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A Passion for Democracy

Elena Panova

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A Passion for Democracy*

Elena Panova†

Résumé / *Abstract*

Les théories de vote supposent qu'un électeur reçoit de la satisfaction ou du « warm glow » quand il vote. Le « warm glow » est créé par la confiance d'une bonne action envers ses compatriotes. Qu'est-ce qui fait croire à un électeur qu'il fait une bonne action envers ses compatriotes en votant pour telle ou telle plateforme politique? Leurs propres votes sont un signal naturellement disponible. Nous proposons un modèle dynamique de vote avec des informations asymétriques dans lequel la majorité fournit un signal positif sur le choix de vote. Ce signal augmente la confiance de l'électeur en ses choix de vote, et par conséquent, son « warm glow » de votes prochains. Les électeurs qui ne peuvent pas distinguer quelle plateforme politique est supérieure essaient d'imiter le vote de majorité afin de bâtir la confiance en ses choix de vote et s'impliquer dans le processus démocratique. Ils votent surtout selon les informations publiques disponibles, cependant, sans « herding ». Nous trouvons ces effets dans l'équilibre unique de notre jeu de vote.

Mots clés : vote expressif, vote habituel, complémentarités au vote, self-signaling, information publique et vote, tendance au statu qo.

Theories of voter turnout assume that an active voter receives a warm glow from doing a good deed to like-minded compatriots. What tells him that he is doing them a good deed by voting for this or that candidate or policy? Their own votes are naturally available feedback. We propose a dynamic model of voting with asymmetric information in which being among the majority provides a voter with a positive feedback on his voting decision, increasing his self confidence, hence, his warm glow from voting in the future. The voters who cannot tell which policy is superior, try to pool with a majority (so as to get involved in the democratic process). They vote much according to the available public information, however, without herding. We find these effects in the unique equilibrium of our voting game.

Keywords: *expressive voting, habitual voting, complementarities in voting, self-signaling, public information and the vote, status quo bias.*

***Codes JEL** : D03, D71, D72, D82, D83, P16*

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† CIRANO, CIRPÉE and Department of Economics, Université du Québec à Montréal, C. P. 8888, Succursale Centre-Ville, Montréal (Québec) H3C 3P8, Canada; e-mail: panova.elena@uqam.ca.

1 Introduction

Many collective decisions are made by simple majority vote. A sizable literature describes information aggregation properties of this procedure: if the voters maximize the probability that their most preferred candidate or policy wins, then the outcome of a large election with private information is such as if all voters shared their information (Feddersen and Pesendorfer, 1997).

However, if a voter's goal is to maximize the electoral fortunes of his most preferred alternative, he has weak incentives to vote, because the turnout is costly and his individual vote is most likely not decisive (Downs, 1957; Riker and Ordeshook, 1968; Ledyard, 1984; Palfrey and Rosenthal, 1983, 1985; Myerson, 1998). Still, many people vote.[1] A sizable empirical literature describes regular patterns of turnout and voting (Blais, 2000). Explaining these patterns is a major challenge for political theory.

The existing theories follow classic economic approach in assuming that a voter has rational preferences policy alternatives (see surveys by Aldrich 1993; Feddersen, 2004; Dhillon and Peralta, 2002; Merlo, 2006; Geys, 2006; and ours at the end of section 2). He votes so as to maximize the electoral fortunes of his most preferred alternative, if he votes in the first place. He is motivated to vote by either the sense of civic duty or a pleasure of expressing his policy preferences. Using Andreoni's terminology, he experiences a warm glow from the very act of voting.[2] Still, he may abstain due to either relatively high turnout cost (a common place) or relatively weak information (Feddersen and Pesendorfer, 1996).

Let us inquire into the warm glow effect. A voter experiences warm glow

[1] For example, the average turnout in the US from 1968 to 2008 is 55,58% in Presidential elections and 46.63% in Congressional elections (U.S. Census Bureau).

[2] James Andreoni introduced “warm-glow” for the pleasure from charitable giving. Most theories accommodate positive turnout by assuming a warm glow from the act of voting. An alternative idea is to focus a voter's attention on the situation in which he is pivotal. Ferejohn and Fiorina (1974) assume that a voter minimizes his regret would he fail to provide the decisive support to his most preferred candidate or policy. Regret-minimization objective re-appears in a voting game by Li and Majumdar (2010).

when he sees his vote as a good deed. Such normative evaluation, however, is a difficult task. What tells a voter that he is doing a good deed to his compatriots by voting for this or that candidate or policy? Their own votes are naturally available feedback.

We propose a dynamic model of voting with asymmetric information in which being among the majority provides a voter with a positive feedback on his voting decision, increasing his self confidence, hence, his warm glow from voting in the future. Our assumptions are as follows (we discuss them later). The voters with common values select public policy by simple majority vote. They vote twice. Each time, a stochastic state of Nature determines the superior policy. Only a minority of voters is informed about the state. A voter's warm glow is proportional to his confidence in supporting the superior policy. He knows whether he is informed or not during the first vote, but he forgets it by the time of the second vote. His posteriors, called self confidence, depend on his past voting behavior and the majority outcome.

If the voters would vote only once, the informed voters would vote for the superior policy and the uninformed voters would abstain. However, a dynamic aspect of our game moves this behavior out of equilibrium: Suppose the informed voters vote and the uninformed voters abstain. Then, each uninformed voter would like to deviate and vote in attempt to pool with the informed voters: if he succeeds, he builds a positive self confidence allowing him to receive the warm glow from voting in the future.

We find the unique equilibrium of our voting game in which the informed voters vote for the superior policy, and the uninformed voters try to imitate this behavior. Public information, if available, guides the uninformed voters. They vote much on public information, however, without herding. The better public information, the more coherent their votes, and the less informative the majority outcome. However, information aggregation remains nonnegative.

Empirical relevance Our equilibrium has relevant features. First, it accommodates so called "habitual voting": Both the instrumental variables analysis of the National Election Studies data by Green and Shachar (2000) and a randomized field experiment by Gerber, Green, and Shachar (2003) find that voting in one election causes nearly 50 percentage point increase in the propensity to vote in the next election.[3] Here, an active voter is likely to receive a positive feedback from his peers, which motivates him to vote again. An abstainer is guaranteed no such feedback, so he continues to abstain.

Second, our equilibrium is consistent with conformity in voting. Several studies from Bartels (1988) to Cloutier et al. (2010) describe electoral bandwagons. Coleman (2004) finds conformity in voting during elections in the US and Western Europe over most of the twentieth century, as well as during recent elections in Eastern Europe and Russia. Tyran (2004) finds conformity in voting in a laboratory experiment.[4] Our voter would like to conform with a majority, because it increases his confidence in voting.

Third, a high equilibrium turnout by poorly informed voters comports nicely with a sizable evidence of "voter ignorance": a low factual knowledge shown in polls (for example, about the distribution of the state budget);[5] biased beliefs regarding economic policies (Caplan, 2007);[6] disagreement as to which policies are appropriate (Bénabou, 2008). However, the uninformed voters have a weaker motivation to vote than the informed voters. This comports nicely with a positive effect of being informed on the propensity to vote:

[3] "Habitual voting" comports nicely with the cohort effect: Firebaugh and Chen (1995) find that "disenfranchisement had enduring pernicious effects on Nineteenth Amendment women but not on their postamendment daughters and granddaughters."

[4] The subjects vote upon charitable donation of their endowments under two treatments. In treatment one, a subject donates his endowment if and only if the proposal is accepted by a required quorum. In treatment two, he donates only if and only if both the proposal is accepted and he voted for it. Under both treatments, the subjects tend to support the donation if they expect the other subjects to support it.

[5] *The Fiscal Times*, "Voter Ignorance Threatens Deficit Reduction," February 4, 2011.

[6] Caplan describes four major biases: underestimation of the market efficiency (antimarket bias), underestimation of benefits from international trade (antiforeign bias), association of prosperity with employment rather than with production (make-work bias), pessimism about economic conditions (pessimistic bias).

Wolfinger and Rosenstone (1980) show that 4 years of schooling increase the propensity to vote by 4 to 13 percentage points (presumably, more educated voters have better information). Lassen (2005) finds the causal effects of being informed on the turnout.

Finally, the effect of public information on equilibrium voting behavior seems to be relevant: There is a growing evidence that political news is influential (Della Vigna and Kaplan, 2007; Gerber, Karlan, and Bergan, 2006; Enikolopov, Petrova, Zhuravskaya, 2011).[7] In a laboratory experiment by Ladha (1995), the subjects playing the role of committee members rely much on public signals. A possible reason for the observed policy persistence[8] is that the status quo is seen as a signal on the appropriate public policy (a majority has selected the appropriate policy yesterday, and this policy is likely to remain appropriate today).

Roadmap The paper is organized as follows: Section 2 reviews related literature. Section 3 presents the basic voting game. Section 4 describes its unique equilibrium. Section 5 presents comparative static analysis with respect to the precision of public information. Section 6 presents a natural extension of the basic game to an overlapping generation game with an infinite horizon which accommodates policy persistence. Section 7 outlines three main directions for the future research. Technical proofs are in the Appendix.

[7] Della Vigna and Kaplan (2006) find that Republicans gained votes in US towns which introduced Conservative Fox News Channel between October 1996 and November 2000. In the randomized field experiment by Gerber, Karlan, and Bergan (2006) subscription for a new press outlet increased the probability of voting Democratic in 2005 Virginia gubernatorial election. Enikolopov, Petrova and Zhuravskaya (2011) find that during 1999 parliamentary elections in Russia exposure to news from the only independent TV channel decreased the aggregate vote for the government party and increased the combined vote for major opposition parties.

[8] For examples of inefficient policy persistence, see Coate and Morris (1999), Fernandez and Rodrick (1991). For a survey of relevant theories, see Mitchell and Moro (2006).

2 Related literature

Modelling Approach Our approach builds on series of behavioral models by Bénabou and Tirole. Recall, our key assumption is that the players remember their actions but they forget their types. Such imperfection of memory is proposed by Bénabou and Tirole (2002) to model behavioral effects of cognitive dissonance - one of the most prominent ideas in psychology (see Harmon-Jones et al., 2009 for a survey and an evidence of functional or action-based motivation behind dissonance processes; and recent randomized field experiments by Mullainathan and Washington, 2007 and by Gerber, Huber and Washington, 2009 for an evidence of cognitive dissonance in voting).

Bénabou and Tirole (2006) and Bénabou (2008, 2009) analyze large games in which the players simultaneously manipulate the extend to which they remember the initial information about the underlying state of the world. Their cognitive strategies are complementary. The reason is that they affect their tomorrow's actions which exhibit positive spillovers. The players commonly sustain either more realistic or more illusory beliefs, and they act accordingly. Our game is similar in that the voters commonly influence their tomorrow's beliefs through their today's behavior. However, there are two major differences. First, there is no direct manipulation of memory. A voter forgets his initial information, and he receives two recalling signals, namely, his yesterday's behavior and the majority outcome. Second, there are no direct spillovers. A voter's warm glow is equal to his confidence in his voting decision. He would like to vote in the same way as a majority in order to increase his self-confidence.

Classic theories of the vote Classic theories of the vote assume that a voter has rational policy preferences. Let us divide these theories into two groups by their assumption regarding voter uncertainty about policy alternatives.

The first group of theories assumes that a voter knows his most preferred alternative, and supports it if he votes. These theories focus on the turnout and outcomes. In early models the turnout decision results from a simple comparison of individual warm glow benefit with the turnout cost (the calculus of voting). The warm glow effect is associated either with fulfillment of a civic duty (Riker and Ordeshook, 1968) or with a pleasure of expressing policy preferences through the vote (Brennan and Buchanan, 1984).[9]

Later contributions model voter interaction. Pivotal-voter models emphasize small pivot probabilities in large elections, hence, the essence of the warm glow for participation (Riker and Ordeshook, 1968; Ledyard, 1984; Palfrey and Rosenthal, 1983, 1985; Myerson, 1998). Group-based theories divide the electorate into a finite number of groups with private interests. In Uhlaner (1989), Shachar and Nalebuff (1999), and Morton (1987, 1991) group leaders mobilize voters in their groups. Voter turnout depends on mobilization cost. In group-utilitarian models by Harsanyi (1977), Feddersen and Sandroni (2006), and Coate and Conlin (2004) a voter projects his behavior on the other voters like him. He follows behavioral rule which is optimal for his group if the voters like him follow the same rule. The rule prescribes the voters with sufficiently low turnout costs to vote for the group's preferred candidate, and it allows the voters with a higher turnout cost to abstain. The rule is more demanding, the more similar the groups' sizes, which comports nicely with higher turnout in closer elections.

The second group of theories assumes that a voter is uncertain as to which alternative is the best. In Matsusaka (1995), Degan (2006), and Degan and Merlo (2011) a voter chooses his behavior regardless of the other voters. If he votes, he supports the alternative which is most likely the best. He votes if he is sufficiently confident in his voting choice.[10] He may increase his confidence

[9]These seem to be relevant aspects of voter motivation. For recent statistical analysis of poll data see Carlsson and Johansson-Stenman (2010). For concrete examples, read voter reports on their motivation during the last three US Presidential elections on http://freakonomics.blogs.nytimes.com: "*I always pick up my dog's poop,*" reminiscent of Riker and Ordeshook; "*I enjoy reading about policy and politics and voting is my way of picking a team,*" reminiscent if Brennan and Buchanan.

[10]Voter confidence either increases the warm glow from voting (Matsusaka, 1995) or

through costly information acquisition (Matsusaka, 1995; Degan, 2006).

In Feddersen and Pesendorfer (1996) a voter's behavior depends not on his confidence *per se*, but on informational asymmetries between him and the other voters. He conditions his behavior on the situation in which he is pivotal, reminiscent of a bidder in a common value auction. The uninformed voter tend to abstain, so as not to jam the votes by the informed voters. More precisely, the uninformed voters participate just enough to eliminate ideological bias created by partizan voters.

Our work is complementary to these theories: They assume the warm glow from participation. We model the warm glow effect and describe the voting behavior accordingly. Our approach allows us to accommodate "irrational" voting behavior described at the end of section 1. Between the above two groups of theories, our work is more related to the second group through its emphasis on the importance of voter information or confidence for participation.[11] The first group of theories devotes increasing attention to private values, while we do not address in this paper.

Empirical links Empirically relevant features of our equilibrium relate us to three economic literatures: Namely, "habitual voting" relates us to adaptive models by Bendor, Diermeier, and Ting (2003) and Fowler (2006). They assume that voting today affects the future propensity to vote according to a given rule. A voter's turnout stochastically depends on his propensity

it decreases the turnout cost (Degan, 2006; Degan and Merlo, 2011). Degan (2006) and Degan and Merlo (2011) consider unidimensional policy space. A voter knows the location of his most preferred policy, but he is uncertain about the locations of competing policy platforms. An active voter receives some warm glow (he fulfills his civic duty). His cost of voting is equal to the probability that he supports the alternative which is not the closest to his "bliss point". The voters at the extremes of ideological spectrum are more confident in their choices than centrally located voters. Therefore participation among the extreme voters is relatively high.

[11] "*In 2004 I voted because I strongly supported one of the candidates...I did not vote in 2008 because I did not like either candidate.*" (http://freakonomics.blogs.nytimes.com).

to vote. Naturally, the insights are sensitive to the choice of the rule.[12] We model turnout and voting decisions without assuming functional dependencies.

Conformity in voting relates us to models by Callander (2008), Rotemberg (2009), and Shuessler (2000), mainly to Callander's work.[13] He assumes direct benefit from being on the winners' side, which creates the social multiplier effect, hence, the multiplicity of equilibria.[14] In some of these equilibria information aggregation is negative. In our game, complementarities in voting are endogenous, and information aggregation is nonnegative.

The effect of public signal on information aggregation relates us to the literature on the social value of public information pioneered by Morris and Shin (1992). They show that public information has an ambiguous effect on the welfare when there are strategic complementarities in players' actions. Here, recall, complementarities are endogenous.

3 Basic model

The voters with common values select public policy by a simple majority rule. There are two successive votes, indexed with $t = 1, 2$.[15]

Policy alternatives There are two alternative policies: "0" and "1". The efficient policy is equal to the state variable x_t which is drawn before

[12] Fowler (2006) proposes reinforcement rule which has a higher empirical relevance than the Bush-Mosteller rule used by Bendor, Diermeier and Ting (2003).

[13] Shuessler (2000) and Rotemberg (2009) assume complementarities in voting: In Shuessler's model, voting is a way to identify yourself with a group of people voting in the same way. The identification benefit is a $\cap$-shape function of the group's size. In Rotemberg's model, a voter votes in order to let the like-minded voters know that he shares their policy preferences, because they are happy to know it and he cares for them.

[14] Scheinkman (2008) and Postlewaite (2010) overview a sizable literature on social multiplier effect.

[15] Timing of the events is summarized at the end of this section.

each vote from the diffuse Bernoulli distribution:

$$\Pr(x_t = j) = \tfrac{1}{2},\ j = 0, 1. \tag{1}$$

For now, we assume that states x_1 and x_2 are not correlated.[16] Policy winning vote t is denoted with a_t.

Voter types and signals There is a continuum of voters with a mass of unity, indexed by $i \in [0, 1]$. At the start of the game, Nature draws type θ^i by voter i: *informed* ($\theta^i = 1$), with probability α; or *uninformed* ($\theta^i = 0$), with probability $1 - \alpha$. Most voters are uninformed, that is, $\alpha < \frac{1}{2}$.[17]

Before vote t, voter i receives private signal σ_t^i on the state x_t. If he is informed, his signal is perfect; if he is uninformed, his signal is diffuse:

$$\sigma_t^i = \theta^i x_t + (1 - \theta^i) z_t^i, \tag{2}$$

where variable z_t^i is an independent draw from distribution (1).

Voter information and strategies during vote 1 Before vote 1, the voters receive public signal σ of quality q on the state x_1:[18]

$$\Pr(x_1 = 0 \mid \sigma = 0) = \Pr(x_1 = 1 \mid \sigma = 1) = q \geqslant \tfrac{1}{2}. \tag{3}$$

Hence, information set by voter i is

$$\Omega_1^i = \left\{\theta^i, \sigma, \sigma_1^i\right\}. \tag{4}$$

Given information (4), voter i can take one of the following actions: (i) vote for policy "0" ($v_1^i = 0$); (ii) vote for policy "1" ($v_1^i = 1$); (iii) abstain from voting ($v_1^i = \varnothing$). Hence, his pure strategy is mapping

$$v_1(\theta^i, \sigma, \sigma_1^i) : \ \{0, 1\}^3 \to \{\varnothing, 0, 1\}. \tag{5}$$

[16] Section 7 extends the game to an infinite number of elections with correlated states.

[17] Recall the evidence of voter ignorance cited in the Introduction.

[18] We assume that the voters receive public signal only before vote 1. Our insights are qualitatively robust if the voters receive public signal before each vote. However, the uninformed voters have weaker incentives to vote.

Voter information and strategies during vote 2 Voting behavior v_1^i stays in the memory by voter i, but not his type θ^i or signals σ and σ_1^i.[19] Everyone can see public policy a_1 chosen by a majority. However, it remains unclear to anyone whether it is efficient or not, as state x_1 remains hidden. Hence, information set by voter i during vote 2 is

$$\Omega_2^i = \left\{v_1^i, a_1, \sigma_2^i\right\}, \tag{6}$$

and his pure voting strategy is mapping

$$v_2(v_1^i, a_1, \sigma_2^i): \ \{\varnothing, 0, 1\} \times \{0, 1\}^2 \to \{\varnothing, 0, 1\}. \tag{7}$$

Posteriors $\Pr\left(\theta^i = 1 \mid \Omega_2^i\right)$ by voter i are called *self confidence*.

Voter objectives Following classic theories of the vote, we assume that an active voter receives a warm glow from participation. He experiences the warm glow when he votes his private signal (thereby, he expresses his deep-seated opinion).[20] The warm glow is equal to the subjective probability of supporting the efficient policy less that of supporting the inefficient policy, akin to Matsusaka (1995).[21] Given the large size of the electorate, we isolate "instrumental" objectives: a voter's payoff is equal to the warm glow less the turnout cost.[22] For now, we assume that the turnout cost is arbitrary small, taking it null for notational convenience.[23] Hence, date t payoff by voter i is

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[19] Recall references to the literature on cognitive dissonance in section 2.

[20] This assumption is made to iolate herding on public signal.

[21] We model action-based motivation behind cognitive dissonance processes, building on Harmon-Jones et al. (2009). We have considered some alternative assumptions about voter objectives. We found similar insights assuming that a voter's warm glow is equal to his satisfaction with his vote $\Pr\left(v_1^i = x_1 \mid \Omega_2^i\right)$. We found multiple equilibria if a voter's warm glow is equal to his self confidence, depending on the strategy played by the informed voters.

[22] Instrumental objectives, if introduced, influence the voting behavior if and only if they have lexicographic superiority. The game with such objectives has the equilibrium described below. However, it is not unique.

[23] Section 5 introduces a higher turnout.

equal to

$$U(v_t^i, \Omega_t^i) = \begin{cases} \Pr\left(v_t^i = x_t \mid \Omega_t^i\right) - \Pr\left(v_t^i = 1 - x_t \mid \Omega_t^i\right) \text{ if } v_t^i = \sigma_t^i; \\ 0, \text{ otherwise.} \end{cases} \tag{8}$$

Sequence of events

Nature draws voters' types. The voters learn their types.

Date 1.

a. Nature draws: state x_1, public signal σ and private signals σ_1^i. The voters receive their signals.

b. Vote 1 takes place.

The voters forget their types and signals.

Date 2.

a. Nature draws state x_2 and private signals σ_2^i. The voters receive their signals.

b. Vote 2 takes place.

4 Equilibrium of the game

This section describes the unique symmetric[24] Perfect Bayesian Equilibrium of the game. Note first of all, that the warm glow experienced by an active voter is equal to his confidence in his voting choice: by equations (2) and (8),

$$U(v_t^i, \Omega_t^i) = \Pr\left(\theta^i = 1 \mid \Omega_t^i\right). \tag{9}$$

Now, consider the votes in the reversed order. During vote 2, voter i maximizes his immediate warm glow. He votes his signal if his self confidence is positive, and he abstains from voting otherwise. Formally, by equation (9),

$$v_2^i = \sigma_2^i \text{ if } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0;\; v_2^i = \varnothing \text{ if } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) = 0. \tag{10}$$

[24] The agents of the same type with the same signals play the same strategy. We focus on symmetric equilibria following Mayerson's argument that identity of every voter can hardly be assumed a common knowledge.

Self confidence by voter i depends on two signals retained from vote 1: his voting behavior v_1^i and the majority outcome a_1.[25] Indeed, by Bayes rule,

$$\Pr\left(\theta^i = 1 \mid \Omega_2^i\right) = \frac{\alpha \Pr\left(v_1^i, a_1 | \theta^i = 1\right)}{\alpha \Pr\left(v_1^i, a_1 | \theta^i = 1\right) + (1-\alpha) \Pr\left(v_1^i, a_1 | \theta^i = 0\right)}. \tag{11}$$

If voter i is informed, his self confidence is positive (trivially, he voted as an informed voter yesterday). Therefore, he votes his signal:

$$\text{if } \theta^i = 1, \text{ then } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0 \text{ and } v_2^i = \sigma_2^i. \tag{12}$$

Hence, the informed voters increase the vote margin for the efficient policy by α. The votes by the uninformed voters, if any cast, "cancel out" because their signals have no systematic component. The efficient policy wins:

$$a_2 = x_2. \tag{13}$$

Now, consider vote 1. A voter maximizes his intertemporal warm glow from voting (today and tomorrow). His today's voting behavior affects his self confidence, and thereby, his tomorrow's warm glow. Without accounting for this effect, the informed voters would vote their signals, and the uninformed voters would abstain from voting. However, if all voters behave in this way, an uninformed voter would like to deviate and vote, no matter how: with probability $\frac{1}{2}$ he pools with the informed voters today, wins thereby perfect self confidence, hence, the highest warm glow tomorrow. More generally, under full separation of types an uninformed voter is tempted to imitate behavior by the informed voters. Therefore, this situation is out of equilibrium. In equilibrium, there must be some pooling of types:[26]

$$\operatorname{Im}(v_1(1, \sigma, \sigma_1^i)) \cap \operatorname{Im}(v_1(0, \sigma, \sigma_1^i)) \neq \varnothing. \tag{14}$$

This insight has two implications: First, the informed voters vote their signals:

$$v_1(1, \sigma, \sigma_1^i) = \sigma. \tag{15}$$

[25] His new signal σ_2^i is irrelevant because the states x_1 and x_2 are independent.

[26] We use standart notation $\operatorname{Im}(v_1(\theta^i, \sigma_1, \sigma_1^i)) = \left\{v_1^i \mid v_1^i = v_1(\theta^i, \sigma_1, \sigma_1^i)\right\}$.

Thereby, they immediately benefit from the highest warm glow. Second, the uninformed voters participate in voting, at least to some extend: otherwise, they would separate from the informed voters.

Notation 1 (the uninformed voters' strategy):

$$v_1(0, \sigma, \sigma_1^i) = \begin{cases} \sigma, \text{ with probability } p_\sigma; \\ 1-\sigma, \text{ with probability } p_{1-\sigma}; \\ \varnothing, \text{ with probability } 1 - p_\sigma - p_{1-\sigma}. \end{cases} \tag{16}$$

The uninformed votes may be pivotal only if their votes are sufficiently coherent, hence, biased towards one of the policies. Trivially, they cannot be biased towards different policies at once. Therefore, the majority outcome is efficient at least for one realization of the state variable (information aggregation is nonnegative):

$$a_1 = j \text{ in state } x_1 = j \text{ for at least one } j \text{ in set } \{0, 1\}. \tag{17}$$

The following sections describe three possible outcomes of vote 1, called informative, uninformative, and semiinformative.

Informative equilibrium Consider the first possibility: the outcome of vote 1 is efficient:

$$a_1 = x_1. \tag{18}$$

Consider optimization by an uninformed voter, without loss of generality, voter i. If he abstains today ($v_1^i = \varnothing$), then his immediate payoff is null; his self confidence remains null:

$$\Pr\left(\theta^i = 1 \mid v_1^i = \varnothing, a_1\right) = 0, \tag{19}$$

and he abstains tomorrow once again ($v_2^i = \varnothing$). If he votes today, no matter how, he pays an arbitrary small turnout cost without receiving any warm glow immediately. However, with probability $\frac{1}{2}$ he pools with the majority. Thereby, he wins self confidence equal to

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = j\right) = \tfrac{\alpha}{\alpha + (1-\alpha)p_j}, \tag{20}$$

which is his tomorrow's warm glow from voting. Hence, the uninformed voters go vote:[27]

$$p_\sigma + p_{1-\sigma} = 1. \tag{21}$$

How do they vote? They must randomize between voting for different policies: if they all vote for the same policy, this policy wins which is generically inefficient (equation (18) is false in one of the states). Hence, they must be indifferent between voting for different policies. This is true if and only if their expected self confidence does not depend on the way in which they vote:

$$\tfrac{q\alpha}{\alpha+(1-\alpha)p_\sigma} = \tfrac{(1-q)\alpha}{\alpha+(1-\alpha)p_{1-\sigma}}. \tag{22}$$

By equations (21) and (22), the voting probabilities are:

$$p_\sigma = q + \tfrac{\alpha}{1-\alpha}(2q-1) \text{ and } p_{1-\sigma} = 1 - q - \tfrac{\alpha}{1-\alpha}(2q-1). \tag{23}$$

Notably, the uninformed voters are inclined to vote more according to the public signal than against it:

$$p_\sigma - p_{1-\sigma} = (2q-1)\,\tfrac{1+\alpha}{1-\alpha} > 0,$$

and this difference is increasing in the signal's precision. If the public signal is correct ($\sigma = x_1$), the uninformed voters only increase the margin for victory of the efficient policy. However, if the public signal is false ($\sigma = 1 - x_1$), they create some support for the inefficient policy. The outcome remains efficient if and only if the public signal is sufficiently weak:

$$q \leqslant \tfrac{2\alpha+1}{2(1+\alpha)}. \tag{24}$$

Proposition 1 *The game has the following equilibrium, call it informative. During vote 1, the informed voters vote their signals. The uninformed voters play voting strategy described by set of equations (23). The efficient policy wins, as described by equation (18). During vote* 2*, the winners of vote* 1 *vote*

[27] Naturally, sufficiently high turnout cost indices abstention: see section 5.

their signals; the losers abstain. The efficient policy wins, once again. The informative equilibrium exists if and only if the public signal is sufficiently weak, as described by inequality (24).

Uninformative equilibrium Consider the second possibility. The outcome of vote 1 coincides with the public signal:

$$a_1 = \sigma \text{ for any } x_1. \tag{25}$$

The analysis is much similar to the above. The uninformed voters turn out to vote, as described by equation (21), because their self confidence is null if they abstains and it is positive otherwise:

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma\right) = \tfrac{\alpha q}{\alpha q + (1-\alpha) p_\sigma}; \tag{26}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma\right) = \tfrac{\alpha (1-q)}{\alpha (1-q) + (1-\alpha) p_{1-\sigma}}. \tag{27}$$

They play a mixed voting strategy.[28] Hence, different pure strategies expected self confidence:

$$\tfrac{\alpha q}{\alpha q + (1-\alpha) p_\sigma} = \tfrac{\alpha (1-q)}{\alpha (1-q) + (1-\alpha) p_{1-\sigma}}. \tag{28}$$

By equations (21) and (28), the uninformed voters vote according to the public signal with probability which is equal to the signal's quality:

$$p_\sigma = q \text{ and } p_{1-\sigma} = 1 - q. \tag{29}$$

They choose policy σ in any state, as described by equation (25) if and only if the signal is sufficiently strong, namely,

$$q \geqslant \tfrac{1}{2(1-\alpha)}. \tag{30}$$

[28] Equation (25) accommodates only one pure strategy: vote according to the public signal; however, if all uninformed voters vote in this way each of them would like to differentiate.

Proposition 2 *The game has the following equilibrium, call it uninformative. During vote* 1*, the informed voters vote their signals. The uninformed voters vote on the public signal with probability equal to the signal's quality, as described by set of equations (29). The majority outcome coincides with the public signal. During vote* 2*, the voters vote their signals, and the outcome is efficient. The uninformative equilibrium exists if and only if the public signal is sufficiently strong, as described by inequality (30).*

Semi-informative equilibrium Informativeness constraint (17) leaves three more possibilities for the outcome of vote 1. We consider them one by one.

(i) The majority outcome is not the public signal in any state:

$$a_1 = 1 - \sigma \text{ for any } x_1. \tag{31}$$

Such outcome is deterministic, hence, "uninformative". The corresponding posteriors and behavior by the uninformed voters are described by equations (26), (27) and (29) Then, however, outcome a_1 is equal to the public signal, at least when the signal is true. Hence, outcome (31) cannot be sustained in equilibrium.

(ii) The majority outcome is efficient if the public signal is false, and it is uncertain otherwise:

$$\text{if } \sigma = 1 - x_1 \text{ then } a_1 = x_1;\ \Pr(a_1 = x_1 \mid \sigma = x_1) < 1. \tag{32}$$

Such situation may realize only if the uninformed voters tend to vote against the public signal ($p_{1-\sigma} > p_\sigma$). Then, however, each of them would like to deviate and vote on the public signal, so as to increase his expected self confidence. Hence, outcome (32) cannot be sustained in equilibrium either.

(iii) The only remaining possibility is that the majority outcome is efficient if the public signal is true, and it is uncertain otherwise:

$$\text{if } \sigma = x_1 \text{ then } a_1 = x_1;\ \Pr(a_1 = x_1 \mid \sigma = 1 - x_1) < 1. \tag{33}$$

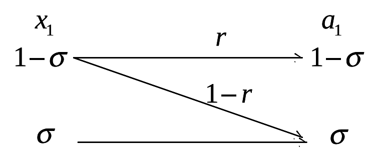

Figure 1: the state and the outcome.

as illustrated on figure 1. The uncertainly is due to a close-tie vote:

$$\alpha = (1-\alpha)\left(p_{\sigma} - p_{1-\sigma}\right). \tag{34}$$

Notation 2 (tie-breaking rule):

$$\Pr\left(a_1 = 1-\sigma \mid x_1 = 1-\sigma\right) = \ r, \ \textit{where } 0 < r < 1. \tag{35}$$

Parameter r measures the informativeness of the majority outcome a_1.

To create a tie, the uninformed voters must play strategy

$$p_{\sigma} = \tfrac{1}{2(1-\alpha)}, \ \ p_{1-\sigma} = \tfrac{1-2\alpha}{2(1-\alpha)}. \tag{36}$$

The appropriately chosen tie-breaking rule

$$r(q) = \frac{1-2q+2\alpha q\left(1+\alpha q-\sqrt{(1+\alpha q)^2-2q}\right)}{(1-q)(1-2q(1-\alpha))} \tag{37}$$

keeps them indifferent between voting for different policies. The stronger the public signal, the easier it is win by voting on the signal and to loose by voting against it. A nosier outcome:

$$\tfrac{dr(q)}{dq} < 0 \tag{38}$$

guarantees that self confidence acquired by winning on the side of the signal decreases, and that by loosing on the opposite side increases, so that the above indifference is preserved.

Proposition 3 *The game has the following equilibrium, call it semiinformative. During vote* 1*, the informed voters vote their signals. The uninformed* *voters play voting strategy described by equations (36). The majority outcome is decreasingly informative in the precision of public signal, as described by inequality (38). During vote* 2*, a voter votes his private signal unless he previously voted on the public signal and lost. The efficient policy wins. Semi-informative equilibrium exist if and only if the public signal is stronger than described by inequality (24), but weaker than described by inequality (30).*

Note that the conditions on parameter q for propositions 1 to 3 are mutually exclusive. At the same time, they completely cover the parameter space.

Corollary *Propositions* 1 *to* 3 *describe the unique equilibrium of the game, depending on the precision of the public signal.*

5 Comparative statics

This section presents comparative static analysis with respect to the precision of public signal, which is measured by parameter q. We partition the parameter space into three intervals, as illustrated on Figure 2: in the lower interval (24) the equilibrium is informative; in the upper interval (30) the equilibrium is uninformative; in the interim interval the equilibrium is semiinformative.

We first analyze information aggregation. By equation (18), it is perfect in the lower interval. By inequality (38), it is decreasing down to null in the interim interval. By equation (25), it is null in the upper interval.

Information aggregation *Information aggregation decreases (nonstrictly) in the precision of the public signal, as illustrated on Figure* 2*-b.*

The intuition behind this insight is transparent: The stronger the public signal, the more uniformed voters vote on it (see Figure 2-a). They introduce noise in the majority outcome when the signal is both sufficiently strong and false.

Now, consider the instrumental efficiency of the outcome. In the informative equilibrium the outcome is efficient, as described by equation (18).

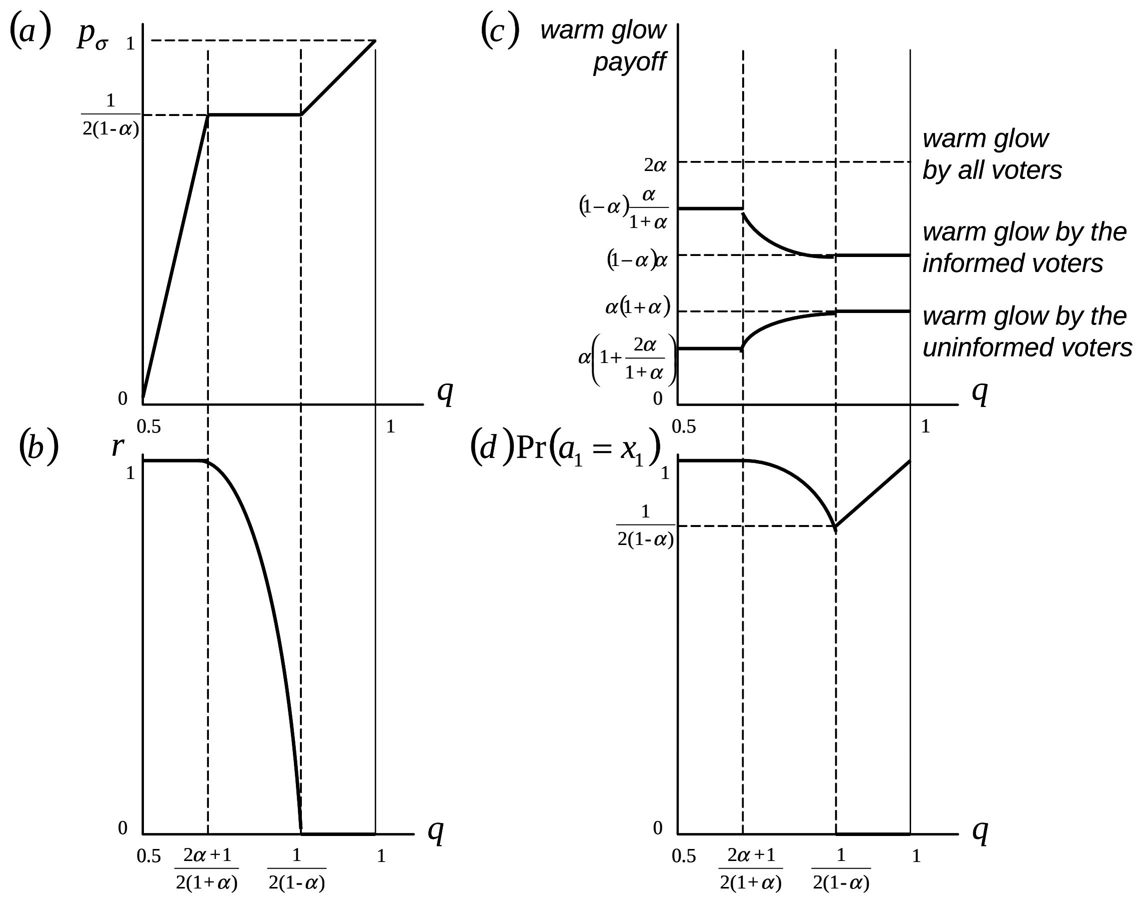

Figure 2: comparative statics. (a) Vote on public signal; (b) Information aggregation; (c) Welfare; (d) Instrumental efficiency.

In the semiinformative equilibrium, the outcome is efficient with probability $q + (1 - q)r$: it is efficient for sure when the public signal is true, and with probability r otherwise. Reinforcement of the public signal creates two controversial effects. On the one hand, the outcome becomes less likely to be efficient if the public signal is false (recall, r decreases in q). On the other hand, such situation becomes less likely. The former effect, however, is stronger, hence, the efficiency of the outcome decreases: $\frac{d}{dq}(q + (1 - q)r) < 0$. In the uninformative equilibrium, only the latter effect is present: the outcome is efficient if and only if the public signal is correct which is more likely, the stronger the signal.

Instrumental efficiency *The expected efficiency of the majority outcome is* $^{-}\cup$ *shaped in the precision of public signal, as depicted on Figure* 2*-d.*

In our game, a voter cares not for the instrumental efficiency which he cannot affect anyway, but for his warm glow from participation. Therefore, the welfare is equal to the warm glow experiences by all voters. The warm glow experienced by each type of voters depends on their self-confidence. Consequently, it depends on separation of types: the informed voters benefit from a clearer separation, the uninformed voters loose. In the informative equilibrium, the informed voters separate from the uninformed losers. In the semiinformative equilibrium, this separation decreases in the precision of public signal. In the uninformative equilibrium, different types pool completely.

Welfare *The expected warm glow by the informed voters decreases (non-strictly) in the precision of the public signal. The opposite is true for expected warm glow by the uninformed voters. Commonly experienced warm glow is twice proportional to the mass of the informed voters, no matter how precise the public signal. These patterns are depicted on Figure* 2*-c.*

Voter turnout and vote margin Consider vote 1. The informed voters have stronger incentives to participate than the uninformed voters.

However, this difference is not reflected in the turnout: all the uninformed voters pay an infinitely small turnout cost for a chance to build a positive self confidence and enjoy voting in the future. Suppose now that the turnout cost is sufficiently high to prevent some uninformed voters from participation.

Notation 3. *Denote the turnout cost with ψ.*

Suppose that it lies in interval

$$\tfrac{1}{4} < \psi < \tfrac{1}{3}. \tag{39}$$

The left inequality guarantees that participation by uninformed voters is sufficiently low so that they introduce no noise in the outcome, that is, the informed equilibrium is sustained for any q. The right inequality guarantees that their participation is positive, no matter how weak the public signal.[29]

Let us describe the turnout by the uninformed voters (the informed voters participate uniformly). When the public signal is sufficiently weak, namely,

$$q < \tfrac{1-2\psi}{1-\psi}, \tag{40}$$

the uninformed voters randomize among three feasible voting behaviors. They vote according to the public signal with probability

$$p_\sigma = \tfrac{\alpha}{1-\alpha} \tfrac{q-\psi(1+q)}{\psi(1+q)}, \tag{41}$$

they vote against the signal with probability

$$p_{1-\sigma} = \tfrac{\alpha}{1-\alpha} \tfrac{1-q-\psi(2-q)}{\psi(2-q)}, \tag{42}$$

and they abstain with the complementary probability. The stronger the public signal, the weaker their incentives to vote against it: $\frac{dp_{1-\sigma}}{dq} < 0$, and the stronger their incentives to vote on it: $\frac{dp_\sigma}{dq} < 0$. However, when too

[29] When ψ lies above threshold $\frac{1}{3}$, the uninformed voters abstain if $q < \frac{\psi}{1-\psi}$. Otherwise, their support for policy "σ" is described by the least of 1 and the right-hand-side of equation (41), and their support to policy "$1-\sigma$" is described by the most of 0 and the right-hand-side of equation (42). The difference in these voting probabilities remains below thershold $\frac{\alpha}{1-\alpha}$ as q approaches 1 if and only if ψ lies below threshold $\frac{1}{4}$.

many uninformed voters vote for the same policy, the expected self confidence from voting on the side of this policy is not sufficiently high to cover the future turnout cost. Therefore, increasingly many uninformed voters abstain: $\frac{d(p_{1-\sigma}+p_\sigma)}{dq} < 0$.

When the public signal is stronger than described by inequality (40), the uninformed voters do not vote against the signal: $p_{1-\sigma} = 0$. They vote on the signal with probability p_σ given by equation (41) and they abstain with the complementary probability. Thus, their turnout increases in q: $\frac{d(p_{1-\sigma}+p_\sigma)}{dq} = \frac{dp_\sigma}{dq} > 0$.

Voter turnout *Voter turnout is $\cup$-shaped in the precision of the public signal for the turnout cost in interval (39): decreasing if the public signal is sufficiently weak as described by inequality (40); and increasing otherwise.*

The margin of victory increases in the quality of public signal when the signal is correct, and the opposite is true when the signal is false. However, the signal is likely to be correct. Therefore, the expected margin of victory increases in the precision of public signal.

Margin of victory *The expected margin of victory increases in the precision of the public signal for the turnout cost in interval (39).*

A sizable empirical literature describes systematic variations in voter turnout. One established correlation is higher turnout in "closer" races (Blais, 2000).[30] We accommodate this correlation by relating each the voter turnout and the vote margins to the precision of public information: increasingly strong public information in favour of one alternative decreases the turnout and extends the margins. The sufficient conditions are: moderate

[30] This correlation is found to be weak but significant by numerous studies: 10 percentage point increase in the vote margin is associated with no more than 2 percentage points decrease in the turnout (Blais, 2006). For a dissenting view see, for example, Ashworth, Geys and Heyndels, (2006). They find non-monotonic relationship between the vote margin and the turnout.

turnout cost and “not too strong” public signal, as described by inequalities (39) and (40).

6 Policy persistence

Proposition 3 shows that sufficiently strong public signal in favor of one political alternative brings a majority of voters on its side. A natural extension of our basic model shows that this effect may explain the observed policy persistence or status quo bias in majoritarian politics.

Consider an overlapping generation game with an infinite horizon. Each generation lives for two periods and plays the basic game. For simplicity, the voters receive no exogenous public information ($q = \frac{1}{2}$). However, they observe the history of the majority outcomes. The state variable x_t follows Markov process:

$$\Pr(x_0 = 0) = \Pr(x_0 = 1) = \tfrac{1}{2}; \tag{43}$$

$$\Pr(x_{t+1} = 0 \mid x_t = 0) = \Pr(x_{t+1} = 1 \mid x_t = 1) = \tau \geqslant \tfrac{1}{2}, \tag{44}$$

where parameter τ measures the persistence of the appropriate public policy.

Consider vote 1. The informed voters of the first generation vote their signals. The uninformed voters vote for each policy with probability $\frac{1}{2}$. The outcome is efficient ($a_1 = x_1$). Note that it signals the future state x_2:

$$\Pr(x_2 = 0 \mid a_1 = 0) = \Pr(a_1 = 1 \mid x_2 = 1) = \tau. \tag{45}$$

Consider vote 2. The old winners vote their signals; the old losers abstain, as described by proposition 1. Altogether, they create margin α for the efficient policy x_2. Behavior by the young voters depends on their beliefs regarding the outcome. Suppose, they believe that there will be no reform anyway. Then, their expected self confidence is given by equations (26)-(27) with q being replaced for τ. Making the same replacement in equations (29), we find that the informed young voters (mass α) vote their signals; while the uninformed young voters (mass $1 - \alpha$) vote for status quo with probability τ

and for the reform with probability $1-\tau$. The status quo is indeed maintained in any state x_2 if and only if

$$(2\tau - 1)(1-\alpha) > 2\alpha.$$

Consider vote t. Suppose no reform took place since the start of the game, regardless of variations in the underlying state. The status quo still signals the appropriate policy:

$$\Pr(x_t = a_1 \mid a_1) - \Pr(x_t = 1 - a_1 \mid a_1) = (2\tau - 1)^t.$$

Suppose that the voters still believe that the status quo will be maintained in any state. Then, the uninformed young voters increase the vote margin for the status quo by $(2\tau - 1)^t (1-\alpha)$. The uninformed old voters do not affect the vote margin. The informed voters (old and young) increase the vote margin for the efficient policy by 2α. As a result, the status quo is maintained if and only if

$$(1-\alpha)(2\tau - 1)^t > 2\alpha. \tag{46}$$

That is, if and only if it has been maintained since not too long, so that it remains sufficiently strong signal on the appropriate policy. It remains such a signal the longer, the higher the persistence of the appropriate public policy τ: for any τ there exist threshold

$$T = \max\left\{t \mid (1-\alpha)(2\tau - 1)^t > 2\alpha\right\} \tag{47}$$

such that inequality (46) is true if and only if $t \leqslant T$.

Proposition 4 *Consider an overlapping generation game with an infinite horizon. Each generation lives for two periods and plays the basic game without an exogenous public information ($q = \frac{1}{2}$). The state variable follows Markov process described by equations (43) and (44). The game has an equilibrium in which the same public policy is maintained for T successive periods regardless of the underlying state, where T is described by equation (47). In period $T+1$, a reform takes place, if it is appropriate.*

7 Conclusion

We have proposed a model of voting in large referenda or elections. Our approach allows us to accommodate voting behavior which cannot be seen as an expression of rational policy preferences. We see three main directions for the future research:

Mainly, we would like to extend the model to formation of private values or partizan identity, relating ourselves to group-based models of the vote. These models do not explain how voters identify with their groups. In our game extended to private values vote for the same alternative may induce such an identification.

Next, we would like to model small elections in which the voters care not only for their warm glow from participation, but also for the outcome which their vote might affect. This creates endogenize cost of uninformed participation, along the lines of Feddersen and Pesendorfer (1996). Notably, a laboratory experiment by Feddersen et al. (2009) shows that voting behavior depends on the size of the election.

Finally, we hope that our model of the vote may help to analyze other activities involving many participants (such as trading in financial markets or contributing to open course projects).

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A Appendix

A.1 Proof of proposition 1

By equations (2) and (8), $U(\sigma_t^i, \Omega_t^i) = \Pr\left(\theta^i = 1 \mid \Omega_t^i\right)$, $U(v_t^i, \Omega_t^i) = 0$ for $v_t^i \neq \sigma_t^i$. Therefore, equation (9) is true.

1. Consider vote 2. By equation (9), the voting behavior is described by set of equations (10).

Let us prove statement (12): If $\theta^i = 1$ then v_1^i lies in $\mathrm{Im}(v_1(1, \sigma, \sigma_1^i))$ and so $\Pr\left(v_1^i, a_1 \mid \theta^i = 1\right) > 0$. By equation (11), $\Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0$.

By equation (2),

$$\int\limits_{i:\ \theta^i=0,\ \Pr\left(\theta^i=1|\Omega_2^i\right)>0} \sigma_2^i di = \int\limits_{i:\ \theta^i=0,\ \Pr\left(\theta^i=1|\Omega_2^i\right)>0} z_2^i di = 0. \tag{48}$$

Equation (48) and statement (12) imply equation (13).

2. Consider vote 1.

Let us prove statement (14). Suppose, it is false:

$$\mathrm{Im}(v_1(1, \sigma, \sigma_1^i)) \cap \mathrm{Im}(v_1(0, \sigma, \sigma_1^i)) = \varnothing \text{ in either state } x_1. \tag{49}$$

$$\text{Then, } \Pr\left(\theta^i = 1 \mid v_1^i \in \mathrm{Im}(v_1(0, \sigma, \sigma_1^i)), a_1\right) = 0, \tag{50}$$

$$\text{and } \Pr\left(\theta^i = 1 \mid v_1^i \in \mathrm{Im}(v_1(1, \sigma, \sigma_1^i)), a_1\right) = 1. \tag{51}$$

The expected payoff by voter i is equal to:

$$U(v_1^i, \Omega_1^i) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \Omega_2^i) = \begin{cases} \theta^i \sigma_1^i + E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) & \text{if } v_1^i = \sigma_1^i; \\ -\theta^i \sigma_1^i + E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) & \text{otherwise.} \end{cases} \quad (52)$$

By equations (50) and (51), maximization of payoff (52) implies that v_1^i lies in $\mathrm{Im}(v_1(1, \sigma, \sigma_1^i))$ for any i, which contradicts to hypothesis (49).

Let us prove equation (15). By statement (14),

$$\Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) < 1. \quad (53)$$

Therefore, if $v_1^i \neq \sigma_1^i$ then

$$U(v_1^i, \left\{1, \sigma, \sigma_1^i\right\}) + \max_{v_2^i} U(v_2^i, \left\{v_1^i, a_1\right\}) = \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) < 1.$$

At the same time,

$$U(\sigma_1^i, \left\{1, \sigma, \sigma_1^i\right\}) + \max_{v_2^i} U(v_2^i, \left\{\sigma_1^i, a_1, \sigma_2^i\right\}) = 1 + \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) \geqslant 1.$$

Let us prove equation (21). By Bayes rule, and equations (18) and (15),

$$\Pr\left(\theta^i = 1 \mid v_1^i = \varnothing, a_1\right) = 0, \quad (54)$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = 1 - j\right) = 0, \quad (55)$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = j\right) > 0. \quad (56)$$

By equations (54)-(56):

$$U(\varnothing, \{0, \sigma, \sigma_1^i\}) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \{\varnothing, a_1\}) = E_{date\ 1} \Pr\left(\theta^i = 1 \mid \varnothing, a_1\right) = 0,$$
$$U(v_1^i, \{0, \sigma, \sigma_1^i\}) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \{v_1^i, x_1\}) = \tfrac{1}{2} \Pr\left(\theta^i = 1 \mid v_1^i, v_1^i\right) > 0 \text{ for } v_1^i \neq \varnothing.$$

Let us prove inequality (17). Suppose that $a_j \neq x_j$ for both j. Then, both inequalities: $\alpha \leqslant (1-\alpha)(p_j - p_{1-j})$ and $\alpha \leqslant (1-\alpha)(p_{1-j} - p_j)$ must be true. However, the sum of these inequalities is false: $\alpha \leqslant 0$.

Let us prove set of equations (23). Equation (18) implies

$$\alpha \geqslant (1-\alpha)\max\{p_\sigma - p_{1-\sigma}, p_{1-\sigma} - p_\sigma\}. \tag{57}$$

Recall that $\alpha < \frac{1}{2}$. Therefore, inequality (57) requires $0 < p_\sigma < 1$. So, the uninformed voters must be indifferent between voting "σ" and "$1-\sigma$". By set of equations (52), this is equivalent to

$$E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i = \sigma_1^i, a_1\right) = E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i = 1 - \sigma_1^i, a_1\right). \tag{58}$$

Using Bayes rule, we find equation (20). By equations (20) and (55), equations (58) and (22) are equivalent. Set of equations (23) solves the system of equations (21) and (22) for the voting probabilities.

Let us prove inequality (24). By set of equations (23) and inequality (57), equation (18) is true if and only if inequality (36) is met.

A.2 Proof of proposition 2

Using Bayes rule, we find equations (26)-(27). The objective function by voter i is described by set of equations (52). Once again, equation (21) is true. In equilibrium, the uninformed voters must be indifferent between different voting strategies, as described by equation (28). Thereby, we find voting probabilities (29). Given these probabilities, equation (25) is true conditional on inequality (30).

A.3 Proof of proposition 3

1. Suppose the outcome of vote 1 is such as described by equations (33) and (35). By Bayes rule, we find posteriors

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = 1-\sigma\right) = \frac{\alpha}{\alpha + p_{1-\sigma}(1-\alpha)}; \tag{59}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = \sigma\right) = \frac{\alpha(1-q)(1-r)}{\alpha(1-q)(1-r) + p_{1-\sigma}(1-\alpha)(q+(1-q)(1-r))}; \tag{60}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = \sigma\right) = \frac{\alpha q}{\alpha q + p_\sigma (1-\alpha)(q+(1-q)(1-r))}; \tag{61}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = 1-\sigma\right) = 0. \tag{62}$$

Once again, voting delivers a positive expected self-confidence, while abstention delivers null self-confidence. Hence, equation (21) is true. By equations (21) and (34), we find voting probabilities (36). Substituting them in equations (59)-(61), we find:

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = 1-\sigma\right) = 2\alpha; \tag{63}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = \sigma\right) = \frac{2\alpha(1-q)(1-r)}{q+(1-q)(1-r)-2\alpha q}; \tag{64}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = \sigma\right) = \frac{2\alpha q}{2\alpha q + q + (1-q)(1-r)}. \tag{65}$$

2. The uninformed voters should be indifferent between voting for different policies, as described by equation

$$\begin{gathered} \frac{2\alpha q}{2\alpha q+q+(1-q)(1-r)} \left(q + (1-q)(1-r)\right) = \\ = 2\alpha r(1-q) + \frac{2\alpha(1-q)(1-r)}{q+(1-q)(1-r)-2\alpha q} \left(q + (1-q)(1-r)\right). \end{gathered} \tag{66}$$

Using notation

$$x = q + (1-q)(1-r), \tag{67}$$

we rewrite equation (66) as

$$2xq\left(x\left(1-\alpha\right)-2\alpha^2 q\right)=x^2-(2\alpha q)^2\text{, or, equivalently,}$$

$$x^2\left(2q(1-\alpha)-1\right)-(2\alpha q)^2 x+(2\alpha q)^2=0. \tag{68}$$

and solve it for x. We find the following roots:

$$x_+(q)=\tfrac{2\alpha q}{2q(1-\alpha)-1}\left(\alpha q+\sqrt{\left(\alpha q+1\right)^2-2q}\right)\text{ and} \tag{69}$$

$$x_-(q)=\tfrac{2\alpha q}{2q(1-\alpha)-1}\left(\alpha q-\sqrt{\left(\alpha q+1\right)^2-2q}\right). \tag{70}$$

We are only interested in real roots. Furthermore, they must lie in the interval $(q,1)$, so that r given by equation (67) lies in the interval $(0,1)$.

3. Suppose that inequality (30) is true. Let us prove that both roots (69) and (70) are real, but they lie at least as high as 1, hence, no semiinformative equilibrium.

First, note that discriminant $\left(\alpha q+1\right)^2-2q$ decreases in q:

$$\tfrac{\partial((\alpha q+1)^2-2q)}{\partial q}=2\alpha\left(\alpha q+1\right)-2=2(\alpha\left(\alpha q+1\right)-1)<2(\tfrac{1}{2}\left(\tfrac{1}{2}q+1\right)-1)<0,$$

and it is positive at $q=\frac{1}{2(1-\alpha)}$:

$$\left.\sqrt{\left(\alpha q+1\right)^2-2q}\right|_{q=\frac{1}{2(1-\alpha)}}=\tfrac{\alpha}{2(1-\alpha)}.$$

Therefore, both roots (69) and (70) are real.

Second, by inequality (30),

$$2q\left(1-\alpha\right)-1>0\text{ and }\alpha q\ >\sqrt{\left(\alpha q+1\right)^2-2q}.$$

Hence, both roots (69) and (70) are positive, root (69) is the highest: $x_+(q) > x_-(q)$.

Let us prove that the smallest root (70) is no lower than 1. Equation (68) is equivalent to

$$F(x,q) = 2q(1-\alpha)x^2 - (2\alpha q)^2 x + (2\alpha q)^2 - x^2 = 0. \tag{71}$$

$$\frac{\partial F(x,q)}{\partial q} = 2x^2(1-\alpha) + (2\alpha)^2\, 2q(1-x) =$$
$$= \tfrac{2}{q}\left(2q(1-\alpha)x^2 - (2\alpha q)^2 x + (2\alpha q)^2 - q(1-\alpha)x^2\right) =$$
$$= \tfrac{2}{q}x^2\left(1 - q(1-\alpha)\right) > 0, \text{ and } \frac{\partial F(x,q)}{\partial x} = 2x\left(2q(1-\alpha)-1\right) - (2\alpha q)^2.$$

By equation (70),

$$\text{By equation (70), } \frac{\partial F(x,q)}{\partial x} = 2\alpha q\left(\alpha q - \sqrt{(\alpha q+1)^2 - 2q}\right) - (2\alpha q)^2 =$$
$$= -2(\alpha q)^2 - 2\alpha q\sqrt{(\alpha q+1)^2 - 2q} < 0.$$

$$\text{By the implicit function theorem, } \frac{dx_-(q)}{dq} = -\frac{\frac{\partial F(x,q)}{\partial q}}{\frac{\partial F(x,q)}{\partial x}} > 0. \tag{72}$$

By inequality (72),

$$x_-(q) \geqslant x_-\left(\tfrac{1}{2(1-\alpha)}\right) = 1.$$

4. Suppose inequality (24) is true. Let us prove that if equation (68) has real roots, then one of them is negative, and the other one lies below q. Hence, no semiinformative equilibrium once again.

Note that for any q below the upper threshold (30),

$$2q\,(1-\alpha) - 1 < 0 \text{ and } \alpha q \ < \sqrt{(\alpha q+1)^2 - 2q}. \tag{73}$$

Therefore $x_+(q) < 0$ and $x_-(q) > 0$. By inequality (72),

$$x_-(q) \leqslant x_-\left(\tfrac{2\alpha+1}{2(1+\alpha)}\right) = q. \tag{74}$$

5. It remains to consider the interim interval

$$\tfrac{2\alpha+1}{2(1+\alpha)} < q < \tfrac{1}{2(1-\alpha)}. \tag{75}$$

By inequalities (73), equation (68) has the unique positive root (70). By equation (67),

$$r(q) = \tfrac{1-x_-(q)}{1-q}, \tag{76}$$

which is equivalent to equation (37).

5.1. Let us prove inequality (38). By equation (76), it is equivalent to

$$\tfrac{dx_-(q)}{dq} > \tfrac{1-x_-(q)}{1-q}. \tag{77}$$

By the implicit function theorem (recall equation (68)),

$$\tfrac{dx_-(q)}{dq} = -\tfrac{x_-^2(q)(1-\alpha)+(2\alpha)^2(1-x_-(q))q}{(2q(1-\alpha)-1)x_-(q)-2(\alpha q)^2} = \tfrac{x_-^2(q)(1-\alpha)+(2\alpha)^2(1-x_-(q))q}{2\alpha q\sqrt{(\alpha q+1)^2-2q}}. \tag{78}$$

Thereby, inequality (77) is equivalent to

$$x_-^2(q)(1-q)(1-\alpha) > (1-x_-(q))\left(\sqrt{(\alpha q+1)^2-2q}-2\alpha(1-q)\right)2\alpha q. \tag{79}$$

According to the second inequality in set (73), inequality (79) follows from

$$x_-^2(q)(1-q)(1-\alpha) > (1-x_-(q))\left(1-2\alpha(1-q)\right)2\alpha q. \tag{80}$$

By the first inequality in set (75) and inequality (72), inequality (74) is inverted. Therefore, inequality (80) follows from inequality

$$q^2(1-q)(1-\alpha) > (1-q)\left(1-2\alpha(1-q)\right)2\alpha q,$$

which is equivalent to the first inequality in set (75).

5.2. By inequalities (38) and (75), tie-breaking rule given by equation (37) lies in set $(0,1)$:

$$r\left(\frac{2\alpha+1}{2(1+\alpha)}\right)=1;\ \lim_{q\longrightarrow\frac{1}{2(1-\alpha)}} r(q)=0.$$

A.4 Comparative statics

Information aggregation Follows from propositions 1-3.

Instrumental efficiency Straightforward algebra shows that

$$\tfrac{d}{dq}\left(q+(1-q)r\right)<0.$$

Welfare 1. Consider the informative equilibrium described by Proposition 1. We use equations (20) and (23) to find the voters' expected payoffs. The uninformed voters who for policy σ receive payoff $\frac{\alpha}{q(1+\alpha)}$ with probability q. The uninformed voters who vote for policy $1-\sigma$ receive a higher payoff $\frac{\alpha}{(1-q)(1+\alpha)}$ with a lower probability $1-q$. Either way, the common expected payoff is equal to:

$$E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=E\left(U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=\tfrac{\alpha}{1+\alpha}. \quad (81)$$

Payoff by the informed voters depends on whether public signal is true or false. It is equal to $1+\frac{\alpha}{q(1+\alpha)}$ if the signal is true, and to $1+\frac{\alpha}{(1-q)(1+\alpha)}$ if the signal is false. The expected payoff by the informed voters is equal to

$$E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)=1+E\left(U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)=1+\tfrac{2\alpha}{1+\alpha}. \quad (82)$$

2. Consider the uninformed equilibrium described by Proposition 2. By equations (26), (27) and (29), all voters receive payoff α during vote 2. The informed voters however, also receive payoff 1 during vote 1. Hence,

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = E\left(U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = \alpha; \tag{83}$$

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + E\left(U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + \alpha. \tag{84}$$

The uninformed voters benefit from pooling (compare equations (81) and (83)). The informed voters loose (compare equations (82) and (84)).

3. Consider the semiinformative equilibrium described by Proposition 3. By equations (37) and (63)-(65), the uninformed voters receive payoff

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = \alpha\left(1 + \alpha q - \sqrt{(\alpha q + 1)^2 - 2q}\right), \tag{85}$$

which is increasing in q:

$$\frac{\partial}{\partial q}\left(E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right)\right) = \alpha\left(\alpha + \frac{\left(1-\alpha-\alpha^2 q\right)\sqrt{(\alpha q+1)^2-2q}}{(\alpha q+1)^2-2q}\right) > 0.$$

The informed voters receive payoff

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + \alpha + (1-\alpha)\left(\sqrt{(\alpha q + 1)^2 - 2q} - \alpha q\right), \tag{86}$$

which is decreasing in q:

$$\frac{\partial}{\partial q}\left(E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right)\right) = -(1-\alpha)\left(\alpha + \frac{\left(1-\alpha-\alpha^2 q\right)\sqrt{(\alpha q+1)^2-2q}}{(\alpha q+1)^2-2q}\right) > 0.$$

4. By equations (81)-(85) and (86), the common payoff is equal to $\alpha E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)+$

$$+(1-\alpha)E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=2\alpha. \tag{87}$$

Voter turnout and vote margin Suppose that equilibrium is informative. Then, self confidence is described by equations (20) and (55).

Notations: Let

$$V(p_\sigma,q)=q\left(\tfrac{\alpha}{\alpha+(1-\alpha)p_\sigma}-\psi\right)-\psi.$$

be the expected payoff by an uninformed voter who votes for policy σ, $V(p_{1-\sigma},1-q)$ be the expected payoff by an uninformed voter who votes for policy $1-\sigma$.

1. Suppose $\psi\geqslant\frac{1}{2}$. Then, $V(p_\sigma,q)<V(0,1)<0$ and $V(p_{1-\sigma},1-q)<V(0,\frac{1}{2})<0$. Therefore, $p_\sigma=p_{1-\sigma}=0$. Hence, if $\psi\geqslant\frac{1}{2}$ the uninformed voters abstain. This is consistent with outcome $a_1=x_1$.

2. Suppose $\frac{1}{3}\leqslant\psi<\frac{1}{2}$. By inequalities

$$V(p_{1-\sigma},1-q)<V(0,\tfrac{1}{2})<0, \tag{88}$$

$p_{1-\sigma}=0$.

If $V(p_\sigma,q)>0$ then $p_\sigma=1$. However, if $p_\sigma=1$ then $a_1\neq x_1$: a contradiction.

If $V(p_\sigma,q)<0$ then $p_\sigma^i=0$; $V(0,q)<0$ if and only if $q<\frac{\psi}{1-\psi}$.

$V(0,\frac{\psi}{1-\psi})=0$, $dV(0,q)/dq>0$, therefore, $V(0,q)>0$ for any $q>\frac{\psi}{1-\psi}$. Hence, it must be $V(p_\sigma,q)=0$, which is equivalently to equation (41). Note that

$$\frac{dp_\sigma}{dq} = \frac{1}{\psi(1+q)^2} > 0. \tag{89}$$

To summarize, the uninformed voters do not vote contrary to the public signal, that is, $p_{1-\sigma} = 0$. If $q < \frac{\psi}{1-\psi}$, they do not vote according to the public signal either, that is, $p_\sigma = 0$. If $q \geqslant \frac{\psi}{1-\psi}$, they support policy σ the more, the stronger the public signal, as described by equation (41) and inequality (89). However, the efficient policy wins at a positive margin even if the public signal is false:

$$\frac{\alpha}{1-\alpha}\frac{q-\psi(1+q)}{\psi(1+q)} < \frac{\alpha}{1-\alpha} \text{ for } \psi \geqslant \frac{1}{4}, \tag{90}$$

hence for $\psi \geqslant \frac{1}{3}$.

3. Suppose $\frac{1}{4} \leqslant \psi < \frac{1}{3}$.

3.1. Suppose $q \geqslant \frac{1-2\psi}{1-\psi}$. Then, inequality (88) is true, and so $p_{1-\sigma} = 0$. By step 2, p_σ is given by equation (41). It increases in q (inequality (89)), but lies below threshold $\frac{\alpha}{1-\alpha}$ for any q (by inequality (90)).

3.2. Suppose $q < \frac{1-2\psi}{1-\psi}$.

3.2.1. Let us prove by contradiction that there is some abstention, that is, $p_\sigma + p_{1-\sigma} < 1$. Suppose equation (21) is true. Then, both inequalities

$$q\left(\frac{\alpha}{\alpha+(1-\alpha)p_\sigma} - \psi\right) \geqslant \psi \text{ and } (1-q)\left(\frac{\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} - \psi\right) \geqslant \psi \tag{91}$$

must be true, where p_σ is given by equation

$$q\left(\frac{\alpha}{\alpha+(1-\alpha)p_\sigma} - \psi\right) - (1-q)\left(\frac{\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} - \psi\right) = 0, \tag{92}$$

guaranteeing the uninformed voters' indifference between voting "σ" and "$1-\sigma$".[31] Adding up inequalities (91), we find inequality

$$\frac{q\alpha}{\alpha+(1-\alpha)p_\sigma} + \frac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \geqslant 3\psi. \tag{93}$$

By equation (92), inequality (93) is equivalent to

$$\frac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \geqslant \psi(2-q). \tag{94}$$

Comparing equations (22) and (92), we find that $p_{1-\sigma}$ lies higher than that in the set of equations (23), that is,

$$p_{1-\sigma} \geqslant 1-q-\frac{\alpha}{1-\alpha}(2q-1). \text{ Therefore,} \tag{95}$$

$$\frac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \leqslant \frac{\alpha}{1+\alpha}. \tag{96}$$

By inequalities (94) and (96),

$$\frac{\alpha}{1+\alpha} \geqslant \psi(2-q). \tag{97}$$

However, $\frac{\alpha}{1+\alpha} < \frac{1}{3}$, because $\alpha < \frac{1}{2}$. At the same time, $\psi(2-q) < \frac{\psi}{1-\psi}$ for any $q < \frac{1-2\psi}{1-\psi}$ and $\frac{\psi}{1-\psi} \geqslant \frac{1}{3}$ for any $\psi \geqslant \frac{1}{4}$. Hence, inequality (97) is false: a contradiction.

3.2.2. $V(0,1-q) > 0$ for any $q < \frac{1-2\psi}{1-\psi}$. By step 3.2.1, the equilibrium is characterized by equations:

$$V(p_{1-\sigma},1-q) = V(p_\sigma,q) = 0.$$

[31] Recall that if the uninformed voters play a pure voting strategy the equilibrium is uninformative.

Hence, p_σ is given by equation (41), and $p_{1-\sigma}$ is given by equation (42). Note that these voting probabilities are consistent with outcome $a_1 = x_1$:

$$p_\sigma - p_{1-\sigma} = \tfrac{\alpha}{1-\alpha}\tfrac{1}{\psi}\tfrac{2q-1}{(1+q)(2-q)} \leqslant \tfrac{\alpha}{1-\alpha}$$

if and only if

$$q \leqslant \tfrac{\sqrt{4+9\psi^2}+\psi-2}{2\psi}, \tag{98}$$

Straightforward algebra shows that $\frac{\sqrt{4+9\psi^2}+\psi-2}{2\psi} > \frac{1-2\psi}{1-\psi}$ for any $\psi \geqslant \frac{1}{2} - \frac{\sqrt{3}}{6}$, hence for any $\psi \geqslant \frac{1}{4}$.

Note that the uninformed voters' turnout is decreasing in the quality of public signal:

$$\tfrac{\partial}{\partial q}\left(p_\sigma + p_{1-\sigma}\right) = \tfrac{\alpha}{1-\alpha}\tfrac{1}{\psi}\left(\tfrac{1}{(1+q)^2} - \tfrac{1}{(2-q)^2}\right) < 0.$$

4. Suppose $\psi < \frac{1}{4}$. Let us show that there exist q such that the informed equilibrium is not supported. Consider $q \geqslant \frac{1-2\psi}{1-\psi}$. By step 2, $p_{1-\sigma} = 0$. If $p_\sigma = 1$ then $a_1 = \sigma$, which is generically different from x_1. If $p_\sigma < 1$ then p_σ is given by equation (41). Hence, $p_\sigma > \frac{\alpha}{1-\alpha}$ for any $q \geqslant \frac{2\psi}{1-2\psi}$. Hence, the informative equilibrium is not supported.

5. Let us show that the expected margin of victory, denote it

$$MV = q(\alpha + (1-\alpha)(p_\sigma - p_{1-\sigma})) + (1-q)(\alpha - (1-\alpha)(p_\sigma - p_{1-\sigma})), \tag{99}$$

increases in the quality of public signal. Let us rewrite equation (99) as

$$MV = \alpha + (1-\alpha)(p_\sigma - p_{1-\sigma})(2q-1). \tag{100}$$

Consider q outside interval (40). By equations $p_{1-\sigma} = 0$ and (100)

$$MV = \alpha + (1-\alpha)\, p_\sigma \,(2q-1)\,. \tag{101}$$

By inequality (89), MV increases in q. Now, consider q inside interval (40). By equations (41), (42) and (101),

$$MV = \alpha + (1-\alpha)\, p_\sigma \,(2q-1)\,.$$

Straightforward algebra shows that

$$\tfrac{\partial MV}{\partial q} = \tfrac{\alpha(2q-1)\left((1-q)^2+q^2+4\right)}{\psi(1+q)^2(2-q)^2} > 0.$$

A.5 Proof of proposition 4

See the main text.