Conceptually, growth forecasts may often be constructed as the sum of two components: the growth rate of potential output plus a term that reflects the speed at which deviations from potential naturally dissipate. Of course, as the planning horizon grows longer, the business cycle becomes relatively small when compared to the forecast growth in real GDP or potential GDP, so the distinction between these measures becomes less important.

transparent and widely-used method for the estimation of potential output, as is discussed below.

The next section of the paper provides a selective review of the macroeconomic literature on output growth uncertainty. The data are described thereafter, focusing on three collections of output growth forecasts. This is followed by an explanation of the methodology used to measure the degree of forecast uncertainty, to measure potential output and to decompose forecast uncertainty into its components. The penultimate section presents the empirical results, while the final section discusses and summarizes the results. A statistical annex provides a more comprehensive breakdown of the statistical properties of the forecast errors.

Literature Review

There is a long-standing tradition in macroeconomic research of examining output growth uncertainty. To be precise, the relevant uncertainty is the probability distribution of future real growth *conditional* on available information. This is commonly assessed by examining the historical performance of growth forecasts from formal models or those made by institutions or professional forecasters. Many papers have also attempted to compare such forecasts; important recent examples include Chauvet and Potter (2012), Edge, Kiley and Laforte (2010). As is typical in this literature, both consider the problem of forecasting US real GDP growth over relatively short horizons (from 1-2Q ahead in the former to 1-8Q in the latter) and with an emphasis on the comparing the root-mean-squared forecast error (RMSFE) across various forecasting models and forecasts. Edge and Gürkanyak (2010) summarize the results as follows

> *... the DSGE model in an absolute sense did a very poor job of forecasting. The Greenbook and time series model forecasts similarly did not capture much of the realized changes in GDP growth and inflation ... There is a moderate amount of nowcasting ability and then almost nothing beginning with one quarter ahead forecasts. Thus, the forecast comparison is not one of one good forecast relative to another; all three methods of forecasting are poor and combining them does not lead to much improvement either.*[6]

While they provide few statistical measures of forecast uncertainty aside from RMSFE, they provide scatterplots of forecast versus measured output growth (see Figure 1) which graphically illustrates the poor predictive performance of the model they find has the best forecast performance. Chauvet and Potter (2012) find similar results (although they stress that this reflects quite different behaviour during "normal" periods and recessions.) Similar conclusions are also found from other measures of forecast uncertainty; for example, van Norden and Galbraith (2011) and Lahiri and Wang (2012) consider probability forecasts for GDP contractions and find little evidence of predictive power for downturns except at the shortest horizons.

[6] Edge and Gürkanyak (2010), p. 2-3.

While such results are perhaps the most common strand in the macroeconomic literature on forecast uncertainty, they fall short of providing useful guidance for Québec fiscal policy in several respects. First and foremost, they examine forecasts of US growth rather than those for the Québec economy. In addition, the forecasts horizons that they consider are too short relative to those typically used for fiscal management. Related to this is the fact that they consider forecasts of quarterly or annual real output *growth*; for fiscal planning it would be more useful to consider forecasts of the future *level* of GDP (or equivalently, of cumulative growth over the forecast horizon.) Finally, as noted above, the majority of these studies consider only simple measures of overall forecast uncertainty (such as root-mean-squared forecast errors, or RMSFEs) rather than trying to characterize the degree of risk for different levels of certainty (i.e. the complete probability density of forecast errors.)

Other, smaller strands of the literature attempt to remedy some of these shortcomings. Several institutions have started to produce regular density forecasts for output growth, including the Bank of England, the FOMC of the US Federal Reserve Board, the UK Office of Budget Responsibility and Canada's Office of the Parliamentary Budget Officer. Examples of the "fan charts" summarizing the forecast uncertainty for real GDP growth are shown in Figures 2 through 5. They show forecasts produced at different dates (from December 2006 through mid-2010) for different countries and using different methods (those for the FOMC are based on simulations of a calibrated large-scale macroeconomic model while those for the UK OBR are based on the historical dispersion of forecast errors from UK Treasury forecasts.) Despite this, they show broadly similar features. In all cases, the fan widens rapidly from the most recently available observations to reach a 90% confidence interval roughly 6% wide at a 2-year forecast horizon. That interval approximates the unconditional 90% confidence interval for real GDP growth, implying that these density forecasts purport to convey little or no information about potential variations in real growth at a two year horizon. To that extent, their results are quite consistent with the US RMSFE results discussed above.[7]

A third strand of the literature has examined the reliability of government fiscal forecasts. Much of this work has focussed on forecasts of fiscal policy variables (deficits, structural deficits and debt levels) rather than the underlying economic growth forecasts, and the focus is typically on the detection of forecast bias rather than assessing the overall degree of forecast uncertainty.[8] Kliessen and Thornton (2012), for example, provide a recent analysis of forecasts from the US Congressional Budget Office in the US. However, relatively more work has been done in recent

[7] Due to the short of span of density forecasts available for analysis, very little formal analysis of their forecast performance is available. The oldest and most-studied are those of the Bank of England: Galbraith and van Norden (2012) find they have no significant forecasting ability at horizons of 1Q or beyond.

[8] An exception is Busby and Robson (2013), which examines the performance of Canadian governments in forecasting their year-ahead spending and revenues.

years on EU and Euro-zone member countries.[9] Cimadomo (2011) provides a comprehensive survey of this literature. Both find widespread and significant evidence of forecast bias, with forecasts consistently tending to be overly optimistic (e.g. underestimating deficits, debts and overestimating surpluses and economic growth.) Since 1994, the Canadian Department of Finance has benchmarked their economic projections against those of a panel of private sector forecasters[10] (see Table 1.) Evaluations of its performance include PEAP-CIRANO (2005), O'Neill (2005) and Mühleisen et al. (2005), who also find evidence of forecast bias in both government and private sector forecasts (although the Canadian government forecasts tended to be consistently too pessimistic.) These reports also provided estimates of the standard deviation of real GDP forecast errors, which are largely consistent with the size of forecast errors documented in the US literature.

Data

Our data consist of collections of economic forecasts as well as the corresponding Statistics Canada estimates of the economic outcomes. Of various sources of forecasts we considered, few provided consistent and regular long-term forecasts for the Québec economy of the key variables examined below.[11] Of those which did, the forecasts of the Conference Board of Canada offered the longest span of forecast vintages. We also examine a smaller set of longer-term forecasts obtained from the Ministère des Finances et de l'Économie du Québec. All of these forecasts were prepared annually and consist of forecasts for annual values of several series.

Forecasts from the Ministère des Finances et de l'Économie du Québec

These forecasts (henceforth known as the MFQ forecasts) were prepared by the Ministère des Finances et de l'Économie as part of its annual September long-term forecast exercise.[12] Our sample covers the fourteen consecutive forecasts 1994-2007.[13] In addition to some historical values, the data included forecasts for up to seven years ahead for seven variables

- Total Population
- Population age 15 or over

[9] In addition to the very recent interest in these issues as a result of the fiscal crisis in the Euro zone, there has been interest in these issues for some time in Europe due to the fiscal rules for participating member states in the Euro zone's Stability and Growth Pact.

[10] Published at http://www.fin.gc.ca/pub/psf-psp/index-eng.asp.

[11] As explained in the next section, the variables were chosen to allow a decomposition of potential output into its commonly-analysed components.

[12] The author would like to thank the Ministère des Finances et de l'Économie du Québec for making the forecasts available for analysis.

[13] All but the 2007 forecast (which was made in November) were made in September of the corresponding year.

- Participation rate
  (The fraction of the Population age 15 or over in the Labour Force)
- Unemployment rate
  (The fraction of the Labour Force unemployed)
- Total Employment
- GDP
- Real GDP

Forecasts from the Conference Board of Canada for Québec

These forecasts (henceforth known as the CBC QC forecasts) were prepared by the Conference Board of Canada as part of its annual *Provincial Outlook Long-Term Economic Forecast*, published each spring. Forecasts from seventeen different years were collected, covering the forecasts published from 1992 to 2008. In addition to some historical values, the data included the full published range of forecasts, typically covering 15 to 25 years of projections for five variables[14]

- Population age 15 or over
- Participation rate
  (The fraction of the Population age 15 or over in the Labour Force)
- Growth rate of the Labour Force
- Growth rate of Total Employment
- Growth rate of Real GDP

Forecasts from the Conference Board of Canada for Canada

These forecasts (henceforth known as the CBC CA forecasts) were prepared by the Conference Board of Canada as part of its annual *Canadian Outlook Long-Term Economic Forecast*, published each spring. These forecasts covered the same years, variables and forecast horizons as the provincial forecasts described above.

Although some forecasters now forecast potential output for Québec, such forecasts have begun too recently to allow for a direct evaluation of their longer-term performance. As discussed in the next section of the paper, we instead use alternative methods to impute values of potential output and examine their implied uncertainty.

Prior to analysis, all three forecast collections were transformed to allow for a direct comparison of five series of interest.

- Population age 15 or over (N)
  This series was available in all three forecast collections. It was simply converted to logs.
- Participation Rate (P)
  This series was available in all three forecast collections. It was simply converted to logs.

[14] The author would like to thank the Conference Board of Canada for sharing their forecast data.

- Employment Rate (M)
  For the MFQ forecasts, this series was constructed as the complement of the unemployment rate (since the two must sum to 100%.)
  For the CBC forecasts, growth rate forecasts for the labour force and employment only permit calculation of an index of the employment rate.
  In both cases, the calculated rate was converted to logs.
- Real GDP (Y)
  This series was available in the MFQ forecast collection and was simply converted to logs.
  For the CBC forecasts, growth rate forecasts for real GDP were cumulated to form an index of forecast real GDP, which was then converted to logs.
- Productivity (Q)
  Implied labour productivity was simply calculated as the difference between the log real GDP series (y) and the log of total employment (calculated as $n + p + m$, where lower-case letters denote logs.)
  Because the underlying CBC forecasts only allow calculation of indices of some of these quantities, this measure of labour productivity is also only an index.

Methodology

To gauge the uncertainty associated with forecasts of real GDP, we simply examine the size of the forecast errors associated with the above forecasts. As noted in the previous section, we have only a relatively small number of forecasts (less than 20) to analyse. Furthermore, due to the overlapping nature of multi-period forecasts, we would expect the forecast errors to be strongly correlated across consecutive forecast vintages. This effectively precludes reliable formal hypothesis testing of the properties of the observed forecast errors. Instead, we provide a graphical and statistical description of the dispersion of the forecast errors.

Forecast Errors

The first step in calculating the forecast errors associated with the forecasts in our collection is to convert the log-level series described in the previous section into forecasts of cumulative growth. We analyse forecasts of cumulative growth rather than levels for two reasons. First, since some of the underlying forecasts are for growth rates, our log-level series are indices rather than true levels. This means that we can only estimate their forecast error up to a constant. Secondly, published real GDP estimates are rebased from time to time, which means that outcomes may never be published in the same units as the forecast. Working with forecast cumulative growth allows us to sidestep these problems.

Specifically, for some forecast

$F(t,t+j)$ = a forecast published in year t for the log-level of a series in year $t+j$

we calculate the cumulative growth over the forecast horizon as

$GR(t,t+j) = F(t,t+j) - F(t,t-1)$

For example, in September 2000 MFQ forecast, the year 2000 figure for Y was a forecast since complete data for that year was clearly not yet available. However, preliminary estimates for 1999 had already been released. We therefore measure cumulative growth relative to the MFQ's year-2000-reported value for 1999.

In all cases, we calculated forecast errors by comparing forecasts of the above cumulative changes to those later reported by Statistics Canada. Note that by using officially revised figures published long after the forecasts, they may incorporate many revisions that were not part of the information originally available at the time that the forecasts were made and may sometimes be quite different from published preliminary estimates. However, as these revised figures represent the best currently-available estimate, they also provide the most realistic measure of the overall forecast errors and forecast uncertainty facing policymakers. Data revision is also expected to be small relative to the forecast errors, particularly at longer forecast horizons.[15]

Potential Output

As discussed in the Introduction, it would be useful to have some measure of the uncertainty associated with forecasts of potential output growth. However, potential output is not directly observed and different estimation methods can sometimes provide sharply divergent estimates. Trying to establish which of these methods is best is a task beyond the scope of the current paper, as is trying to gauge the risk caused by using the wrong estimator of potential output. Instead, we try to quantify the forecast uncertainty associated with a specific model of potential output.

We begin by using the HP filter to decompose our series into cycles and trends.[16] The HP filter is by far the most common technique used to estimate potential output, in part because of the ease of its calculation. Also known as the Whittaker-Henderson smoothing spline, the HP filter produces trends that closely resemble those produced by band-pass filters designed to isolate business cycle frequencies.[17] State-space models may also produce trends very similar to those of the HP filter.[18]

An important weakness of the HP filter arises, however, when it is used to analyse points close to the end of a data series. In particular, the spectral properties of the filter change and the

[15] For comparison, one MFQ expert suggested that revisions in annual estimates of Québec real GDP are on the order of 0.4% while those in Canadian real GDP are on the order of 0.2%. Below in Table 2, we find that the root-mean squared error of current-year GDP estimates are more than double that size.

[16] See Hodrick and Prescott (1997). Consistent with their suggested use of $\lambda=1600$ for quarterly data, we use $\lambda=400$ for the annual data we analyze.

[17] See Henderson (1924), Whittaker (1923) and Baxter and King (1999).

[18] See Harvey and Jaeger (1993) and Morley, Nelson and Zivot (2003).

resulting trends become more volatile and deviations from the raw series become smaller and less persistent.[19] As a result, when using the HP filter to estimate trends close to the end of sample it is recommended that the data series be extended by "padding" the available observations with forecasts.[20] We do so using the MFQ and CBC forecasts, HP filtering each individual vintage to recover an estimate of the perceived HP trend and its forecast values. Specifically, we apply the HP filter to the series $\{ F(t,1),..., F(t,t+H)\}$ for each available vintage t where H is the maximum forecast horizon available. In Figure 6 we show an example of the unfiltered (blue) and filtered (red) series for the MFQ forecast published in 2000. As expected, deviations from trend are larger and more volatile prior to 2000. During the forecast period, series tend to converge towards their trends as one would expect. This is consistent with forecasts where variables are expected to converge to their long-run trends over the medium-to-long term.

We calculate "forecast" errors for the potential output and other HP-filtered trends in the manner described above for other series; that is, we replace forecasts with actual outcomes and calculate the difference.[21] However, since potential output (and its components) is never directly observed, we are not capturing all of the error associated with its forecast. Instead we are simply measuring the degree to which additional information allows us to improve our estimate of potential. The resulting "revision" we measure captures only part of the forecast uncertainty, and so should be considered an underestimate of the "true" uncertainty.[22]

Decompositions

As noted above, the five variables in our forecast collections may be expressed as

$$y = n + p + m + q \tag{1}$$

where lowercase letters denote logs and all variables are expressed as deviations from a base year value. Furthermore, if we use "*" to denote the values associated with "potential" or "trend" levels,

$$y^* = n + p^* + m^* + q^* \tag{2}$$

[19] See St. Amant and van Norden (1997), Orphanides and van Norden (2002) and Mise, Kim and Newbold (2005).

[20] See Mise, Kim and Newbold (2005).

[21] To be precise, let X^t be the $(T+H)$ x 1 vector containing all values of a series as published at time t plus H forecasts also made at time t. Let $HP(.)$ be the HP filter function such that the trend of the series X^t is $X^{*t} = HP(X^t)$. The "forecast error" or revision in the trend is calculated as $X^{*T} - X^{*t} = HP(X^T) – HP(X^t)$ where X^T is the 2012 data vintage. The sole exception to this rule is the case of potential output (which is not directly HP-filtered) where the revision is calculated as the sum of the revision in its components. See the discussion, below.

[22] See Orphanides and van Norden (2002) and Cayen and van Norden (2005).

where we assume that population is always at its "potential" level so that $n = n^*$.[23] We can also combine these two relationships to obtain

$$y = n + p^* + m^* + q^* - (y^*-y) \tag{3}$$

where the last term is the deviation of output from potential output (also known as the "output gap" or *g*.)

We use equations (1) and (3) to understand the different factors contributing to the uncertainty in real GDP forecasts, while (2) is used to better understand the sources of uncertainty in forecasts of potential output. In addition to decomposing the mean forecast errors in y and y^*, we decompose their forecast-error variances as well. This requires a means of allocating the covariances among the various components. We use the Cholesky factorization (commonly used in the analysis of VAR systems), which requires us to select an ordering for the components.[24] In all cases, we rank population ahead of all other variables as we consider it to be exogenous with respect to the economic variables. When using (3), we rank the output gap after all the other variables as we consider the trend or potential levels of the other variables to be exogenous with respect to the business cycle. For the remaining three variables, we consider two alternative orderings; (p,m,q), which ranks the labour market variables ahead of the productivity variable, and the reverse ordering (q,m,p). Comparing the results of these orderings provides some insight into the robustness of the decompositions to alternative reasonable decompositions.

Results

We now consider the behaviour of the forecast errors from the three different forecast datasets. As discussed above, it should be kept in mind that results are based on a relatively small number of outcomes and we make no attempt here to determine whether or not the patterns we find are significant in a strict statistical sense. This applies in particular to the longer-horizon forecasts where the overlapping nature of the forecasts reduces the effective number of independent outcomes observed.[25]

[23] Note that because population is not detrended, estimated potential output will not be exactly equal to the HP-filtered trend in output.

[24] The Cholesky factorization of a symmetric positive semi-definite matrix Σ is an upper-triangular matrix Λ such that $\Lambda \cdot \Lambda' = \Sigma$. The square of the *j*th element of $\Lambda \cdot \iota$ (where ι is a conformable vector of ones) is then interpreted as the contribution of the *j*th variable to the overall variance. This decomposition is not invariant to the ordering of the columns and rows of Σ; covariances are attributed to the variable in any pair which is listed first. This effectively treats the first-listed variable as exogenous with respect to the second variable in any pair.

[25] The MFQ forecasts (1994-2007) also do not cover precisely the same vintages as the two sets of CBC forecasts (1992-2008).

We begin by examining the results for real and potential output growth before turning to consider their components. In addition to the main tables discussed below, comprehensive tables of descriptive statistics may be found in the annex to the paper.

Growth Uncertainty

Figure 7 and Table 2 summarize the performance of the forecasts for real GDP growth. Evidence of forecast bias is mixed. Average forecast errors for the MFQ forecasts were always quite close to zero, as was the case for the CBC-CA forecasts at all but the 6-year horizon.[26] The CBC-QC forecasts tended to have a positive error of about 1% for all horizons beyond 2 years, suggesting a degree of excessive optimism. The Figure shows that each of these averages encompass a broad dispersion of forecast errors, with forecasts in any given vintage tending to track consistently above or below realized GDP. For example, the relative strong growth Québec experienced in the late 1990s tended to create a string of negative forecast errors, whereas the recession starting in 2008 was followed by a series of positive forecast errors.

The lower panel of Table 2 reports the root-mean squared forecast errors (RMSFE) for the three sets of real output growth forecasts. All three sets of forecasts show similar results, with the RMSFE increasing monotonically with the forecast horizon by about 1% per year until the 3-yr horizon and more slowly thereafter. [27] The RMSFE for cumulative growth is just under 1% for a nowcast (i.e. estimates for the current year), 3% at a 2-year horizon, and 5% at a 5-year horizon.[28]

Figure 8 and Table 3 give comparable results for potential output growth. Results for the two Québec forecasts were similar, with mean forecast errors consistently positive (indicating forecasts of potential output that were too optimistic on average) and monotonically increasing with the forecast horizon. For the Canadian forecast, however, mean forecast errors consistently close to zero at all forecast horizons. In all three cases, the RMSFE increased linearly with the forecast horizon, with the MFQ forecast having somewhat higher average squared errors than the CBC-QC, which in turn had somewhat higher average squared errors than the CBC-CA. Figure 9 shows that this linear increase was also evident in each vintage of the forecast errors, reflecting the influence of the smoothing used to construct the measures of the potential output.

[26] If anything, the MFQ forecasts over this period tended to be slightly pessimistic, with average forecast errors less than zero at all forecast horizons. Such pessimism would be similar to that discussed above for Canadian Ministry of Finance forecasts but unlike the more common finding of excessive government optimism.

[27] If true, this would be consistent with output that returns to a trend growth path over the medium to long term, so that forecast uncertainty eventually levels off as the forecast horizon becomes large. However, due to the very limited number of independent observations generating this result, the above results could also be reasonably explained by chance.

[28] If errors are normally distributed with mean zero, we should then expect our current estimates and forecasts of GDP to be wrong by ±RMSFE just over 30% of the time.

Comparing the RMSFEs for potential output to that for real output growth, we see that the former is smaller and that the difference between the two is appreciable even at a forecast horizon of 6 years. This underlines the potential importance of cyclical factors for forecasting, even in the medium to longer run. We provide further evidence on this point below, where we examine the sources of the forecast errors that we have studied to this point.

Sources of Uncertainty

To better understand the sources of output growth forecast errors, we now turn to the decomposition framework discussed above which relates forecast errors in output growth *y* to those in population *n*, participation rates *p*, employment rates *m* and labour productivity *q* as expressed in equation (1). Similarly, (2) relates the forecast errors in potential output to those in population and the trends of the other three variables, while (3) relates output growth to those variables plus the change in the output gap (*g*). For each case, we analyse two types of evidence. First, we consider plots of the various forecast errors to allow for a comparison of their relative sizes. Second, we perform the Cholesky decomposition of the variance-covariance matrix of forecast errors as described above to understand their relative contributions to overall forecast errors for real and potential output.

Figure 9 presents forecast errors for the various components of real output growth in (1). There are several apparent similarities across the three forecast data sets. First, cumulative forecast errors in *n* and *m* are relatively smaller than those in *p* and especially *q*; this difference increases with the forecast horizon. The range of forecast errors was greatest for *q*, particularly at longer forecast horizons. The size of the forecast errors was roughly comparable across datasets, with the CBC Canada forecast having somewhat smaller errors for *p* while the MFQ errors for *q* seemed somewhat smaller.

Results of the Cholesky variance decomposition for these forecast errors are presented in Table 4. The table shows four distinct panels, one for each variable in the decomposition. Each panel shows the fraction of the forecast error variance in y that is can be explained by the indicated variable at a given forecast horizon. Results for each data set are given in each panel. Panels for all variables other than *n* also give results for two different orderings of the Cholesky decomposition: NPMQ and NQMP.

The results of the variance decomposition were generally somewhat sensitive to the data set examined and even more so to the ordering used in the decomposition. For example, *p* could explain well over half of the variance of longer-horizon output growth forecast errors in the MFQ forecasts, but never more than half of that in the CBC-CA forecasts. However, a change in the ordering of the decomposition could reduce its importance by more than three-quarters.

The most robust feature of the variance decomposition was the dominant role of productivity (*q*) forecast errors in contributing to overall output growth forecast errors at short horizons. They accounted for at least 60% of the forecast error variance for nowcasts, independent of the model or the variable ordering; when placed second in the ordering, this increased to 75%. As horizons lengthened, however, their role both diminished and became more ambiguous. With a favourable ordering, they could still account for almost half of the forecast error variance at the longest horizons, but in the case of the MFQ forecasts a different ordering could reduce this to less than 10%.

The dominance of productivity forecast errors at short horizons implies that other variables only appeared to play a significant role as forecast horizons lengthened. The role of population shocks ranged from just over a quarter of the overall forecast error variance in MFQ forecasts, to one third in the CBC-QC forecasts and just over one-half in the CBC-CA forecasts. Given the relatively small size of the forecast errors for *n* discussed above, this implies that these errors must have important correlations with forecast errors in other variables.

For all the other variables in the MFQ forecasts, results were very sensitive to the ordering of the variables, implying important correlations between their forecast errors. As one of the most extreme examples, forecast errors in *q* could explain over 70% or less than 10% of the forecast error variance in *y* at a 4-year horizon. When listed ahead of the labour market variables, however, q's share of the overall forecast error variance tended only to fall from over 2/3 at the shortest horizons to just less than ½ at the longest, implying that it can interpreted as playing a major role. The relatively dominant role played by errors in *q* in the previous figure may therefore be misleading to the extent that such errors appear to be highly correlated with errors in *p* and *m*. The reasons for this correlation are unclear, but it appears this is unlikely to be due to cyclical fluctuations in these variables as the correlation appears to be relatively more important at longer forecast horizons. The decomposition of potential output may also provide insight into this correlation.

Figure 10 compares the forecast errors for the decomposition of potential output given in (2). The first point to note is that the forecast errors for *n* are identical to those shown in the previous figure; changes in appearance are entirely due to the change in scale which is now almost half that in Figure 9. Forecast errors in *n* are now larger on average than those in *p** or *m**, while those in *q** remain as large as or larger than those of any other component and increase rapidly with forecast horizon. The relatively small contribution of trend movements in *m** in all three sets of forecasts are consistent with only minor movements in equilibrium rates of unemployment in Québec and Canada during the period covered by the forecasts.

The MFQ forecast errors differ somewhat from those of the two CBC forecasts in having relatively larger errors in the two labour market variables *m** and *p** and somewhat smaller errors in *q**. The MFQ errors in *q** appear to show a consistent bias towards overly optimistic forecasts, with realized labour productivity growth trends falling short of forecast in all but one or two of the fourteen vintages analysed.[29] These errors are more than enough to account for the mean positive forecast errors in *y** previously noted in Table 3. In the case of the CBC-QC

[29] Further statistics are provided in Table A10, which shows a mean forecast error of 2.35% over a six-year forecast horizon. Mean errors for the CBC-QC forecast were smaller and the errors varied more across forecast vintages. Mean forecast errors for *p** and *m** were also larger (in absolute value) for the MFQ forecasts than for the CBC-QC or the CBC-CA forecasts. However, these were much smaller than those for *q** and had opposite signs, roughly cancelling each other out.

forecasts, however, the mean forecast errors in *y** are roughly the same size and sign as those in *q**.

Table 5 provides the results of the Cholesky variance decomposition of (2). Perhaps unsurprisingly, results for trend forecasts were less variable across forecast horizons than those examined above. The contribution of forecast errors in *m** were also quite minor in all possible cases, consistent with the interpretation that equilibrium unemployment rates were relatively stable over this period. More surprisingly, errors in *n* sometimes played an important role in some cases, particularly for the CBC-CA forecasts where they accounted for between one-third and one-half of overall forecast error variance. In the forecasts for Québec, however, they more typically accounted for between a 10% and a one-quarter share. Results for Québec and Canada also differed for the remaining two variables. Regardless of the ordering of the labour market variables, *p** played a minor role in the Canadian forecast with *q** accounting for roughly one-third to one-half of the overall forecast error variance. For the Québec forecasts, however, results were highly sensitive to the ordering of the variables, with contributions frequently changing by 50% or more. This implies that forecast errors in *q** were highly correlated with those in *p**, and this effect was particularly pronounced in the MFQ forecast errors.[30]

Figure 11 shows the forecast errors associated with (3). The components shown for *n*, *p** and *q** are identical to those shown in Figure 10 (aside from a change of scale) while the estimated forecast errors for *g* are new.[31] In each case, the range of forecast errors for *g* is as large as or larger than those for any other component. The relative importance of *g* is particularly pronounced at shorter horizons, as the errors for the other components tend to increase smoothly with forecast. In the case of the MFQ forecasts, we also see that negative forecast errors for *g* tend to be much larger than positive forecast errors.[32] These effectively cancel the mean forecast errors in *q**, giving the near-zero mean forecast errors in *y* described above in Table 2.

Table 6 provides the results of the Cholesky variance decomposition for (3). It is interesting to compare these results to those in Table 5 to better understand the role of cyclical and trend factors in explaining overall growth forecast errors. Recall also that *g* was always placed last in the decomposition ordering so its results are invariant to the ordering of the other variables. The Table shows that *g* is the most important component of *y* at very short horizons, but as expected this importance falls rapidly towards near-zero levels at longer horizons. This comes largely at the expense of *p** or *q**, depending on which is placed first in the decomposition

[30] Figure 10 confirms that this is a negative correlation, with the most overoptimistic forecasts of *q* corresponding to negative errors in the forecasts for *p*.

[31] To conserve space, we omit the results for the trend employment rate, which was the smallest of the four components shown above in (3).

[32] As shown in Table A11, *g* has a negative mean whose size steadily increases with forecast horizon to reach -2% at a horizon of 6 years. Recall that a negative output gap implies that output is below trend. A negative *g* therefore implies that the output gap was worse than forecast.

ordering. Forecast errors in *g* also explain a much smaller fraction of the overall forecast errors in *y* in the MFQ forecasts than in those for either of the private sector forecasts, with *p** and sometimes *m** playing comparable roles in explaining errors in nowcasts. The results for *n* are identical to those previously presented in Table 4 and discussed above.

Conclusions

This paper has reviewed evidence on the nature, size and composition of the forecast errors for three historical datasets comprised of government and private sector forecasts of real GDP growth for the Québec and Canadian economy. Any such exercise has its natural limitations; in this case they include the limited time span covered by the forecasts relative to the forecast horizons under study. Comparisons between the government and private sector forecasts also make no adjustment for the different time periods covered by the different forecasts or the fact that the forecasts were prepared at different times of year. Forecasts of potential output growth were imputed rather than collected, and their calculated forecast errors are likely to overstate the apparent accuracy of such forecasts. For all these reasons, the analysis presented here is simply intended to be descriptive of the available evidence and suggestive rather than definitive.

Root-mean-squared forecast errors for cumulative output growth tended to increase linearly with the forecast horizon, reaching roughly 5% of output at a 5-year horizon. Forecasts of potential output growth showed lower RMSFEs, ranging from 2.2% for the CBC Canadian forecast to 3.6% for the MFQ forecast. This result is somewhat surprising given that the government forecasts are produced later in the year and so have the advantage of slightly more information. Typical distributional assumptions imply that forecast errors will commonly fall outside the range of ± 1 RMSFE and occasionally outside ±2 RMSFE. For risk management purposes, this would imply planning for a range of real output growth outcomes that vary by 10%-20% at the five-year horizon, or potential output growth that varies by roughly half those amounts.

Evidence of forecast bias varied across the different forecasts examined here. The CBC Canadian forecasts showed little or no evidence of bias, while their Québec forecasts showed a tendency towards excessive optimism that increased with the forecast horizon. Although the MFQ forecasts growth forecasts had mean forecast errors close to zero at all horizons (and, if anything, have been slightly pessimistic over the period analysed) this reflected roughly offsetting errors in forecast trend productivity growth (which tended to be too optimistic) and movements in the implied output gap.

The importance of the different sources of forecast errors varied considerably across forecasts, horizons and variable ordering. However, errors in the rate of employment were minor in almost all cases. Errors in forecasting labour productivity growth were often important contributors, but these errors were often correlated with errors in participation rate forecasts making the results sometimes extremely sensitive to the order of the variables in the decomposition. Population growth errors tended to become more important at longer horizons and were most important in the Canadian forecast and least important in the MFQ forecast. Output gap forecasts played a key role in accounting for overall growth forecast errors at the shortest horizons but their importance quickly faded as forecast horizons increased.

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Figures

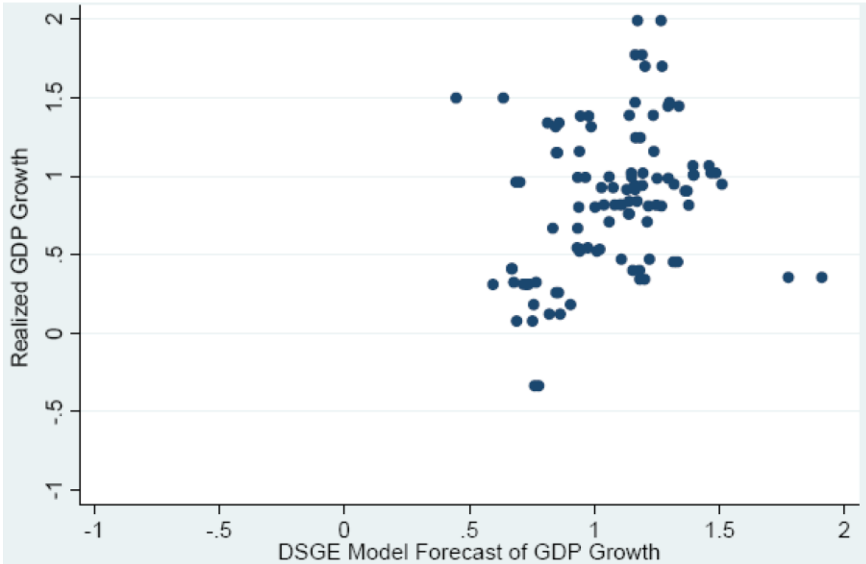
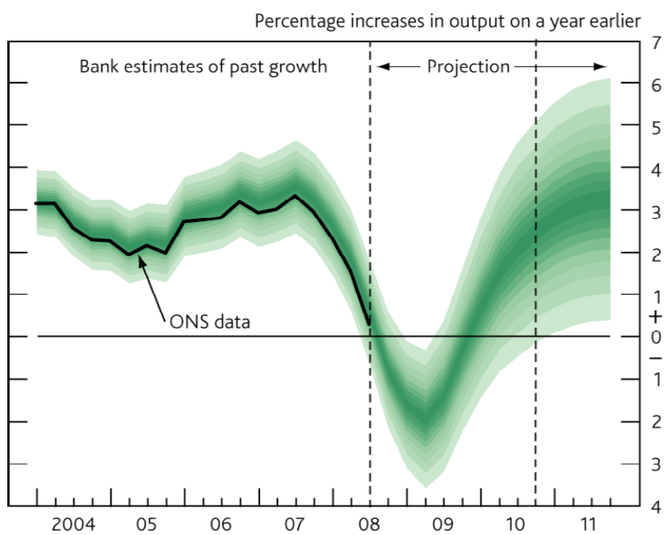


Figure 1: 4Q-ahead DSGE Model Forecasts
Source: Edge and Gürkaynak (2010)

Chart 1 GDP projection based on market interest rate expectations



The fan chart depicts the probability of various outcomes for GDP growth. To the left of the first vertical dashed line, the distribution reflects the likelihood of revisions to the data over the past; to the right, it reflects uncertainty over the evolution of GDP growth in the future. If economic circumstances identical to today's were to prevail on 100 occasions, the MPC's best collective judgement is that the mature estimate of GDP would lie within the darkest central band on only 10 of those occasions. The fan chart is constructed so that outturns are also expected to lie within each pair of the lighter green areas on 10 occasions. Consequently, GDP growth is expected to lie somewhere within the entire fan on 90 out of 100 occasions. The bands widen as the time horizon is extended, indicating the increasing uncertainty about outcomes. See the box on page 39 of the November 2007 *Inflation Report* for a fuller description of the fan chart and what it represents. The second dashed line is drawn at the two-year point of the projection.

Figure 2: Bank of England GDP Fan Chart
Source: Bank of England *Inflation Report* - November 2008

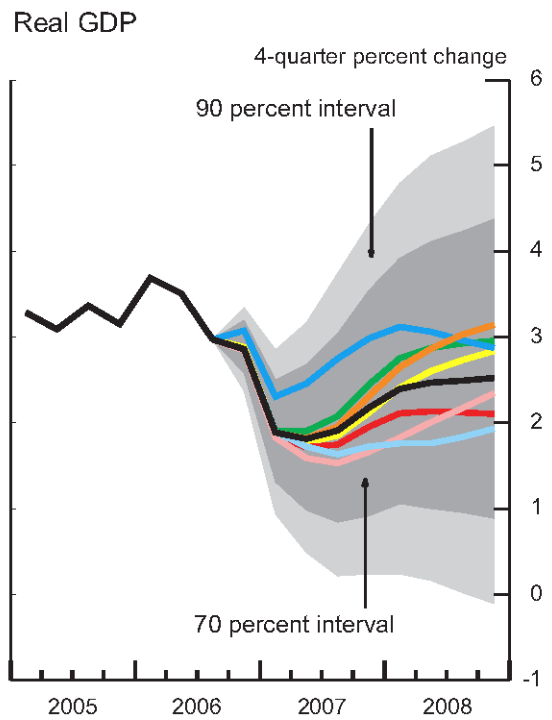


Figure 3: Federal Reserve Board Staff Fan Chart
Source: FOMC Greenbook - 12 December 2006

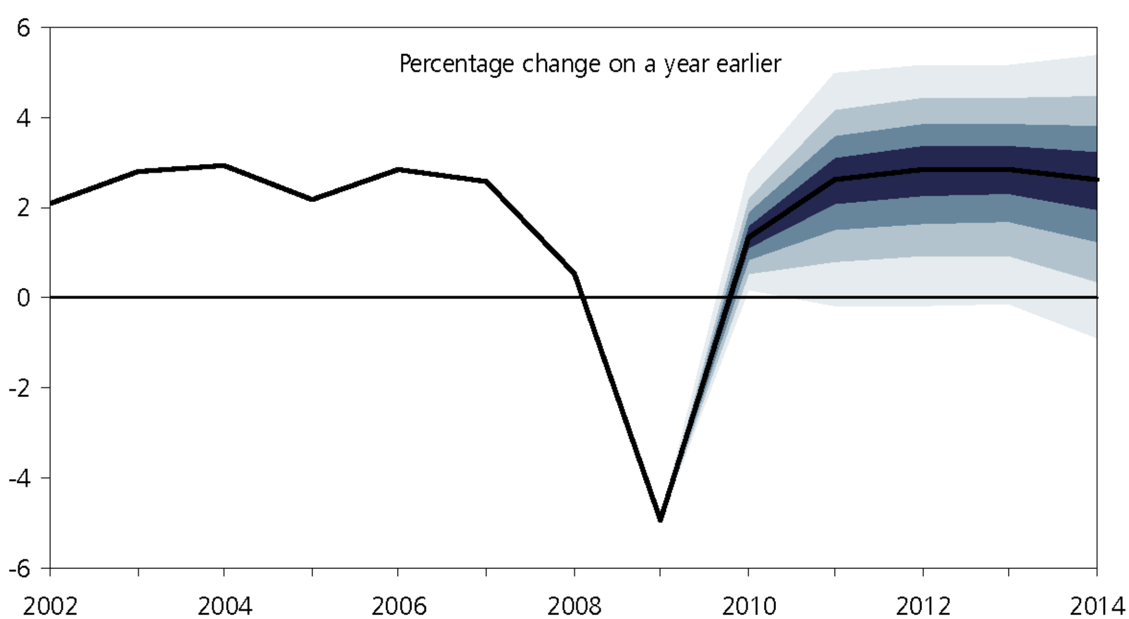


Figure 4: UK Office for Budget Responsibility Fan Chart
Source: Office for Budget Responsibility, *Pre-Budget Forecast* – June 2010

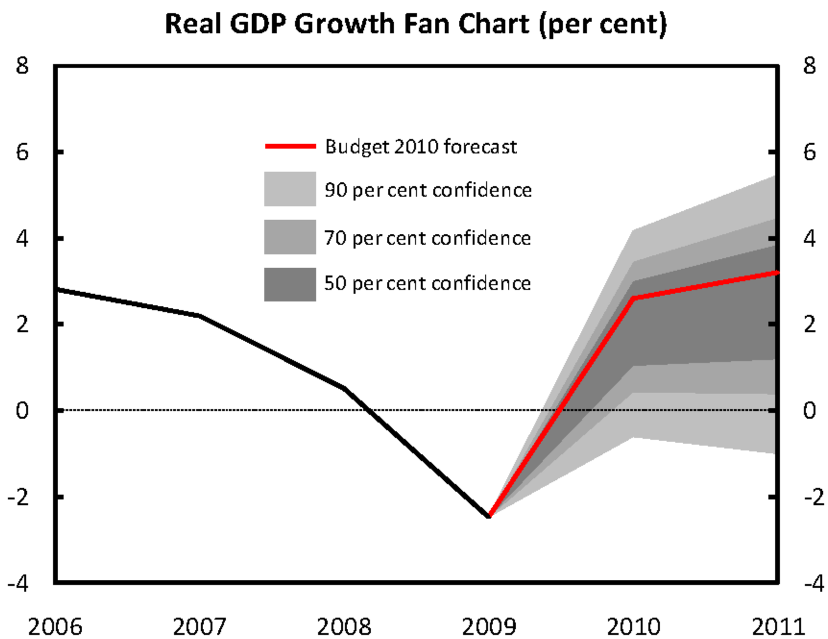


Figure 5: PBO Fan Chart
Source: Office of the Parliamentary Budget Officer (2010)

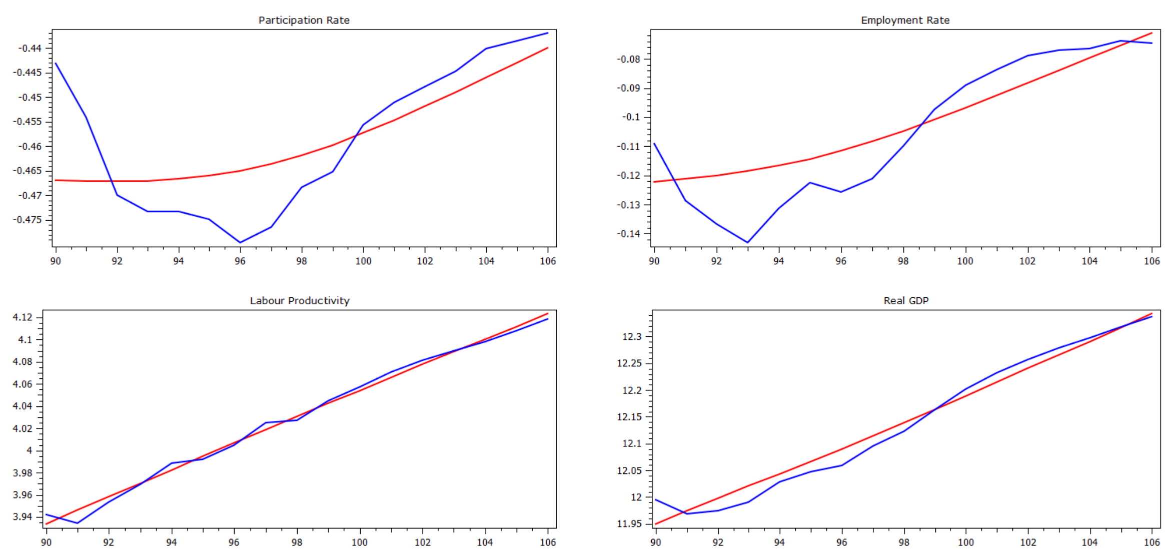


Figure 6: Observations, Forecasts and HP Trends, MFQ 2000

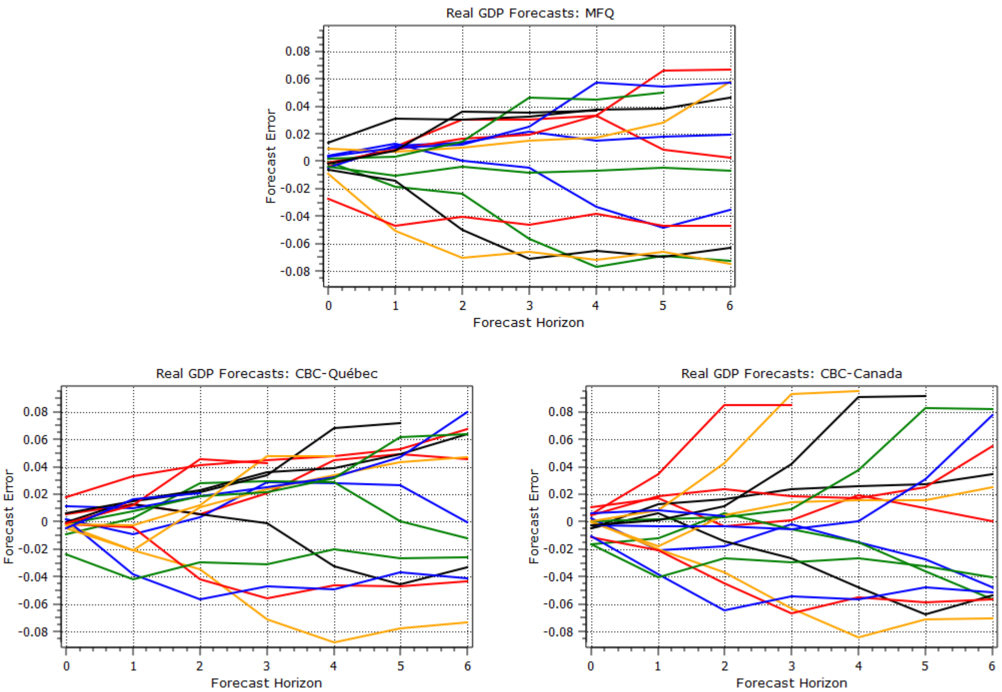


Figure 7: Real GDP Forecast Errors

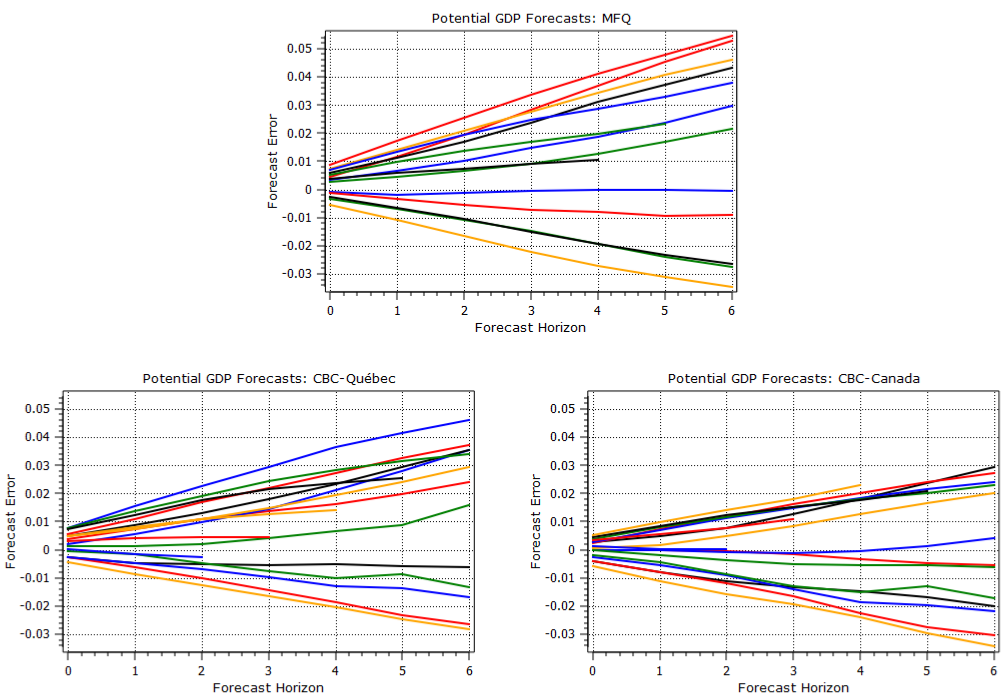


Figure 8: Potential Output Forecast Errors

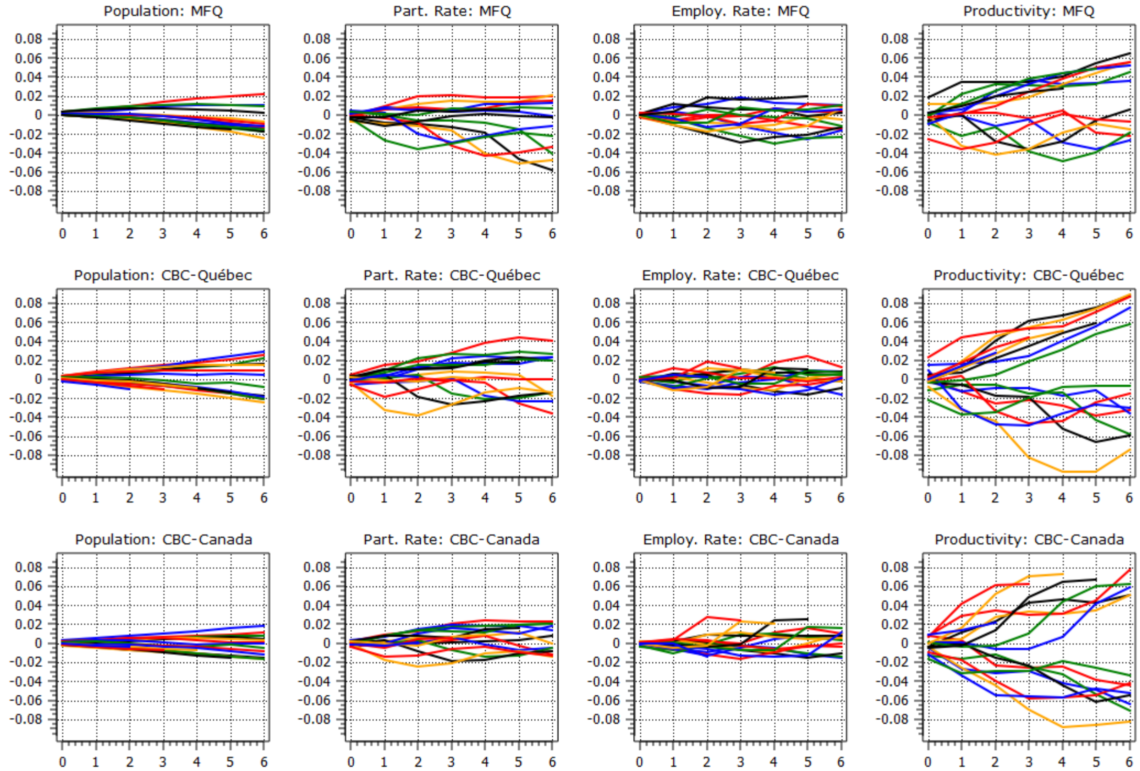


Figure 9: Sources of Real GDP Growth Uncertainty

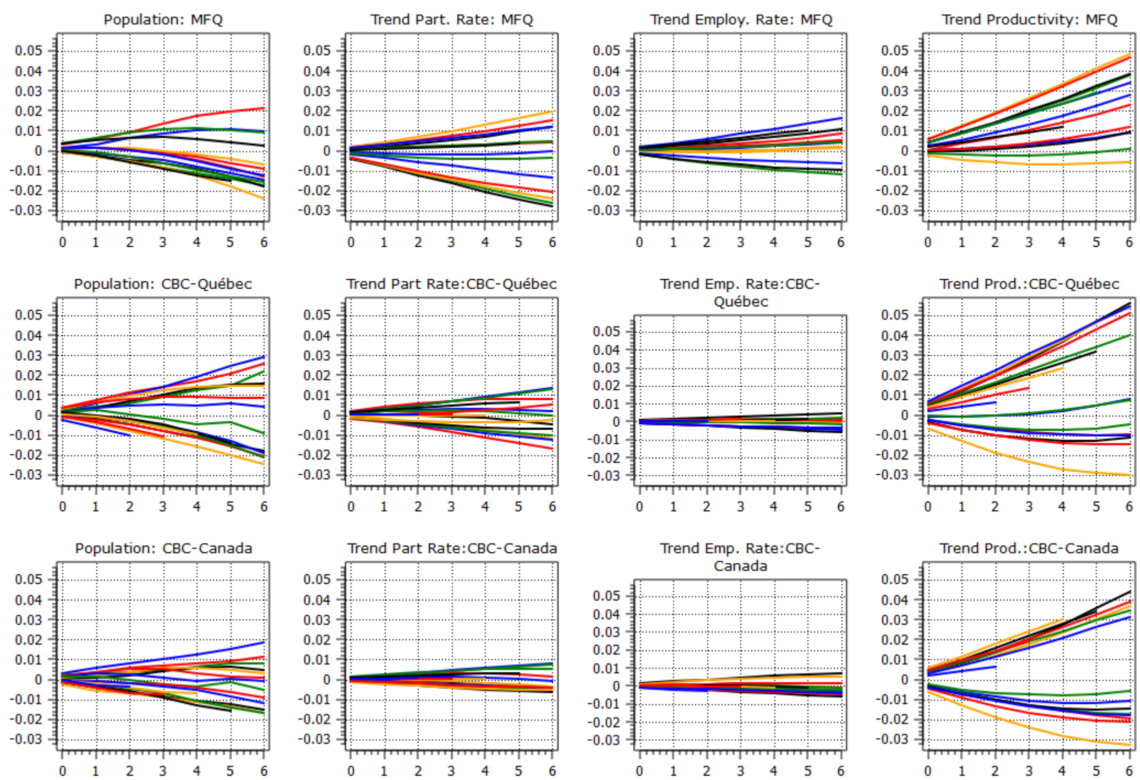


Figure 10: Sources of Potential Output Growth Uncertainty

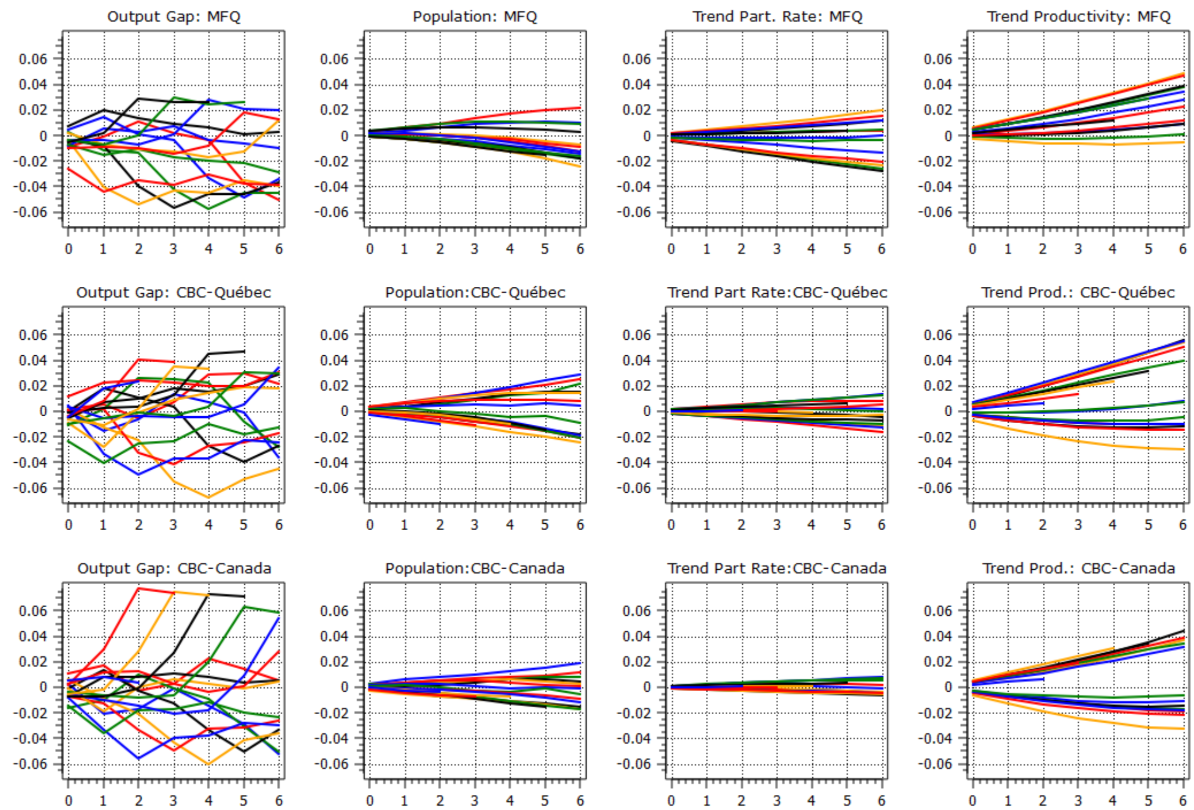


Figure 11: Sources of Real GDP Growth Uncertainty

Tables

Table 1: Uncertainty Estimates from PEAP-CIRANO (2005)

Measures of Forecasting Difficulty

	Standard Deviation		Standard Deviation/Mean		Lagged Correlation	
	1984-1993	1994-2003	1984-1993	1994-2003	1984-1993	1994-2003
Real Growth	2.53	1.59	0.96	0.45	0.90	0.88
Unemployment Rate	1.48	1.17	0.15	0.14	0.97	0.93
Employment Growth	1.75	0.86	1.12	0.42	0.96	0.80
CPI Inflation	1.32	1.00	0.33	0.54	0.86	0.76
Nominal GDP Growth	3.23	2.55	0.55	0.48	0.94	0.85
3-Month T-Bill Rate	2.44	1.45	0.27	0.33	0.89	0.89
10-Year Benchmark Bond Yield	1.55	1.28	0.16	0.21	0.87	0.95
Current Account Balance	8.87	18.31	-0.47	2.99	0.86	0.87
Exchange Rate	5.29	3.87	0.07	0.06	0.98	0.88
Labour Income Growth	2.79	1.90	0.46	0.42	0.96	0.87
Corporate Profit Growth	20.44	21.25	6.38	1.35	0.84	0.84
Total Business Investment Growth	7.50	5.82	2.97	1.09	0.89	0.88
U.S. Real GDP Growth	1.91	1.33	0.57	0.41	0.85	0.88

Table 2: Real GDP Growth Forecasts

Horizon	0	1	2	3	4	5	6
Mean FE							
MFQ	-0.2%	-0.3%	-0.2%	-0.2%	-0.1%	-0.3%	-0.4%
CBC-QC	-0.1%	0.0%	0.5%	0.9%	1.1%	1.2%	1.1%
CBC-CA	-0.2%	-0.4%	-0.1%	0.2%	0.0%	-0.4%	-0.8%
RMSFE							
MFQ	0.9%	2.3%	3.1%	4.0%	4.5%	4.9%	5.2%
CBC-QC	0.9%	2.0%	2.9%	3.9%	4.6%	4.9%	5.1%
CBC-CA	0.8%	2.0%	3.3%	4.5%	5.0%	5.1%	5.4%

Table 3: Potential Output Forecast Errors

Horizon	0	1	2	3	4	5	6
Mean FE							
MFQ	0.2%	0.5%	0.7%	0.9%	1.1%	1.4%	1.6%
CBC-QC	0.2%	0.4%	0.6%	0.8%	1.0%	1.2%	1.3%
CBC-CA	0.0%	0.1%	0.1%	0.2%	0.2%	0.1%	-0.1%
RMSFE							
MFQ	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.6%
CBC-QC	0.4%	0.8%	1.2%	1.6%	2.1%	2.5%	2.9%
CBC-CA	0.3%	0.6%	1.0%	1.3%	1.7%	1.9%	2.2%

Table 4: Variance Decomposition - EQ 1

	Population			Participation Ratio					
	MFQ	CBC-QC	CBC-CA	MFQ		CBC-QC		CBC-CA	
Horizon				NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	9%	2%	12%	1%	8%	1%	9%	7%	6%
1	8%	13%	25%	21%	16%	15%	24%	4%	10%
2	12%	32%	41%	29%	17%	12%	18%	4%	7%
3	18%	27%	39%	40%	13%	24%	10%	11%	4%
4	21%	24%	40%	51%	8%	39%	9%	23%	4%
5	28%	32%	56%	57%	10%	36%	16%	24%	4%
6	29%	37%	53%	61%	14%	41%	16%	18%	4%

	Employment Rate						Labour Productivity					
	MFQ		CBC-QC		CBC-CA		MFQ		CBC-QC		CBC-CA	
Horizon	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	29%	7%	3%	2%	13%	6%	61%	75%	94%	87%	68%	76%
1	29%	11%	23%	13%	32%	3%	42%	66%	50%	51%	38%	62%
2	30%	7%	18%	7%	25%	5%	28%	64%	38%	43%	30%	47%
3	29%	2%	23%	1%	29%	2%	13%	68%	26%	63%	21%	55%
4	21%	0%	20%	1%	15%	3%	6%	71%	18%	66%	22%	53%
5	10%	1%	4%	7%	1%	8%	5%	60%	28%	45%	20%	33%
6	1%	10%	0%	2%	0%	5%	8%	47%	21%	45%	29%	37%

Table 5: Variance Decomposition - EQ 2

	Population			Participation Ratio					
	MFQ	CBC-QC	CBC-CA	MFQ		CBC-QC		CBC-CA	
Horizon				NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	28%	19%	36%	65%	6%	33%	8%	21%	3%
1	21%	19%	39%	73%	4%	39%	9%	17%	3%
2	16%	19%	48%	79%	4%	45%	9%	17%	3%
3	11%	28%	52%	83%	4%	53%	7%	14%	3%
4	10%	28%	46%	84%	4%	55%	7%	12%	3%
5	10%	26%	46%	85%	3%	52%	8%	9%	3%
6	10%	15%	35%	85%	3%	59%	7%	11%	3%

	Employment Rate						Labour Productivity					
	MFQ		CBC-QC		CBC-CA		MFQ		CBC-QC		CBC-CA	
Horizon	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	0%	10%	5%	1%	2%	1%	7%	56%	44%	72%	40%	59%
1	0%	9%	1%	1%	5%	1%	5%	65%	41%	71%	38%	57%
2	1%	9%	1%	1%	4%	1%	5%	71%	35%	71%	32%	49%
3	1%	9%	0%	1%	2%	0%	5%	76%	19%	63%	32%	45%
4	1%	9%	0%	1%	4%	0%	5%	78%	17%	64%	38%	50%
5	2%	7%	1%	1%	9%	0%	3%	80%	21%	65%	36%	50%
6	2%	7%	2%	0%	15%	0%	3%	80%	24%	77%	39%	62%

Table 6: Variance Decomposition - EQ 3

	Population			Output Gap			Participation Ratio					
	MFQ	CBC-QC	CBC-CA	MFQ	CBC-QC	CBC-CA	MFQ		CBC-QC		CBC-CA	
Horizon							NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	9%	2%	12%	30%	74%	80%	28%	31%	19%	11%	8%	4%
1	8%	13%	25%	39%	53%	60%	52%	11%	34%	30%	11%	4%
2	12%	32%	41%	22%	29%	43%	62%	11%	38%	27%	8%	2%
3	18%	27%	39%	13%	27%	31%	64%	13%	40%	11%	8%	1%
4	21%	24%	40%	9%	17%	23%	65%	10%	43%	7%	15%	6%
5	28%	32%	56%	8%	6%	11%	62%	6%	45%	11%	20%	14%
6	29%	37%	53%	3%	6%	9%	67%	7%	44%	10%	19%	8%

	Employment Rate						Labour Productivity					
	MFQ		CBC-QC		CBC-CA		MFQ		CBC-QC		CBC-CA	
Horizon	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP	NPMQ	NQMP
0	29%	10%	0%	1%	0%	1%	4%	19%	5%	13%	0%	3%
1	0%	3%	0%	0%	0%	1%	0%	39%	0%	5%	4%	10%
2	4%	15%	1%	1%	0%	0%	0%	40%	0%	11%	8%	14%
3	5%	17%	1%	3%	0%	0%	0%	40%	5%	33%	22%	29%
4	5%	15%	4%	6%	2%	0%	0%	45%	11%	46%	20%	31%
5	1%	4%	6%	6%	5%	1%	0%	54%	11%	45%	8%	19%
6	0%	1%	4%	1%	3%	2%	0%	59%	9%	45%	15%	27%

Annex – Statistical Tables

This annex provides additional detail on the statistical behaviour of the forecast errors for the variables and forecast data sets discussed in the body of the paper. The tables presented below provide information on the mean, standard deviation, minimum and maximum forecast errors, as well as the estimated first-order autocorrelation coefficient and the number of observations. This figures are compared for each data set and at each forecast horizon from 0 to 7 years ahead. They are presented for each of the available variables (*y,n,p,m,q*) as well as the imputed forecasts of potential values (*y*, p*,m*,q**). Results for three related forecasts are reported as well:

- *y1** is the estimate of potential output calculated by detrending *y* directly rather than its components *p,m* and *q*.[33]
- *g* = y – y* is the forecast output gap
- *g1 = y – y1** is an alternative estimate of the output gap

[33] The difference between this estimate and the estimate of y* reported in the body of the paper is equal to difference between n and its HP-filtered trend.

Table A1: Forecast Errors for Real GDP (Y)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0020	-0.0007	-0.0024	14	17	17
1	-0.0029	0.0000	-0.0038	14	17	17
2	-0.0019	0.0053	-0.0009	14	17	17
3	-0.0019	0.0092	0.0021	14	16	16
4	-0.0014	0.0110	0.0001	14	15	15
5	-0.0033	0.0121	-0.0043	13	14	14
6	-0.0041	0.0107	-0.0079	12	13	13
		Standard Deviations			Minimum	
0	0.0096	0.0088	0.0077	-0.0277	-0.0240	-0.0164
1	0.0233	0.0205	0.0207	-0.0511	-0.0422	-0.0405
2	0.0324	0.0291	0.0344	-0.0705	-0.0566	-0.0648
3	0.0411	0.0387	0.0461	-0.0714	-0.0715	-0.0665
4	0.0470	0.0459	0.0514	-0.0768	-0.0880	-0.0845
5	0.0507	0.0494	0.0527	-0.0697	-0.0776	-0.0710
6	0.0538	0.0522	0.0560	-0.0746	-0.0731	-0.0705
	1st Autocorrelation Coefficient				Maximum	
0	0.4041	0.8315	0.7824	0.0133	0.0176	0.0105
1	0.7819	0.9580	0.9011	0.0309	0.0335	0.0349
2	0.8856	0.9024	0.7912	0.0363	0.0452	0.0853
3	0.8225	0.8033	0.8145	0.0466	0.0480	0.0933
4	0.8151	0.7569	0.7517	0.0569	0.0684	0.0952
5	-0.5335	0.6642	0.6966	0.0660	0.0721	0.0918
6	0.5280	0.2561	0.5923	0.0670	0.0801	0.0818

Table A2: Forecast Errors for Participation Rate (P)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0008	-0.0006	-0.0002	14	17	17
1	-0.0019	0.0002	0.0001	14	17	17
2	-0.0032	0.0020	0.0023	14	17	17
3	-0.0050	0.0043	0.0043	14	16	16
4	-0.0066	0.0057	0.0052	14	15	15
5	-0.0083	0.0055	0.0049	14	14	14
6	-0.0111	0.0023	0.0034	13	13	13
		Standard Deviations			Minimum	
0	0.0032	0.0030	0.0019	-0.0076	-0.0066	-0.0038
1	0.0099	0.0113	0.0073	-0.0272	-0.0323	-0.0171
2	0.0142	0.0146	0.0105	-0.0364	-0.0380	-0.0249
3	0.0171	0.0167	0.0123	-0.0330	-0.0272	-0.0206
4	0.0200	0.0188	0.0126	-0.0437	-0.0236	-0.0176
5	0.0235	0.0220	0.0133	-0.0512	-0.0255	-0.0125
6	0.0270	0.0239	0.0143	-0.0588	-0.0361	-0.0146
	1st Autocorrelation Coefficient				Maximum	
0	0.8386	0.8885	0.9385	0.0042	0.0043	0.0026
1	0.0273	0.8378	0.8821	0.0094	0.0152	0.0088
2	0.8884	0.8678	0.7095	0.0194	0.0215	0.0153
3	0.9112	0.8628	0.7246	0.0206	0.0278	0.0204
4	0.8651	0.9016	0.6890	0.0178	0.0383	0.0241
5	0.8216	0.9046	0.6803	0.0185	0.0439	0.0234
6	0.5890	0.7985	0.3865	0.0205	0.0401	0.0228

Table A3: Forecast Errors for Population (N)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	0.0010	0.0010	0.0001	14	17	17
1	0.0010	0.0011	-0.0002	14	17	17
2	0.0005	0.0007	-0.0007	14	17	17
3	-0.0006	0.0008	-0.0012	14	16	16
4	-0.0024	0.0007	-0.0016	14	15	15
5	-0.0048	0.0006	-0.0019	14	14	14
6	-0.0068	0.0006	-0.0022	13	13	13
		Standard Deviations			Minimum	
0	0.0015	0.0017	0.0016	-0.0006	-0.0024	-0.0026
1	0.0033	0.0042	0.0033	-0.0027	-0.0065	-0.0055
2	0.0054	0.0070	0.0049	-0.0057	-0.0101	-0.0071
3	0.0076	0.0095	0.0065	-0.0092	-0.0120	-0.0093
4	0.0097	0.0125	0.0081	-0.0126	-0.0158	-0.0131
5	0.0115	0.0158	0.0100	-0.0178	-0.0201	-0.0158
6	0.0134	0.0198	0.0114	-0.0242	-0.0243	-0.0169
	1st Autocorrelation Coefficient				Maximum	
0	0.9916	0.9827	0.9369	0.0036	0.0037	0.0031
1	0.9437	0.9947	0.9444	0.0066	0.0075	0.0059
2	0.8746	0.9835	0.9782	0.0094	0.0111	0.0082
3	0.5052	0.9005	0.9615	0.0137	0.0143	0.0101
4	0.9793	0.8641	0.9635	0.0172	0.0193	0.0125
5	0.9873	0.9510	0.9836	0.0199	0.0243	0.0155
6	0.9952	0.9740	0.9575	0.0214	0.0291	0.0185

Table A4: Forecast Errors for Employment Rate (M)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0003	-0.0006	-0.0002	14	17	17
1	-0.0026	-0.0014	-0.0015	14	17	17
2	-0.0031	-0.0003	-0.0011	14	17	17
3	-0.0041	0.0000	0.0006	14	16	16
4	-0.0044	0.0007	0.0013	14	15	15
5	-0.0034	0.0014	0.0018	14	14	14
6	-0.0031	0.0013	0.0022	13	13	13
		Standard Deviations			Minimum	
0	0.0013	0.0017	0.0016	-0.0030	-0.0029	-0.0034
1	0.0070	0.0064	0.0044	-0.0112	-0.0117	-0.0103
2	0.0112	0.0089	0.0099	-0.0197	-0.0156	-0.0142
3	0.0136	0.0088	0.0115	-0.0292	-0.0162	-0.0169
4	0.0142	0.0098	0.0120	-0.0308	-0.0165	-0.0143
5	0.0140	0.0103	0.0122	-0.0254	-0.0162	-0.0151
6	0.0117	0.0080	0.0102	-0.0229	-0.0166	-0.0151
	1st Autocorrelation Coefficient				Maximum	
0	0.6221	0.6996	0.5979	0.0018	0.0023	0.0020
1	0.7911	0.4906	0.5610	0.0112	0.0112	0.0048
2	0.8688	0.5813	0.4558	0.0188	0.0177	0.0273
3	0.5560	0.6030	0.6056	0.0188	0.0114	0.0243
4	0.3088	0.5293	0.4741	0.0167	0.0169	0.0244
5	0.5031	0.3629	0.5266	0.0192	0.0237	0.0250
6	0.4432	0.6752	0.5805	0.0101	0.0130	0.0165

Table A5: Forecast Errors for Labour Productivity (Q)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0019	-0.0005	-0.0021	14	17	17
1	0.0006	0.0001	-0.0022	14	17	17
2	0.0039	0.0028	-0.0014	14	17	17
3	0.0077	0.0042	-0.0016	14	16	16
4	0.0119	0.0039	-0.0047	14	15	15
5	0.0152	0.0045	-0.0090	13	14	14
6	0.0187	0.0065	-0.0112	12	13	13
		Standard Deviations			Minimum	
0	0.0100	0.0099	0.0077	-0.0256	-0.02185	-0.0161
1	0.0193	0.0211	0.0212	-0.0362	-0.03762	-0.0336
2	0.0248	0.0329	0.0343	-0.0419	-0.04787	-0.0550
3	0.0290	0.0431	0.0448	-0.0388	-0.08258	-0.0702
4	0.0316	0.0505	0.0502	-0.0486	-0.09751	-0.0888
5	0.0349	0.0578	0.0539	-0.0397	-0.09758	-0.0864
6	0.0357	0.0630	0.0603	-0.0265	-0.07442	-0.0828
	1st Autocorrelation Coefficient				Maximum	
0	0.3731	0.9410	0.9034	0.0180	0.0224	0.0091
1	0.6658	0.9658	0.9414	0.0340	0.0438	0.0412
2	0.3467	0.9367	0.8907	0.0342	0.0502	0.0607
3	0.4163	0.9152	0.8921	0.0380	0.0609	0.0704
4	0.3111	0.8518	0.8307	0.0436	0.0672	0.0730
5	0.7153	0.6925	0.7836	0.0546	0.0747	0.0671
6	0.8803	0.6409	0.7236	0.0647	0.0890	0.0773

Table A6: Forecast Errors for Potential Output (Y*)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	0.0025	0.0023	0.0004	14	17	17
1	0.0046	0.0039	0.0007	14	17	17
2	0.0068	0.0056	0.0011	14	17	17
3	0.0091	0.0078	0.0017	14	16	16
4	0.0114	0.0099	0.0015	14	15	15
5	0.0138	0.0117	0.0008	13	14	14
6	0.0156	0.0127	-0.0006	12	13	13
		Standard Deviations			Minimum	
0	0.0045	0.0039	0.0033	-0.0054	-0.0043	-0.0059
1	0.0090	0.0075	0.0066	-0.0109	-0.0086	-0.0114
2	0.0136	0.0110	0.0098	-0.0166	-0.0126	-0.0158
3	0.0183	0.0146	0.0136	-0.0222	-0.0163	-0.0195
4	0.0228	0.0187	0.0175	-0.0271	-0.0204	-0.0241
5	0.0282	0.0225	0.0201	-0.0311	-0.0246	-0.0296
6	0.0335	0.0269	0.0232	-0.0346	-0.0284	-0.0344
	1st Autocorrelation Coefficient				Maximum	
0	0.9994	0.9992	0.9764	0.0088	0.0077	0.0051
1	0.5616	0.9997	0.9978	0.0174	0.0156	0.0099
2	0.9986	0.9963	0.8822	0.0255	0.0224	0.0141
3	0.9980	0.9565	0.9296	0.0335	0.0295	0.0181
4	0.9990	0.9977	0.9970	0.0410	0.0364	0.0229
5	0.9789	0.9983	0.9977	0.0478	0.0415	0.0241
6	0.9989	0.9974	0.9920	0.0546	0.0459	0.0292

Table A7: Forecast Errors for Trend Output (Y1)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	0.0012	0.0014	0.0002	14	17	17
1	0.0027	0.0031	0.0005	14	17	17
2	0.0047	0.0049	0.0009	14	17	17
3	0.0070	0.0069	0.0013	14	16	16
4	0.0098	0.0087	0.0009	14	15	15
5	0.0127	0.0101	-0.0006	13	14	14
6	0.0152	0.0108	-0.0022	12	13	13
		Standard Deviations			Minimum	
0	0.0049	0.0038	0.0037	-0.0068	-0.0058	-0.0057
1	0.0099	0.0078	0.0075	-0.0133	-0.0116	-0.0115
2	0.0149	0.0119	0.0114	-0.0192	-0.0173	-0.0172
3	0.0198	0.0164	0.0157	-0.0243	-0.0226	-0.0226
4	0.0245	0.0210	0.0196	-0.0287	-0.0273	-0.0275
5	0.0301	0.0255	0.0222	-0.0321	-0.0312	-0.0317
6	0.0357	0.0298	0.0249	-0.0347	-0.0342	-0.0350
	1st Autocorrelation Coefficient				Maximum	
0	0.9999	0.9998	0.9503	0.0068	0.0066	0.0054
1	0.9761	0.9997	0.9964	0.0142	0.0135	0.0112
2	0.9994	0.9995	0.9237	0.0220	0.0207	0.0170
3	0.9990	0.9539	0.9019	0.0302	0.0281	0.0226
4	0.9995	0.9951	0.9970	0.0387	0.0355	0.0277
5	0.8579	0.9993	0.9994	0.0475	0.0428	0.0259
6	0.9999	0.9999	0.9995	0.0564	0.0498	0.0313

Table A8: Forecast Errors for Trend Participation Rate (P*)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0008	0.0000	0.0001	14	17	17
1	-0.0015	0.0000	0.0002	14	17	17
2	-0.0022	-0.0001	0.0001	14	17	17
3	-0.0027	-0.0003	0.0000	14	16	16
4	-0.0030	-0.0005	-0.0002	14	15	15
5	-0.0032	-0.0009	-0.0006	14	14	14
6	-0.0037	-0.0018	-0.0014	13	13	13
		Standard Deviations			Minimum	
0	0.0022	0.0012	0.0008	-0.0041	-0.0017	-0.0011
1	0.0045	0.0025	0.0015	-0.0082	-0.0037	-0.0022
2	0.0068	0.0038	0.0022	-0.0123	-0.0059	-0.0032
3	0.0092	0.0052	0.0030	-0.0165	-0.0084	-0.0040
4	0.0116	0.0067	0.0038	-0.0205	-0.0111	-0.0049
5	0.0140	0.0083	0.0046	-0.0244	-0.0139	-0.0057
6	0.0169	0.0098	0.0052	-0.0280	-0.0167	-0.0064
	1st Autocorrelation Coefficient				Maximum	
0	0.9998	0.9689	0.9952	0.0020	0.0021	0.0013
1	0.9999	0.9027	0.9901	0.0044	0.0040	0.0025
2	0.9999	0.9846	0.9698	0.0070	0.0056	0.0035
3	0.9999	0.9959	0.9990	0.0098	0.0070	0.0045
4	0.9998	0.9979	0.9986	0.0130	0.0090	0.0057
5	0.9996	0.9978	0.9990	0.0163	0.0113	0.0069
6	0.3952	0.9967	0.9995	0.0199	0.0136	0.0082

Table A9: Forecast Errors for Trend Employment Rate (M*)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	0.0001	0.0000	0.0000	14	17	17
1	0.0002	0.0000	0.0000	14	17	17
2	0.0005	0.0000	-0.0001	14	17	17
3	0.0010	-0.0001	-0.0001	14	16	16
4	0.0016	-0.0002	-0.0002	14	15	15
5	0.0024	-0.0004	-0.0003	14	14	14
6	0.0027	-0.0007	-0.0005	13	13	13
		Standard Deviations			Minimum	
0	0.0012	0.0004	0.0006	-0.0021	-0.0007	-0.0010
1	0.0024	0.0009	0.0012	-0.0042	-0.0015	-0.0020
2	0.0036	0.0013	0.0018	-0.0061	-0.0023	-0.0030
3	0.0048	0.0018	0.0022	-0.0079	-0.0032	-0.0032
4	0.0059	0.0022	0.0028	-0.0095	-0.0041	-0.0041
5	0.0070	0.0027	0.0033	-0.0107	-0.0051	-0.0050
6	0.0079	0.0030	0.0038	-0.0117	-0.0060	-0.0059
	1st Autocorrelation Coefficient				Maximum	
0	0.9997	0.9956	0.9960	0.0019	0.0007	0.0012
1	0.9964	0.9099	0.9359	0.0039	0.0015	0.0024
2	0.9993	0.9980	0.9995	0.0061	0.0022	0.0036
3	0.9990	0.9984	0.9998	0.0084	0.0029	0.0048
4	0.9435	0.9984	0.9990	0.0109	0.0035	0.0058
5	0.9998	0.9993	0.9992	0.0136	0.0041	0.0066
6	1.0000	0.8253	0.8849	0.0165	0.0045	0.0072

Table A10: Forecast Errors for Trend Labour Productivity (Q*)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	0.0022	0.0012	0.0002	14	17	17
1	0.0049	0.0029	0.0008	14	17	17
2	0.0080	0.0049	0.0018	14	17	17
3	0.0114	0.0074	0.0029	14	16	16
4	0.0152	0.0100	0.0035	14	15	15
5	0.0197	0.0124	0.0035	13	14	14
6	0.0235	0.0147	0.0035	12	13	13
		Standard Deviations			Minimum	
0	0.0027	0.0043	0.0042	-0.0024	-0.0068	-0.0066
1	0.0053	0.0088	0.0085	-0.0044	-0.0132	-0.0129
2	0.0079	0.0133	0.0129	-0.0058	-0.0189	-0.0189
3	0.0104	0.0181	0.0177	-0.0067	-0.0236	-0.0240
4	0.0128	0.0230	0.0221	-0.0069	-0.0271	-0.0282
5	0.0155	0.0275	0.0255	-0.0065	-0.0292	-0.0311
6	0.0181	0.0317	0.0284	-0.0056	-0.0298	-0.0327
	1st Autocorrelation Coefficient				Maximum	
0	0.9998	0.9936	0.9810	0.0061	0.0070	0.0056
1	0.9999	0.9982	0.9941	0.0125	0.0145	0.0116
2	0.6414	0.9951	0.9777	0.0192	0.0223	0.0179
3	0.9961	0.9878	0.9801	0.0262	0.0304	0.0242
4	0.8377	0.8769	0.8753	0.0335	0.0386	0.0303
5	0.9982	0.8485	0.8593	0.0409	0.0467	0.0354
6	0.9999	0.8277	0.8970	0.0485	0.0560	0.0438

Table A11: Forecast Errors for Output Gap (G)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0045	-0.0030	-0.0028	14	17	17
1	-0.0075	-0.0039	-0.0045	14	17	17
2	-0.0087	-0.0003	-0.0021	14	17	17
3	-0.0111	0.0014	0.0005	14	16	16
4	-0.0128	0.0011	-0.0014	14	15	15
5	-0.0172	0.0003	-0.0050	13	14	14
6	-0.0198	-0.0021	-0.0073	12	13	13
		Standard Deviations			Minimum	
0	0.0082	0.0079	0.0072	-0.0263	-0.0234	-0.0162
1	0.0175	0.0180	0.0177	-0.0437	-0.0406	-0.0361
2	0.0220	0.0229	0.0281	-0.0539	-0.0496	-0.0556
3	0.0264	0.0275	0.0356	-0.0564	-0.0552	-0.0499
4	0.0279	0.0303	0.0373	-0.0575	-0.0676	-0.0604
5	0.0272	0.0292	0.0363	-0.0484	-0.0530	-0.0503
6	0.0254	0.0293	0.0369	-0.0504	-0.0447	-0.0522
	1st Autocorrelation Coefficient				Maximum	
0	0.762387	0.700625	0.740491	0.0075	0.0120	0.0108
1	0.780838	0.689139	0.746149	0.0197	0.0228	0.0293
2	0.788119	0.713062	0.754839	0.0288	0.0407	0.0776
3	0.674916	0.757825	0.788082	0.0298	0.0383	0.0752
4	0.16647	0.777003	0.770177	0.0281	0.0448	0.0731
5	-0.75867	0.593358	0.744652	0.0264	0.0468	0.0711
6	0.241171	0.5786	0.706828	0.0198	0.0341	0.0589

Table A12: Forecast Errors for Output Gap (G1)

Horizon	MFQ	CBCQC	CBCCA	MFQ	CBCQC	CBCCA
		Means			Number Observations	
0	-0.0032	-0.0021	-0.0026	14	17	17
1	-0.0056	-0.0031	-0.0043	14	17	17
2	-0.0065	0.0004	-0.0019	14	17	17
3	-0.0090	0.0023	0.0008	14	16	16
4	-0.0112	0.0023	-0.0008	14	15	15
5	-0.0160	0.0020	-0.0036	13	14	14
6	-0.0193	-0.0002	-0.0057	12	13	13
		Standard Deviations			Minimum	
0	0.0083	0.0078	0.0069	-0.0247	-0.0214	-0.0147
1	0.0172	0.0170	0.0166	-0.0416	-0.0371	-0.0334
2	0.0209	0.0205	0.0262	-0.0513	-0.0458	-0.0527
3	0.0246	0.0248	0.0332	-0.0505	-0.0489	-0.0462
4	0.0253	0.0278	0.0352	-0.0491	-0.0607	-0.0570
5	0.0244	0.0264	0.0339	-0.0421	-0.0464	-0.0476
6	0.0223	0.0257	0.0344	-0.0444	-0.0389	-0.0445
	1st Autocorrelation Coefficient				Maximum	
0	0.6883	0.6394	0.7139	0.0083	0.0128	0.0115
1	0.7492	0.5972	0.7090	0.0204	0.0234	0.0262
2	0.7723	0.6308	0.7363	0.0271	0.0353	0.0725
3	0.6815	0.6996	0.7761	0.0289	0.0308	0.0707
4	0.1929	0.7483	0.7546	0.0288	0.0424	0.0684
5	-0.6513	0.5204	0.7220	0.0223	0.0416	0.0659
6	0.3008	0.5197	0.7014	0.0170	0.0314	0.0553