

2000s-06

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**Série Scientifique**  
*Scientific Series*

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**CIRANO**  
Centre interuniversitaire de recherche  
en analyse des organisations

Montréal

Version révisée, février 2001 / Revised version, February 2001

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# Preemption and Rent Dissipation with Multiple Investments\*

Marcel Boyer<sup>†</sup>, Pierre Lasserre<sup>‡</sup>, Thomas Mariotti<sup>§</sup>, Michel Moreaux<sup>¶</sup>

## Résumé / Abstract

Dans cet article, nous étudions un modèle (duopole) de préemption avec investissements multiples et concurrence instantanée à la Bertrand dans un marché à croissance stochastique. Différentes configurations d'équilibre peuvent apparaître en fonction de l'importance de l'effet d'option réelle. Si le taux de croissance moyen du marché s'approche du taux d'intérêt sans risque, ou si la volatilité de la demande est élevée, l'unique processus d'investissement d'équilibre implique une adoption conjointe à la date socialement optimale. En l'absence de ces conditions, la séquence d'investissement d'équilibre n'est pas optimale et les capacités à long terme des entreprises dépendent des conditions de marché initiales. Cependant, pour un large éventail de valeurs des paramètres pertinents, aucune dissipation de rentes ne se produit à l'équilibre, malgré la concurrence à la Bertrand. Ceci jette quelques doutes sur la robustesse des résultats des modèles de préemption à une période.

*We study a simple duopoly model of preemption with multiple investments and instantaneous Bertrand competition in a stochastically growing market. Different patterns of equilibria may arise, depending on the importance of the real option effect. If the average growth rate of the market is close to the risk free rate, or if the volatility of demand changes is high, the unique equilibrium acquisition process involves joint adoption at the socially optimal date. If these conditions do not hold, the equilibrium investment timing is suboptimal, and the firms' long-run capacities depend on the initial market conditions. However, under a broad set of empirically relevant parameters, no dissipation of rents occur in equilibrium, despite instantaneous Bertrand competition. This casts some doubts on the robustness of one-shot models of preemption.*

**Mots Clés :** Préemption, dissipation de rentes, investissement sous incertitude

**Keywords:** Preemption, rent dissipation, investment under uncertainty

**JEL:** C73, D43, D92, L13

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We wish to thank the participants in the Economic Theory seminar at Caltech for their comments. Financial support from INRA is gratefully acknowledged by the first two authors. Part of this research was completed while the third author benefitted from a STICERD grant. We remain of course solely responsible for the content of this paper.

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## 1. INTRODUCTION

Since the early work of Posner (1975), the idea that rent-seeking behavior by firms leads to dissipation of monopolistic rents has become a recurrent theme in the industrial organization literature. Posner's argument is that firms compete for monopoly power, and that monopoly profits are offset by the total expenditures made by firms to win this contest, thereby adding to the usual deadweight loss. By modeling the contest for monopoly power as a timing game, the game-theoretic literature on preemption in oligopoly has provided a useful framework to assess this argument.<sup>1</sup> The basic intuition is well captured by Fudenberg and Tirole's (1985) analysis of new technology adoption in a duopoly. In their model, two *ex-ante* identical firms can invest in a cost-reducing technology. Early adoption by one firm delays or prevents adoption by the other. If information lags are negligible, the investment game always has a subgame-perfect equilibrium in which any first-mover advantage is completely dissipated by preemptive adoption. The intuition is akin to static Bertrand competition: if one firm enjoyed a strict first-mover advantage in equilibrium, its rival could strictly increase its equilibrium payoff by preempting its investment by an infinitesimal amount of time.<sup>2</sup>

Most analyses of the preemption phenomenon have focused on environments in which each firm can make at most one indivisible investment. This seems to be an important restriction, since it is *a priori* reasonable to believe that rent dissipation is alleviated in the presence of long term relationships between firms. However, as shown by Gilbert and Harris (1984), the full rent-dissipation outcome still obtains at equilibrium when firms can accumulate capacity by building new production plants. In a preemption equilibrium, firms threaten to invest whenever a new plant is profitable, so that each new plant built by any firm earns zero profit. Provided marginal revenue is everywhere positive, this rule can be credibly followed by any firm since it depends only on the total capacity installed so far in the industry, and not on a firm's own existing capacity. Thus long term competition *per se* does not rule out full rent dissipation on new investments.

The objective of this paper is to assess the robustness of this result. To do so, we

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<sup>1</sup>Seminal contributions along these lines include Eaton and Lipsey (1979), Dixit (1980), Gilbert and Newbery (1982), and Fudenberg and Tirole (1985, 1987), among others.

<sup>2</sup>Whether this is the unique equilibrium depends on the specification of payoffs. Basically, if innovation deters imitation for a long period, the preemption equilibrium is the unique subgame-perfect equilibrium. By contrast, if adoption by a firm triggers a fast response by its rival, there exists a continuum of other equilibria. In particular, the optimal joint-adoption date is sustainable as a subgame-perfect equilibrium.

consider the following stylized environment. There are two competing firms. Investment takes the form of irreversible acquisitions of indivisible units of capacity at a constant fixed cost. Market demand is driven by consumers' taste for the output. Specifically, while the total size of the market is finite and fixed, the consumers' willingness to pay for any given quantity of the good fluctuates unpredictably, and tends to increase on average. At each instant of time, firms compete à la Bertrand given their existing capacity. Our main focus is on the timing of investments in a Markov perfect equilibrium of the game, for which one of the natural state variables is the current position of the demand curve.

The trade-off faced by firms when taking their investment decisions is simple (see Smets (1991) for an early analysis). On the one hand, uncertainty and irreversibility entail an option value of delaying investment. On the other hand, the threat of preemption is likely to accelerate investment, thereby mitigating the real option effect. When one unit of capacity suffices to cover the market, full rent dissipation occurs in equilibrium: the real option value of waiting is totally offset by the threat of preemption. However, when multiple investments are required to serve all consumers, the possibility of segmenting capacity prevents the existence of permanent barriers to entry. Indeed, even if a leader firm has already enough capacity to serve all consumers, a laggard firm without capacity can always install a smaller amount of capacity without driving industry profits to zero. This in turn affects the incentives to accumulate capacity for preempting. Indeed, as the size of the market is finite, the type of investment rule considered by Gilbert and Harris—invest as soon as a new plant can earn profits—cannot be indefinitely applied by an incumbent, as its marginal revenue from new investments drops to zero after a finite number of investments—that is, once he has enough capacity to serve all consumers in the market. In the same circumstances, however, marginal revenue from new investments is not necessarily zero for a laggard firm. Thus, contrary to what happens if the market size is infinite and marginal revenue is everywhere positive, firms with different capacities have in general different incentives to invest. The outcome of the game depends then on whether the laggard or the leader has more incentive to preempt its rival.

To illustrate this simple intuition, we solve the model analytically under two additional assumptions. First, we suppose that demand is price inelastic at any date. This assumption has the advantage of keeping the model tractable and close to the unit market case: with constant investment costs per unit, a monopolist would like to perform all his investments

at once. Second, to cut down on the non-essential complex features of the model, we assume that two units of capacity are needed to serve all consumers. However, we do not require investments to take place sequentially.

Under these assumptions, our results are the following. If the average growth rate of the market is large enough relative to the risk-free rate and/or if the volatility of demand changes is high enough, there is an essentially unique Markov perfect equilibrium in which both firms simultaneously invest one unit of capacity at the random time that maximizes industry profits. Since the duopoly exactly duplicates the monopoly outcome, no rent dissipation occurs in equilibrium. The interpretation is that it is never worthwhile for a firm to invest two units of capacity, because the stochastic delay induced for the entry of the second firm is not long enough on average to allow the first firm to recover the fixed cost of a second unit of capacity. While this intuition is close to the cooperative equilibria found by Fudenberg and Tirole (1985), an important difference with their results is that, under the above assumptions, the efficient joint-entry outcome is the *unique* equilibrium outcome in our model.

If on the other hand, the risk-free rate is high relative to the average growth rate of the market, and if the volatility of demand is low enough, then multiple Markov perfect equilibria exist. However, if the initial state of demand is low enough, all these equilibria generate the same outcome: firms simultaneously invest one unit of capacity at the first date at which it becomes worthwhile to preempt one's rival with two units of capacity. This occurs earlier than the optimal monopolistic investment date, implying that some monopolistic rents are dissipated in equilibrium. For a high enough initial state of demand, different patterns of investment are compatible with equilibrium. In particular, there exist equilibria in which firms try to preempt each other with two units of capacity, resulting in over-accumulation of capital compared to the total size of the market.

The paper is organized as follows. Section 2 describes the model. In Section 3, we derive two useful benchmarks: first, the optimal investment policy of a monopolist, and second, the Markov perfect equilibrium of the game when the market can be covered with a single unit of capacity. The Markov perfect equilibria of the multiple investments model are described in Section 4 in the special case where two units are needed to cover the market. Section 5 concludes by discussing the robustness of our results.

## 2. THE MODEL

Consider the following strategic investment problem:

*Market demand.* Time is continuous, and indexed by  $t \geq 0$ . At any date  $t$ , the demand side of the market is described by a price inelastic unit demand:

$$D_t(P) = \begin{cases} 0 & \text{if } P > P_t \\ [0, 1] & \text{if } P = P_t \\ 1 & \text{if } P < P_t \end{cases}, \quad (1)$$

where the total willingness to pay  $P_t$  for the commodity produced by the firms is subject to aggregate demand shocks described by a geometric Brownian motion:

$$dP_t = \alpha P_t dt + \sigma P_t dZ_t, \quad (2)$$

where  $P_0$ ,  $\alpha$ , and  $\sigma$  are positive constants and  $\{Z_t\}_{t \geq 0}$  is a standard Brownian motion.

*Firms.* There are two firms, labelled by  $i, j \in \{1, 2\}$ . Both are risk neutral and discount future revenues and costs at the same constant risk free rate  $r > \alpha$ .<sup>3</sup> Variable costs are normalized to zero. Investment is irreversible and takes place in a lumpy way. Each unit of capacity allows a firm to cover at most a positive fraction  $1/N$  of the market, for some positive integer  $N$ . The cost of each unit of capacity is constant and equal to  $I > 0$ . Capital has no resale value and does not depreciate.

*Competition.* Within each instant  $[t, t + dt)$ , the timing of the game is the following. (i) First, each firm chooses how many units of capacity to invest, given the realization of  $P_t$  and the existing levels of capacity. (ii) Next, each firm quotes a price given its new level of capacity and that of its rival. (iii) Last, consumers choose to which firm to purchase, and productions and transfers take place. Although total demand is price inelastic, each firm faces a perfectly elastic residual demand at the competitor's price.

*Markov strategies.* We focus on Markov perfect equilibria (MPE), in which firms' investment and pricing decisions depend only on the current value of the consumers' reserve price  $p$  and the firms' capital stock measured in units of capacity,  $(n^i, n^j)$ . This rules out any kind of

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<sup>3</sup>If  $\alpha \geq r$ , each firm's value would be maximized by perpetually holding its investment options alive, and investment would never take place.

implicit collusion between the firms when making their pricing decisions: in any period, firms play an equilibrium of the static Bertrand pricing game given their current capacities. This assumption is consistent with the main purpose of our paper, which is to study the dynamics of capital accumulation.

Our definition of Markov strategies and payoffs generalizes Fudenberg and Tirole's (1985) concept of preemption strategies to a stochastic environment with multiple investments. To obtain an adequate continuous time representation of limits of discrete time mixed strategy equilibria, we define a strategy for firm  $i$  as an *intensity function*  $s^i$ , with the interpretation that  $s^i_\nu(n^i, n^j, p) \in [0, 1]$  is the intensity with which firm  $i$  invests  $\nu$  additional units of capacity given the capital stocks  $(n^i, n^j)$  and the state of demand  $p$ . The details of the construction are given in Appendix A. We denote by  $U_s^i(n^i, n^j, p)$  firm  $i$ 's expected discounted value in state  $(n^i, n^j, p)$  given the strategy profile  $s = (s^1, s^2)$ .

There are several points of departure between this model and the environment considered by Gilbert and Harris (1984). First, we allow for stochastic fluctuations of demand. This is because we want to assess the impact of demand uncertainty on firms' investment decisions. Second, instantaneous competition takes place in prices, and not in quantities. This assumption is made for convenience only since making market competition more aggressive can only increase the incentives for preemption. Third, the size of the market is finite and constant through time. That is, aggregate demand shocks are of the kind considered in the real option literature (see *e.g.* Dixit and Pindyck (1994)). "Growth" of demand arises only through taste shocks that affect the willingness to pay of the consumers, and not through mere replication of initial consumers. It is this feature of the model that rules out full rent dissipation when more than one investment is required to cover the market.

### 3. BENCHMARKS

As benchmarks, we study two polar situations. First, we analyze the optimal investment policy of a monopolist. Next, we characterize the MPEs of the investment game when the market can be covered by a single unit of capacity.

#### 3.1. *Optimal Investment Policy in a Monopoly*

Since demand is price inelastic and variable costs are nil, a risk neutral monopolist will optimally invest  $N$  units of capacity simultaneously. Once in place, it extracts all the consumer



surplus by charging a price  $P_t$  at any date  $t$ . The expected discounted value of a monopolist when the current state of demand is  $p$  is thus given by:

$$V_N^M(p) = \sup_{\tau} \left\{ E_p \left( \int_{\tau}^{\infty} e^{-rt} P_t dt - e^{-r\tau} NI \right) \right\}, \quad (3)$$

where the supremum is taken with respect to all stopping times adapted to  $\{P_t\}_{t \geq 0}$ , and  $E_p(\cdot)$  is the expectation operator conditional on  $P_0 = p$ . An optimal stopping time for (3) is to invest when the state of demand reaches the investment trigger:

$$p_N^M = \frac{\beta}{\beta - 1} (r - \alpha) NI, \quad (4)$$

where:

$$\beta = \frac{1}{2} - \alpha/\sigma^2 + \sqrt{(\alpha/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2}. \quad (5)$$

(See *e.g.* Dixit and Pindyck (1994, Chapter 5)). Since  $\beta > 1$ , the investment trigger  $p_N^M$  is above the Marshallian trigger  $p_N^m = (r - \alpha) NI$  at which the value of the firm is equal to zero. Note also that since the monopolist can extract all the social surplus, his investment strategy coincides with that of a benevolent social planner, provided that the risk free rate  $r$  is equal to the social discount rate.

### 3.2. The Investment Game in the Case $N = 1$

If  $N = 1$ , Bertrand competition acts as a permanent barrier to entry. Indeed, if an incumbent firm already holds one capacity unit, price competition following the entry by the challenger would drive the industry profits to zero and therefore prevent it from recovering its investment cost  $I$ . Similarly, if both firms enter simultaneously, subsequent profits are zero. It is also obvious that no firm wants to acquire more than one unit of capacity in an MPE. In other terms, in any MPE  $s$ ,

$$\begin{aligned} U_s^i(0, 1, p) &= 0 \\ U_s^i(1, 0, p) &= E_p \left( \int_0^{\infty} e^{-rt} P_t dt \right) = \frac{p}{r - \alpha} \\ U_s^i(1, 1, p) &= 0, \end{aligned} \quad (6)$$

for any firm  $i$  and state of demand  $p$ . We now determine the firms' equilibrium strategies before any firm has invested yet, *i.e.* in states of the form  $(0, 0, p)$ . First, if  $p < p_1^m$ , it is clearly a strictly dominant strategy for both firms not to invest. Next, if  $p > p_1^M$ , both firms

have no incentive to delay investment, so that any subgame starting at  $(0, 0, p)$  is essentially static. As a result, one can construct many continuation MPEs on  $(p_1^M, \infty)$ . An obvious candidate is that firm  $i$  invests one unit with intensity one, and firm  $j$  optimally reacts by not investing. Alternatively, using equation (A.1) in Appendix A, it is easy to check that if firm  $j$  invests with intensity  $(p - p_1^m)/p$ , all choices of firm  $i$  yield a zero payoff. By symmetry, it follows that there exists a mixed strategy continuation equilibrium yielding both firms a zero payoff. In fact, this strategy profile turns out to be the unique MPE strategy profile on  $(p_1^m, p_1^M)$  as well. Hence the following proposition (all proofs are in Appendix B).

**Proposition 1** *If  $P_0 \in (0, p_1^M)$  then, in any MPE of the Bertrand-Edgeworth game, the expected payoff of each firm at date zero is equal to zero.*

The intuition is straightforward. Suppose that  $P_0 < p_1^m$ . The maximal payoff any firm can achieve is obtained by investing at the first time  $\tau_1^M$  where the state of demand reaches the monopolistic trigger  $p_1^M$ . But if firm  $i$  anticipates that  $j$  will invest at  $\tau_1^M$ , it is better off avoiding being preempted by  $j$  by investing at the first date where the state of demand reaches  $p_1^M - \varepsilon$  for some small  $\varepsilon > 0$ . Reasoning backwards, at any  $p \in (p_1^m, p_1^M)$ , each firm wants to preempt to avoid being preempted later on. At the Marshallian trigger  $p_1^m$ , the rents of the leader and the follower are both equal to zero, so that both firms are indifferent between preempting or staying out of the market. Hence, on any MPE path one firm (possibly chosen randomly) will invest at the first time  $\tau_1^m$  where the state of demand reaches the Marshallian trigger  $p_1^m$ , and the other firm never invest afterwards.

Proposition 1 illustrates in a spectacular way the rent dissipation phenomenon described by Posner (1975) and Fudenberg and Tirole (1987), among others. Here, the threat of Bertrand competition implies that in any MPE, the first firm to invest enjoys a permanent monopoly position. In equilibrium, the real option effect is completely offset by the threat of preemption, so that the monopoly rents—hence the social surplus, since demand is price inelastic—are totally dissipated at the investment date.

#### 4. MULTIPLE INVESTMENTS: THE CASE $N = 2$

In this section, we contrast the previous rent dissipation result with the outcome of the game if multiple units of capacity are necessary to cover the whole market. For simplicity, we focus on the case where two units of capacity are needed to serve all consumers.

#### 4.1. Short Term Competition

If  $N = 2$ , a firm never wants to hold more than two capacity units in equilibrium, so that we may assume that  $n^i, n^j \in \{0, 1, 2\}$ . Let  $\pi(n^i, n^j, p)$  denote firm  $i$ 's instantaneous expected profits in a Nash equilibrium of the Bertrand stage game when the current state of the game is  $(n^i, n^j, p)$ . We have:

**Lemma 1** For  $p \in [0, \infty)$  :

$$(i) \quad \pi(1, 0, p) = \frac{1}{2} p, \quad \pi(2, 0, p) = p;$$

$$(ii) \quad \pi(1, 1, p) = \frac{1}{2} p;$$

$$(iii) \quad \pi(2, 2, p) = 0;$$

$$(iv) \quad \pi(2, 1, p) = \frac{1}{2} p \text{ and } \pi(1, 2, p) = \frac{1}{4} p.$$

The proof of items (i), (ii), (iii) is immediate. In case (iv), where one firm holds one unit of capacity while the other holds two units, the instantaneous Bertrand game has no pure strategy equilibrium. In Appendix B, we show that there exists a unique mixed strategy equilibrium with the above payoffs. In this equilibrium, the firm with one unit of capacity prices more aggressively than its rival. As in Ghemawat (1997), one can interpret this phenomenon as a “stochastic price umbrella” held by the smallest firm.<sup>4</sup> If a firm already holds one unit of capacity while its rival has not invested yet, the profitability of the existing unit, hence the instantaneous profits of the incumbent, does not change following the investment of one unit by its challenger. Similarly, if a firm's capacity is of one unit, the expected profit of its rival are the same whether its own capacity is of one or two units. This property greatly simplifies the analysis of the firms' long term interactions, to which we now turn.

#### 4.2. Long Term Competition: Preliminary Results

To characterize the MPE of the Bertrand-Edgeworth game, we proceed by backward induction on the firms' capacity profiles  $(n^1, n^2)$ . Four cases must be distinguished.

*Case 1:*  $(n^1, n^2) \in \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ . It is clear from Lemma 1 that when both firms already hold at least one unit of capacity, acquiring additional units of capacity is a

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<sup>4</sup>See also Boyer and Moreaux (1988, 1989) for more aggressive pricing and rational rationing by the smaller firm in a duopoly.

strictly dominated strategy from each firm's perspective. Since capital does not depreciate, the investment game is over as soon as one of these positions is reached. Thus, in any MPE  $s$ , firm  $i$ 's continuation payoff is then given by:

$$U_s(n^i, n^j, p) = E_p \left( \int_0^\infty e^{-rt} \pi(n^i, n^j, P_t) dt \right) = \frac{\pi(n^i, n^j, p)}{r - \alpha}. \quad (7)$$

(From now on, we drop the firm index in the expressions of equilibrium value functions when the MPE outcome is unique.)

*Case 2:*  $(n^1, n^2) \in \{(0, 2), (2, 0)\}$ . Suppose now that firm  $i$  already holds two units of capacity, while its rival  $j$  has not invested yet. It is clear that making further investments has no value for the incumbent  $i$ . Moreover, it is not in the interest of the challenger  $j$  to invest more than one unit, since doing so would drive industry profits to zero. The challenger's continuation payoff is thus given by:

$$U_s(0, 2, p) = \sup_\tau \left\{ E_p \left( \int_\tau^\infty e^{-rt} \pi(1, 2, P_t) dt - e^{-r\tau} I \right) \right\}. \quad (8)$$

Replacing  $\pi(1, 2, P_t)$  by its value given by Lemma 1, and proceeding as for (3), we get that an optimal stopping time is to invest when the state of demand reaches the investment trigger:

$$p^C = 4 \frac{\beta}{\beta - 1} (r - \alpha) I. \quad (9)$$

Let  $\tau^C$  denote the corresponding stopping time. Then:

$$U_s(0, 2, p) = E_p \left( \int_{\tau^C}^\infty e^{-rt} \pi(1, 2, P_t) dt - e^{-r\tau^C} I \right) = \left( \frac{p}{p^C} \right)^\beta \left( \frac{\pi(1, 2, p^C)}{r - \alpha} - I \right) \quad (10)$$

for any  $p \in (0, p^C)$ . The value of the incumbent is given by:

$$\begin{aligned} U_s(2, 0, p) &= E_p \left( \int_0^{\tau^C} e^{-rt} \pi(2, 0, P_t) dt + \int_{\tau^C}^\infty e^{-rt} \pi(2, 1, P_t) dt \right) \\ &= \frac{\pi(2, 0, p)}{r - \alpha} + \left( \frac{p}{p^C} \right)^\beta \frac{\pi(2, 1, p^C) - \pi(2, 0, p^C)}{r - \alpha}. \end{aligned} \quad (11)$$

for any  $p \in (0, p^C)$ . In contrast with Cases 1 and 2, where the MPE outcome of the game is uniquely determined, the remaining cases are more problematic to handle.

*Case 3:*  $(n^1, n^2) \in \{(0, 1), (1, 0)\}$ . Suppose now that firm  $i$  has already installed one unit of capacity, while its rival  $j$  has not invested yet. By investing one additional unit, the

incumbent can temporarily increase its profits from  $\pi(1, 0, P_t)$  to  $\pi(2, 0, P_t)$  and delay the challenger's entry until  $\tau^C$ . However, it is not *a priori* clear whether the incumbent can gain from doing so. The only case in which the conclusion is straightforward is when  $p \geq p^C$ . Indeed, from Case 2, it is then a strictly dominant strategy for the challenger to invest immediately one unit. The incumbent has thus no incentives to make further investments.

*Case 4:*  $(n^1, n^2) = (0, 0)$ . If both firms have not invested yet, they may enter with one or two capacity units. As in Case 3, the only situation in which the MPE strategies are straightforward to determine is when the state of demand is  $p \geq p^C$ . Indeed, the same argument implies that investing two units is a strictly dominated strategy, and both firms enter simultaneously with one unit.

It turns out that the equilibrium dynamics in Cases 3 and 4 depends crucially on whether  $r < 2\alpha + \sigma^2$ . To simplify the exposition, we shall first consider the case where this relation holds. We then tackle the analysis in the opposite case.

#### 4.3. *Optimal MPE Trajectories in the Case $r < 2\alpha + \sigma^2$*

The purpose of this section is to show that, in contrast to the benchmark case of Bertrand competition with  $N = 1$ , the investment choices in a MPE may now match the socially optimal trajectory for certain values of the parameters  $r$ ,  $\sigma$  and  $\alpha$ . The following lemma plays a crucial role in establishing this result.

**Lemma 2** *If  $r < 2\alpha + \sigma^2$  then, in any MPE  $s$ ,*

$$U_s(2, 0, \cdot) - I < U_s(1, 1, \cdot). \quad (12)$$

Under the assumption of Lemma 2, the incumbent in Case 3 prefers to be preempted by one unit of capacity rather than preempting itself with one unit. Therefore, the only possible rationale for the incumbent to preempt the challenger would be to prevent it from investing two units at once. However, the challenger never benefits from doing so, since  $\pi(2, 1, \cdot) = \pi(1, 1, \cdot)$ . Thus the preemption incentive of an incumbent is always negative. Since the challenger is not threatened by new investments from its rival, its best response is to invest one unit at the date  $\tau_2^M$  where demand reaches the optimal investment trigger  $p_2^M$ .

Of course, the assumption  $r < 2\alpha + \sigma^2$  is crucial for this result. In the limit deterministic case,  $\sigma = 0$ , this means that the growth rate  $\alpha$  of the consumers' valuation must not be too

small compared to the discount rate  $r$ . This may be interpreted as follows. When  $\sigma = 0$ , the optimal investment policy of a challenger is to invest when  $p^C = 4rI$  is reached, *i.e.* at date  $T^C = \ln(4rI/p_0)/\alpha$ , if we suppose that the incumbent holds two units of capacity at time zero. Clearly, the larger  $\alpha$ , the shorter the time interval over which the incumbent can enjoy full monopoly profits, and thus the weaker the incentives to preempt with two units of capacity. A similar intuition pertains to the stochastic case,  $\sigma > 0$ . Since  $\alpha > 0$ ,  $r < 2\alpha + \sigma^2$  is now equivalent to  $\beta \in (1, 2)$ . This implies that  $p^C = 4(r + \frac{1}{2}\sigma^2\beta)I$  is relatively small, and that the stochastic discount factor  $E_t(e^{-r(\tau^C - t)}) = (p_t/p^C)^\beta$  between any date  $t$  and  $\tau^C$  is relatively large. Again, the reason why it is not worthwhile for a firm to invest two units of capacity is that doing so does not delay sufficiently the entry of the challenger. Moreover, for a given expected growth rate of demand  $\alpha$ , an increase in the volatility  $\sigma$  reduces the preemption incentive. This is a standard real option effect: *ceteris paribus*, an increase in uncertainty makes firms more willing to delay their investment.

Consider now what happens in Case 4 under the assumption of Lemma 2. From Lemma 1,  $\pi(2, 1, \cdot) = \pi(1, 1, \cdot)$  and  $\pi(1, 2, \cdot) \geq \pi(2, 2, \cdot)$ . Therefore,

$$\begin{aligned} U_s(1, 1, \cdot) - I &> U_s(2, 1, \cdot) - 2I \\ U_s(1, 2, \cdot) - I &> U_s(2, 2, \cdot) - 2I. \end{aligned} \tag{13}$$

Hence, if with probability one both firms invests simultaneously in state  $(0, 0, p)$ , each firm is better off investing one rather than two units. Suppose now that one firm preempts its rival with one unit. Lemma 2 implies that it will never reinvests on any MPE path. Taking advantage from  $\pi(1, 1, \cdot) = \pi(1, 0, \cdot)$ , it follows immediately that  $U_s(1, 0, \cdot) = U_s(1, 1, \cdot)$ . We may therefore rewrite (12) as:

$$U_s(2, 0, \cdot) - 2I < U_s(1, 0, \cdot) - I. \tag{14}$$

Intuitively, if  $r < 2\alpha + \sigma^2$  a firm prefers to preempt its rival with one rather than two units. Together with (13), this implies that given the unique continuation MPE outcome previously constructed in Cases 1-3, investing two units of capacity with positive intensity is a strictly dominated strategy in Case 4. It follows immediately that on any MPE path, each firm will invest at most one unit of capacity. To do so, they are both better off waiting until the state of demand reaches the optimal trigger  $p_2^M$ . Hence, we have proved the following result:

**Proposition 2** *If  $r < 2\alpha + \sigma^2$ , there exists an unique MPE outcome. On any MPE path*

both firms simultaneously invest one unit of capacity at the optimal investment date  $\tau_2^M$  and never reinvests thereafter.

Under the assumption of Proposition 2, the unique MPE outcome exactly duplicates the socially efficient outcome, and no dissipation of rents arises in equilibrium: everything happens as if the two firms were two divisions of a monopolistic firm. This is in sharp contrast with the benchmark case, where each firm could cover the whole market with a single unit of capacity. The main differences are that (i) firms may now segment their capacity to serve only a fraction of consumers so that, even if they cannot implicitly collude on prices, there is no permanent barrier to entry, and (ii) covering the whole market may not delay sufficiently the challenger's entry.

If we adopt a reasonable parametrization of  $r = 4\%$ , the assumption of Proposition 2 holds for any  $\alpha \in (0, r)$  as soon as  $\sigma$  is larger than 20%, which is a good approximation of the average standard deviation  $\sigma_{NYSE}$  of the expected rate of return on the New York Stock Exchange. Of course,  $\sigma_{NYSE}$  is based on a diversified portfolio of assets. However, one may argue that it represents a reasonable lower bound for the volatility on the rate of return on the introduction of a new product, or an R&D venture, two typical applications of our model. Overall, the conclusion of Proposition 2 clearly holds for a broad set of realistic parameter values.

#### 4.4. MPE Trajectories in the Case $r > 2\alpha + \sigma^2$

To assess the robustness of Proposition 2, it is instructive to analyze the firm's MPE investment strategies if  $r > 2\alpha + \sigma^2$ . The following lemma compares the incumbent's and the challenger's preemption incentives in Case 3.

**Lemma 3** *In Case 3, the preemption incentive of the incumbent is always less than the preemption incentive of the challenger, i.e.*

$$U_s(2, 0, \cdot) - U_s(1, 1, \cdot) < U_s(1, 1, \cdot) - U_s(0, 2, \cdot). \quad (15)$$

Moreover, if  $r > 2\alpha + \sigma^2$ , there exists two numbers  $p^- \in (0, p_2^M)$  and  $p^+ \in (p_2^M, p^C)$  such that for each  $p \in [0, \infty)$ ,  $U_s(2, 0, p) - I > U_s(1, 1, p)$  if and only if  $p \in (p^-, p^+)$ .

The intuition is as follows. Suppose that the present state of the game is  $(1, 0, p)$ . From the incumbent's perspective, the gain of preempting relative to being preempted is to reap

the additional profits  $\pi(2, 0, P_t) - \pi(1, 1, P_t) = \frac{1}{2} P_t$  until  $\tau^C$ . By contrast, the relative gain of preemption for the challenger is  $\pi(1, 1, P_t) - \pi(1, 2, P_t) = \frac{1}{4} P_t$ , on the whole time interval  $[\tau^C, \infty)$ . Hence in Case 3, the preemption incentive of the challenger is always larger than the incumbent's. However, this does not mean that the challenger will always win the investment race. In particular, if  $r > 2\alpha + \sigma^2$ , there is a whole interval  $(p^-, p^+)$  of current states of demand on which both firms have a positive preemption incentive.

To characterize the MPE outcome in Case 3, it is helpful to determine each firm's stand-alone optimal investment date, *i.e.* the random time at which it would invest if it were not threatened by its rival's preemption (see Katz and Shapiro (1987)). For the challenger, this is obviously equal to  $\tau_2^M$ . For the leader, it is the solution of:

$$\sup_{\tau} \left\{ E_p \left( \int_0^{\tau} e^{-rt} \pi(1, 0, P_t) dt + e^{-r\tau} (U_s(2, 0, P_{\tau}) - I) \right) \right\}. \quad (16)$$

Using Lemma 3, standard computation shows that the solution to (16) is to invest if the current state of demand lies in  $[p_2^M, p^+]$ . The lower bound of this interval coincides with the optimal investment strategy for a myopic incumbent which does not take into account the future entry of the challenger. The intuition is as in Leahy (1993): when computing its stand-alone date, a myopic incumbent overstates by the same amount the value of the investment option and the marginal benefit from investing, leaving the investment rule unaffected. Since demand is price inelastic, the stand-alone investment dates of the challenger and of a myopic incumbent coincide. Of course, if the state of demand is  $p > p^+$ , a non myopic incumbent does not want to invest since its preemption incentive is negative.

As in the benchmark case, it immediately follows that if  $p \in (p_2^M, p^+)$ , both the challenger and the incumbent in Case 3 have no incentive to delay investment, so that any subgame starting at  $(1, 0, p)$  is essentially static. As a consequence, one may construct many continuation MPEs on  $(p_2^M, p^+)$ . Next, on  $(p^+, \infty)$ , there is clearly a unique MPE outcome, namely that the incumbent abstains from investing and the challenger invests immediately one unit. As shown in the following lemma, there exists also a unique MPE continuation strategy profile on  $[0, p_2^M)$ .

**Lemma 4** *Suppose Case 3 holds and  $r > 2\alpha + \sigma^2$ . Then, in any MPE  $\sigma$ :*

- (i) *If  $p \in (p^-, p_2^M)$ , both firms invests one unit with positive intensity, and the equilibrium payoffs are  $U_s(0, 2, p)$  for the challenger and  $U_s(1, 1, p)$  for the incumbent.*



(ii) If  $p \in [0, p^-)$ , neither the incumbent nor the challenger invests. At  $p = p^-$ , the challenger invests with positive intensity and the incumbent abstains.

Thus, the unique equilibrium on  $(p^-, p_2^M)$  is in mixed strategies, and the positive probability of simultaneous entry drives each firm's profit to its follower level, as in Fudenberg and Tirole (1985). Interestingly, since the preemption incentive of the challenger is larger than that of the incumbent, in this mixed strategy equilibrium the incumbent must invest with a greater intensity than its rival. Note finally that to avoid an openness problem at  $p^-$  in the challenger's optimization problem on  $[0, p^-)$ , it is necessary that the challenger enters with probability one at  $p^-$  and that the incumbent optimally abstains to invest. It follows that the challenger's equilibrium payoff at  $p^-$  exhibits a downward jump from  $U_s(1, 1, p^-) - I$  at  $p^-$  to  $U_s(0, 2, p)$  in a right neighborhood of  $p^-$ . This discontinuity obviously arises because of the firms' asymmetric capacities.

To complete the construction of an MPE, we must describe the firms' equilibrium strategies in Case 4, *i.e.* when none invested yet. It is intuitively clear that, for any current value  $p \in (0, p^-) \cup (p^+, \infty)$  of  $P_t$ , investing two units of capacity at once is strictly dominated by investing only one unit of capacity. However, if  $p \in (p^-, p^+)$ , it is not *a priori* obvious whether firms will attempt to invest one or two units of capacity. In fact, both scenarios can occur in equilibrium. The following proposition describes the first type of equilibrium.

**Proposition 3** *If  $r > 2\alpha + \sigma^2$ , there exists an MPE such that, on the equilibrium path, both firms simultaneously invests one unit of capacity at the first date  $\tau_{p^-}$  at which the stochastic process of demand reaches  $p^-$  and never reinvest thereafter.*

The MPE of Proposition 3 generalizes to the case  $r > 2\alpha + \sigma^2$  the unique MPE obtained when  $r < 2\alpha + \sigma^2$ . If the current state of demand is  $p \geq p_2^M$ , the intuition is similar: each firm is ready to invest exactly one unit if it anticipates that its rival will do the same. Things are different if  $p < p_2^M$ . In the case  $r < 2\alpha + \sigma^2$ , each firm was ready to wait until  $\tau_2^M$  before investing because it anticipated that its rival would at most invest one unit, with no impact on its own incentives. By contrast, if  $r > 2\alpha + \sigma^2$ , being preempted by one unit on  $(p^-, p_2^M)$  yields a continuation payoff of  $U_s(0, 2, p)$  since an incumbent firm holding one unit has then an incentive to immediately reinvest another unit. In other terms, there is no difference in terms of payoffs of being preempted with one or two units on  $(p^-, p_2^M)$ . It follows that at each  $p \in (p^-, p_2^M)$ , each firm wants to invest one unit if its rival do the same. It should be

noted that  $\lim_{r \downarrow 2\alpha + \sigma^2} p^- = \lim_{r \downarrow 2\alpha + \sigma^2} p^+ = p_2^M$ , so that as  $r \downarrow 2\alpha + \sigma^2$ , the above equilibrium outcome converges to the optimal MPE derived in Proposition 2.

There are other equilibria, however. For instance, it is easy to construct an equilibrium such that: (i) No firm invests on  $[0, p^-]$ ; (ii) At  $p^-$ , both firms invest one unit with probability one; (iii) on  $(p^-, p_2^M)$ , both firms invest two units of capacity with positive intensity:

$$s_2(0, 0, p) = \frac{U_s(2, 0, p) - 2I - U_s(0, 2, p)}{U_s(2, 0, p)},$$

and abstain from investing with intensity  $s_0(0, 0, p) = 1 - s_2(0, 0, p)$ ; (iv) on  $(p_2^M, \infty)$ , both firms invest one unit with probability one. In this latter equilibrium, the pattern of capacity accumulation depends on the initial state of the economy. If  $P_0$  is low enough ( $P_0 \in [0, p^-]$ ) or high enough ( $P_0 \in [p_2^M, \infty)$ ), then each firm will acquire only one unit of capacity on the equilibrium path. If on the other hand  $P_0$  lies in the intermediary range  $(p^-, p_2^M)$ , there will be over-accumulation of capacity along the equilibrium path, since one firm will enter immediately with two units, and the other firm will follow with one unit at  $\tau^C$ .

## 5. CONCLUDING REMARKS

In a preemption model with multiple investments, a firm has two motives for accumulating capacity. An obvious reason is to take advantage of its current market power. However, it may also invest only to postpone the next investment of its competitor. The interplay between these two motives may give rise to highly complex investment sequences in equilibrium. In our model, different patterns of equilibria may arise, depending on the importance of the real option effect. If the average growth rate of the market is close to the risk free rate, or if the volatility of demand changes is high, then the unique equilibrium acquisition process involves joint adoption at the socially optimal date. If these conditions do not hold, the equilibrium investment timing is suboptimal, and the firms' long-run capacities depend on the initial market conditions. However, under a broad range of empirically relevant parameter values, no dissipation of rents occur in equilibrium, despite instantaneous Bertrand competition. This is in sharp contrast with the outcome of the standard winner-take-all model of preemption under Bertrand competition. One implication of these results is that even if capacity can be only very crudely segmented, the timing of the investment *per se* can be used by firms to reach a cooperative outcome in the absence of any implicit collusion on prices. This suggests that the implications of one-shot models of preemption are likely to

prove very fragile and sensitive to both modeling choices and initial conditions.

As mentioned in the introduction, our conclusions differ also sharply from the full rent dissipation result of Gilbert and Harris (1984). The source of this discrepancy lies in how demand evolves in the two environments. When the market size grows in an unbounded way, firms can threaten to preempt each other on each new investment, leading to complete dissipation of rents. However, if the market size is bounded, and the evolution of demand is driven by consumers' tastes, the extent to which rents are dissipated is bounded.

Our results are likely to be robust to more general formulation of competition in capacity. Consider for instance the case where  $2N$  units of capacity, each costing  $I/N$ , are needed to serve all consumers and suppose that the incumbent has already  $2N$  units. Then, by investing  $N$  units of capacity at  $\tau^C$ , the challenger can guarantee itself the same payoff as in (10). Indeed, when the size of the market is bounded, the worst that can happen to a firm is that its rival has already enough capacity units to serve all consumers. However, if capacity is lumpy and many units are necessary to cover the whole market, the follower's rents from its investments will be bounded away from zero even in these circumstances, and this, independently of the form of short term competition or the size of the market. Moreover, even if there are more than two firms in the industry, the dissipation of rents will at best be partial insofar as investment is lumpy. Thus, even if *some* rent dissipation is likely to occur as a result of preemption, the extent to which this happens has been perhaps overestimated.

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## 6. APPENDIX A

Formally, a Markov strategy for firm  $i$  is a mapping  $s^i$  that associates, to each capacity vector  $(n^i, n^j) \in \{0, \dots, N\} \times \{0, \dots, N\}$  and state of demand  $p \in (0, \infty)$  a probability distribution  $s^i(n^i, n^j, p)$  on  $\{0, \dots, N - n^i\}$ . (Since each unit of capacity can cover a fraction  $1/N$  of the market, it is obvious that firms never wish to acquire more than  $N$  units of capacity.) For each  $\nu \in \{0, \dots, N - n^i\}$ , the quantity  $s_\nu^i(n^i, n^j, p) = s^i(n^i, n^j, p)(\nu)$  will be interpreted as the intensity with which firm  $i$  invests  $\nu$  units in the state  $(n^i, n^j, p)$ . The following regularity conditions are imposed for technical convenience:

- ( $R_1$ ) For each  $i \in \{1, 2\}$  and  $(n^1, n^2) \in \{0, \dots, N\} \times \{0, \dots, N\}$ , the mapping  $s^i(n^i, n^j, \cdot)$  is piecewise continuous and admits everywhere a right limit.
- ( $R_2$ ) For each  $p \in (0, \infty)$ , if  $s_0^i(n^i, n^j, p) = 1$ , then  $s_0^i(n^i, n^j, \cdot) = 1$  in a left neighborhood of  $p$ . If furthermore  $p = \inf\{p' > p \mid s_0^i(n^i, n^j, p') \neq 1\}$ , then for each  $\nu \in \{1, \dots, N - n^i\}$ , the right partial derivative  $\partial_p^+ s_\nu^i(n^i, n^j, p)$  exists and at least one of them is strictly positive.

For each strategy profile  $s = (s^1, s^2)$ , let  $U_s^i(n^i, n^j, p)$  denote the value of firm  $i$  when  $s$  is played and the current state of the game is  $(n^i, n^j, p)$ . Since investment is irreversible and capital does not depreciate, we will be able to compute these value functions recursively from the equilibria of the static Bertrand game. To do so, we distinguish three cases.

*Case 1.* First, if  $p \in A_s(n^i, n^j) = \{p' \in [0, \infty) \mid s_0^i(n^i, n^j, p') s_0^j(n^j, n^i, p') \neq 1\}$ , then at least one firm is active in the state  $(n^i, n^j, p)$ .  $U_s^i(n^i, n^j, p)$  is then defined as:

$$\sum_{(\nu^i, \nu^j) \neq (0,0)} s_{\nu^i}^i(n^i, n^j, p) s_{\nu^j}^j(n^j, n^i, p) (U_s^i(n^i + \nu^i, n^j + \nu^j, p) - \nu^i I) \quad (1)$$

$$\Big/ \left(1 - s_0^i(n^i, n^j, p) s_0^j(n^j, n^i, p)\right).$$

The intuition underlying (A.1) is that, if the firms' intensity of investment is not identically zero in state  $(n^i, n^j, p)$ , with probability one some new investments will take place in this state. With probability  $s_{\nu^i}^i(n^i, n^j, p) s_{\nu^j}^j(n^j, n^i, p) / \left(1 - s_0^i(n^i, n^j, p) s_0^j(n^j, n^i, p)\right)$ ,  $(\nu^i, \nu^j)$  additional units are invested.

*Case 2.* Next, if  $p \in \partial A_s(n^i, n^j) \setminus A_s(n^i, n^j)$ , the intensity of entry of both firms is zero. However, since  $p$  is on the boundary of  $A_s(n^i, n^j)$ , with probability one the first random time  $\tau_s(n^i, n^j, p)$  at which the process of demand shocks reaches  $A_s(n^i, n^j)$  starting from  $p$  is equal to zero. (This is a special case of the 0-1 law for diffusions, see Øksendall (1995, Corollary 9.2)). Using ( $R_1$ ) and ( $R_2$ ),  $U_s^i(n^i, n^j, p)$  can be computed as the limit of  $U_s^i(n^i, n^j, p + \varepsilon)$  when  $\varepsilon \downarrow 0$ , if necessary using a first-order Taylor expansion of (A.1). As in

Fudenberg and Tirole (1985), the interpretation is that there is an “interval of atoms” following  $p$  in at least one firm’s strategy, so with probability one some investments will occur at  $p$ .

*Case 3.* Last, if  $p \notin A_s(n^i, n^j) \cup \partial A_s(n^i, n^j)$ , both firms’ intensity of investment is zero in a neighborhood of  $p$ . Firm  $i$ ’s continuation value is then standardly computed as:

$$E_p \left( \int_0^{\tau_s(n^i, n^j, p)} e^{-rt} \pi(n^i, n^j, P_t) dt + e^{-r\tau_s(n^i, n^j, p)} U_s^i(n^i, n^j, P_{\tau_s(n^i, n^j, p)}) \right), \quad (2)$$

where  $\pi(n^i, n^j, P_t)$  is firm  $i$ ’s payoff in a Nash equilibrium of the static Bertrand game in state  $(n^i, n^j, P_t)$ , and  $\tau_s(n^i, n^j, p)$  is defined as in Case 2.

## 7. APPENDIX B

PROOF OF PROPOSITION 1: Using (A.1), it is straightforward to verify that both firms investing with intensity  $(p - p_1^m)/p$  forms a MPE strategy profile on  $(p_1^m, p_1^M)$ . Moreover, it is easy to see that, for any  $p$  in this interval, if  $s_1^i(0, 0, p) > (p - p_1^m)/p$ , then the unique best response of  $j$  at  $(0, 0, p)$  is  $s_0^j(0, 0, p) = 1$ . Hence this is the unique Markov strategy profile on  $(p_1^m, p_1^M)$  in which both firms are active on this whole interval. Suppose now that there exists a MPE strategy profile  $s$  such that, for some  $p^i \in (p_1^m, p_1^M)$ ,  $s_0^i(0, 0, p^i) = 1$ . Then, from  $(R_2)$ , there exists a maximal interval  $I^i \subset (p_1^m, p_1^M)$  containing  $p^i$  such that for each  $p \in I^i$ ,  $s_0^i(0, 0, p) = 1$ . Since  $I^i \subset (p_1^m, p_1^M)$ , however, it is a strictly dominated strategy for firm  $j$  to invest on the interior of  $I^i$ , since, for each  $p \in \text{int } I^i$ , there exists  $\varepsilon > 0$  such that:

$$p + \varepsilon \in \text{int } I^i \quad \text{and} \quad U_s^j(1, 0, p) - I < \left( \frac{p}{p + \varepsilon} \right)^\beta (U_s^j(1, 0, p + \varepsilon) - I), \quad (\text{B.1})$$

so that firm  $j$  would be strictly better off waiting until  $p + \varepsilon$  before investing. Hence  $s_0^j(0, 0, p) = 1$  for all  $p \in \text{int } I^i$ . Let now  $\underline{p}^i = \inf I^i$  and  $\bar{p}^i = \sup I^i$ . It is obvious from (6) that at least one of the continuation values  $U_s^i(0, 0, \underline{p}^i)$ ,  $U_s^j(0, 0, \bar{p}^i)$  must be strictly less than  $U_s^i(1, 0, \bar{p}^i) - I = U_s^j(1, 0, \bar{p}^i) - I = \bar{p}^i/(r - \alpha) - I$  and similarly at  $\underline{p}^i$ . Suppose for instance that  $U_s^i(0, 0, \bar{p}^i) < U_s^i(1, 0, \bar{p}^i) - I$ . Then, since  $U_s^i(1, 0, \underline{p}^i) < U_s^i(1, 0, \bar{p}^i)$ , there exists some  $\eta > 0$  such that  $\max\{U_s^i(0, 0, \underline{p}^i), U_s^i(0, 0, \bar{p}^i)\} = U_s^i(1, 0, \bar{p}^i) - I - \eta$ . Hence, it follows from (A.2) that:

$$U_s^i(0, 0, p) \leq \left( \frac{p}{\bar{p}^i} \right)^\beta (U_s^i(1, 0, \bar{p}^i) - I - \eta) \quad \forall p \in \text{int } I^i. \quad (\text{B.2})$$

Moreover, by continuity of the mapping  $U_s^i(1, 0, \cdot)$ :

$$\lim_{p \uparrow \bar{p}^i} \left\{ \left( \frac{p}{\bar{p}^i} \right)^\beta (U_s^i(1, 0, \bar{p}^i) - I) / (U_s^i(1, 0, p) - I) \right\} = 1. \quad (\text{B.3})$$

Furthermore,  $s_0^j(0, 0, p) = 1$  for all  $p \in \text{int } I^i$ . It follows then from (B.2) and (B.3) that by deviating and investing with positive intensity in a left neighborhood of  $\bar{p}^i$ , firm  $i$  can get a strictly higher payoff, namely  $U_s^i(1, 0, p) - I$ , than its equilibrium payoff  $U_s^i(0, 0, p)$ , a contradiction. Hence the only MPE strategy on  $(p_1^m, p_1^M)$  is to invest with intensity  $(p - p_1^m)/p$ , which yields an equilibrium payoff of  $U_s^i(0, 1, p) = 0$ . At  $p_1^m$ , both firms are indifferent between preempting or being preempted, since they get a zero payoff in any case. In an MPE, one may have either  $s_1^i(0, 0, p_1^m) = s_0^j(0, 0, p_1^m) = 1$ , *i.e.* firm  $i$  invests and firm  $j$  abstains, or  $s_0^i(0, 0, p_1^m) = s_0^j(0, 0, p_1^m) = 1$ . In all cases, both firms get a zero payoff. ■

PROOF OF LEMMA 1: Let us assume that firm  $i$  has two units of capacity, while firm  $j$  has only one unit. Without loss of generality, we may assume that  $p = 1$ . A strategy profile is then a vector of prices  $(p^i, p^j) \in [0, 1] \times [0, 1]$ . For any strategy profile  $(p^i, p^j)$ , firms' payoffs are given by the following matrix:

$$\pi^i(p^i, p^j) = \begin{cases} p^i & \text{if } p^i < p^j \\ \frac{1}{2} p^i & \text{if } p^i \geq p^j \end{cases}, \quad \pi^j(p^j, p^i) = \begin{cases} \frac{1}{2} p^j & \text{if } p^j \leq p^i \\ 0 & \text{if } p^j > p^i \end{cases}. \quad (4)$$

It is easy to see that the game (B.4) has no pure strategy equilibrium. We look for a mixed strategy equilibrium, described by a pair of cumulative distribution functions  $(F^i, F^j)$  over  $[0, 1]$ . Note first that any  $p^i < \frac{1}{2}$  is a strictly dominated strategy for  $i$ , since  $i$  can secure itself a payoff of  $\frac{1}{2}$  by charging  $p^i = 1$ . Hence necessarily  $\text{supp}(F^i) \subset [\frac{1}{2}, 1]$ , so that one must have  $\text{supp}(F^j) \subset [\frac{1}{2}, 1]$  as well. Define:

$$F^i(p^i) = \begin{cases} 0 & \text{if } p^i \in [0, \frac{1}{2}] \\ 1 - \frac{1}{2p^i} & \text{if } p^i \in (\frac{1}{2}, 1) \\ 1 & \text{if } p^i = 1 \end{cases}, \quad F^j(p^j) = \begin{cases} 0 & \text{if } p^j \in [0, \frac{1}{2}] \\ 2 - \frac{1}{p^j} & \text{if } p^j \in (\frac{1}{2}, 1) \end{cases}. \quad (5)$$

Now, from (B.5), we have  $\pi^j(p^j, F^i) = \frac{1}{2} p^j (1 - F^i(p^j)) = \frac{1}{4}$  for all  $p^j \in [\frac{1}{2}, 1)$ . Moreover, for  $p^j = 1$ , we have  $\pi^j(1, F^i) = \frac{1}{2} \Pr_{F^i}(p^i = 1) = \frac{1}{4}$ . Hence, given the strategy  $F^i$  of firm  $i$ , firm  $j$  is indifferent between all possible prices in  $\text{supp}(F^j) = [\frac{1}{2}, 1]$ . Similarly, for all  $p^i \in [\frac{1}{2}, 1]$ , we have  $\pi^i(p^i, F^j) = p^i (1 - F^j(p^i)) + \frac{1}{2} p^i F^j(p^i) = \frac{1}{2}$ . Hence, given the strategy  $F^j$  of firm  $j$ , firm  $i$  is indifferent between all possible prices in  $\text{supp}(F^i) = [\frac{1}{2}, 1]$ . It follows that  $(F^i, F^j)$  is a mixed strategy equilibrium of the static Bertrand game, with corresponding profits  $(\frac{1}{2}, \frac{1}{4})$ . The proof that this equilibrium is unique is standard (see *e.g.* Ghemawat 1997, p. 39, and the references therein), and is therefore omitted.  $\blacksquare$

PROOF OF LEMMA 2: From (7) and (11), we have, after substitution from Lemma 1:

$$U_s(2, 0, p) - I - U_s(1, 1, p) = \frac{1}{2(r - \alpha)} \left( p - \left( \frac{p}{p^C} \right)^\beta p^C \right) - I \quad \forall p \in [0, \infty). \quad (6)$$

Since  $\beta > 1$ , this quantity attains its global maximum at  $p^* = \beta^{1/(1-\beta)} p^C < p^C$ . From (B.6), straightforward manipulations yield:

$$U_s(2, 0, p^*) - I - U_s(1, 1, p^*) = \left( 2\beta^{\frac{1}{1-\beta}} - 1 \right) I. \quad (7)$$

It is immediate to check that the expression in (B.7) is negative if and only if  $\beta < 2$ , or equivalently  $r < 2\alpha + \sigma^2$  since  $\alpha \geq 0$ . This concludes the proof.  $\blacksquare$

PROOF OF LEMMA 3: From (7), (10), (11) and the fact that  $\pi(2, 0, \cdot) = 2\pi(2, 1, \cdot) = 2\pi(1, 1, \cdot) = 4\pi(1, 2, \cdot)$ , it follows that for each  $p \in [0, \infty)$ :

$$2U_s(1, 1, p) - U_s(0, 2, p) - U_s(2, 0, p) = E_p \left( \int_{\tau^C}^{\infty} e^{-rt} \pi(1, 2, P_s) ds + e^{-r\tau^C} \right). \quad (8)$$

The expression in (B.8) is obviously positive, which proves (15). Next, if  $r > 2\alpha + \sigma^2$ , then from the proof of Lemma 2,  $U_s(2, 0, p^*) - I > U_s(1, 1, p^*)$ . Moreover, the mapping  $U_s(2, 0, \cdot) - U_s(1, 1, \cdot)$  is strictly quasiconcave, and is equal to zero at 0 and  $p^C$ . Hence, by the intermediate value theorem, there exists



$p^-, p^+ \in (0, p^C)$  such that for all  $p \in [0, \infty)$ ,  $U_s(2, 0, p) - I > U_s(1, 1, p)$  if and only if  $p \in (p^-, p^+)$ . Last, from (B.6), we can write:

$$U_s(2, 0, p_2^M) - I - U_s(1, 1, p_2^M) = \left( (1 - 2^{1-\beta}) \frac{\beta}{\beta - 1} - 1 \right) I. \quad (9)$$

It is immediate to check that the expression in (B.9) is positive if and only if  $\beta > 2$ , or equivalently  $r > 2\alpha + \sigma^2$  since  $\alpha \geq 0$ . It follows that  $p_2^M \in (p^-, p^+)$ , as claimed.  $\blacksquare$

PROOF OF LEMMA 4: (i) If the current state of demand is  $p \in (p^-, p_2^M)$ , then  $U_s(2, 0, p) - I > U_s(1, 1, p)$  and  $U_s(1, 1, p) - I > U_s(0, 2, p)$ , so both the incumbent and the challenger have a positive incentive to preempt each other. Proceeding as in the proof of Proposition 1, it is easy to check that the unique MPE strategy profile is for both firms to invest one unit of capacity with respective intensities:

$$s_1(1, 0, p) = \frac{U_s(1, 1, p) - I - U_s(0, 2, p)}{U_s(1, 1, p) - U_s(1, 2, p)} \quad (10)$$

for the incumbent, and:

$$s_1(0, 1, p) = \frac{U_s(2, 0, p) - I - U_s(1, 1, p)}{U_s(2, 0, p) - U_s(2, 1, p)} \quad (11)$$

for the challenger. From (B.10) and (B.11), the associated MPE payoffs are  $U_s(1, 1, p)$  for the incumbent and  $U_s(0, 2, p)$  for the challenger.

(ii) If the current value of demand is  $p \in [0, p^-)$ , an incumbent with one unit of capacity cannot gain from investing a second unit at  $p$  since  $U_s(2, 0, p) - I < U_s(1, 1, p)$ . It follows that the challenger is not threatened by the incumbent on this interval. However, since  $p^- < p_2^M$ , it is suboptimal for the challenger to invest, and its best response is therefore to wait until  $p^-$  is reached. At  $p^-$ , the incumbent is indifferent between preempting and being preempted. Suppose that  $s_1(1, 0, p^-) > 0$  in equilibrium. Then the challenger's payoff at  $p^-$  is  $U_s(1, 1, p^-) - I - \eta$  for some  $\eta > 0$ . Hence, it follows from (A.2) that:

$$U_s(0, 1, p) = \left( \frac{p}{p^-} \right)^\beta (U_s(1, 1, p^-) - I - \eta) \quad \forall p \in [0, p^-). \quad (12)$$

Moreover, by continuity of the mapping  $U_s(1, 1, \cdot)$ :

$$\lim_{p \downarrow p^-} \left\{ \left( \frac{p}{p^-} \right)^\beta (U_s(1, 1, p^-) - I) / (U_s(1, 1, p) - I) \right\} = 1. \quad (13)$$

Furthermore,  $s_0(1, 0, p) = 1$  for all  $p \in [0, p^-)$ . It follows then from (B.12) and (B.13) that by deviating and investing with positive intensity in a left neighborhood of  $p^-$ , the challenger can get a strictly higher payoff, namely  $U_s(1, 1, p) - I$ , than its equilibrium payoff  $U_s(0, 0, p)$ . However, as noted previously, investing at  $p^- - \varepsilon$  is never a best response for any  $\varepsilon > 0$  given that the incumbent does not invest on  $[0, p^-)$ . Therefore, if  $s_1(1, 0, p^-) > 0$ , there is an openness problem in the determination of the challenger's best response in a left neighbourhood of  $p^-$ . Thus in any continuation MPE, the incumbent never invests at  $p^-$ , and the challenger invests with positive intensity at  $p^-$ . Note that in any MPE,  $s_1(0, 1, p^-) > 0 = \lim_{p \downarrow p^-} s_1(0, 1, p)$  and  $s_1(1, 0, p^-) = 0 < \lim_{p \downarrow p^-} s_1(1, 0, p)$ .  $\blacksquare$

PROOF OF PROPOSITION 3: Cases 1-3 have already been analyzed. Suppose that Case 4 holds. Let us prove that there is an MPE in which, for any current state of demand  $p \geq p^-$ , both firms invest

one unit of capacity with intensity  $s_1(0,0,p) = 1$ . Note first that this is obvious if  $p > p_2^M$  since then no firm has an incentive to delay investment. Consider now what happens if  $p \in (p^-, p_2^M)$ . From the discussion of Case 1, if both firms invest simultaneously one unit at  $p$ , their continuation payoff at  $p$  is  $U_s(1,1,p) - I$ . Suppose now that firm  $i$  deviates, and invests one unit with intensity  $s_1^i \in [0,1)$ . (It is clear that investing two units cannot be a best response if firm  $j$  invests one unit.) Then by Lemma 4, firm  $i$ 's continuation payoff at  $p$  is  $s_1^i(U_s(1,1,p) - I) + (1 - s_1^i)U_s(0,2,p)$ . However, from Lemma 3, one has  $U_s(1,1,p) - I - U_s(0,2,p) > U_s(2,0,p) - I - U_s(1,1,p) > 0$  for any  $p \in (p^-, p_2^M)$  so this deviation cannot be profitable. Last, suppose that firm  $j$  never invests before  $p^-$  is reached. Clearly, investing two units is never a best response for firm  $i$ . Moreover, as  $p^- < p_2^M$ , investing one unit at some  $p \in [0, p^-)$  is clearly dominated for firm  $i$  given the continuation equilibrium just constructed for Case 4 on  $[p^-, \infty)$  and a best response is to wait until  $\tau_{p^-}$  to invest. ■

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